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11.Theory of Domination in Graphs

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Graph theory has become the nucleus of tremendous research activity because of its wide applications to different branches like electronics, communications, electrical engineering, computer science, psychology, and sociology. Graph theory is also related to many branches of mathematics, including algebra, linear algebra, numerical analysis, probability, topology, and combinatorics. Any system involving binary relations can be modeled as a graph. Several research articles have been published in graph theory over the last three decades. In these articles, it was observed that certain areas of graph theory have received good attention from researchers in graph theory. A few of them are graph labeling, Coloring of graphs, domination in graphs and algebraic graphs. In recent years, among these areas, the **theory of domination** has become one of the potential research areas in graph theory.

The concept **of domination** was introduced by Claude Berge in 1958. A set $S \subseteq V$is said to be a **dominating set** in a graph G = (V, E) if every vertex in V\S is adjacent to at least one vertex of S and the **domination number of G, denoted by,** is defined to be the minimum cardinality of all dominating sets in G. During the last 3 to 4 decades there has been a tremendous research activity in this area, and this was attributed mainly due to the following reasons:

1) The extensive applications of domination in real-world scenarios and its significance in mathematical modeling.
2) The ability to define numerous domination parameters to address diverse problems.
3) The NP-complete nature of the fundamental domination problem, its intrinsic connection to other NP-complete problems, and the pursuit of polynomial-time solutions for domination in specific graph classes.

The origin of domination problems came from the Queen's problem in the chess game in the 1850s. We know that a queen can move in any direction (vertical, horizontal or diagonal) and any number of squares on the chessboard The chessboard problem is keep a least number of Queens on a chessboard so that each square is controlled by at least one queen. Using graph theory, this problem is represented by treating each square of the chessboard as a vertex. In the resulting graph GGG, two vertices are connected if a queen can move from one corresponding square to the other in a single step. The objective is to find the smallest number of queens needed to cover or dominate all squares on the board, which corresponds to the domination number of the graph. Numerous variations of this problem have been created by altering the dimensions of the chessboard or the types of chess pieces used.

The further development of study of domination in graphs was happened in the late 1950s or 1960s, with books published by Claude Berge, Ore and Harary. One interesting survey paper on domination was published by Cockayne and Hedetniemi in 1977. In 1963, Vizing Conjectured that the domination number of the Cartesian product of two graphs is at least the product of their domination number. So many partial results have been proved on this conjecture, but the conjecture has not yet been generally proved. This is one of the major open problems in the study of domination in graphs. Another good survey on domination articles was conducted by Hedetniemi and Laskar in 1990, containing 400 articles. At the end of 1997, this bibliography grew to cover 1200 articles, indicating the growth of research in domination. In 1995, Hartnell and Rall established **Vizing's Conjecture** for large graphs. Another method to this conjecture is to identify a constant c such that $\gamma(GXH) \geq \gamma(G)\gamma(H)$. This inequality is proved by clark and suen for c = 0.5 in the year 2000.In 2009Bresar and Rall have defined the fair domination number of a graph which is denoted by $\gamma_F(G)$. This concept was used to prove very recent results pertaining to vizing’s conjecture. They have shown that $\gamma(GXH) \geq \min\{\gamma(G)\gamma_F(H), \gamma_F(G)\gamma(H)\}$. Thusvizing’s conjecture is true for all graphs G which satisfy $\gamma(G) = \gamma_F(G)$.

After the invention of the dominating set and domination number, several types of domination parameters were introduced by researchers, which are mainly formed by imposing additional conditions on dominating set S, V\S or on the vertex set V(G). There are more than 75 such domination linked parameters. Some of these types of domination are independent domination, location identification, distance related, connected domination, strong domination, directed domination, global domination and power domination etc.

In addition to the domination problems on the chessboard, domination in graphs has applications in several fields. Domination occurs in location of facility problems, problems in in identifying sets of representatives of a given data, school bus routing problems in transportation networks, land surveying problems, etc. We particularly explain a few of these applications.

1. Facility location problems: The dominating sets in graphs are direct or natural models for facility location problemsin operations research. These are concerned with the location of one or more facilities (e.g., hospitals, fire stations) in such a way that optimizes a specific objective like minimizing transportation costs, providing equitable service to customers and capturing the largest market share.

2. Locating radar stations problem: The problem is fixing several strategic locations that need to be kept under surveillance. The aim is to locate the radar for surveillance at as few locations as possible. We can determine a set of locations where the radar stations are to be placed, similar to find dominating sets.

3. Land surveying: In land surveying, we can use dominating sets to find the least numberof places where a surveyor can stand in order to know the height measurements for an entire region.

Conclusion: The main purpose of this article is to convey theoretical ideas related to domination, particularly in various fields of science and engineering. Young researchers may focus on it in their research with these ideas in graph theoretical concepts and their applications.

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