11.Theory of Domination in Graphs

By

Dr.P.Vijaya Saradhi
Professor
Dept. of Mathematics,
Bapatla Engineering College,
Bapatla.

Graph theory has become the nucleus of tremendous research activity because of its wide applications to different branches like electronics, communications, electrical engineering, computer science, psychology, and sociology. Graph theory is also related to many branches of mathematics, including algebra, linear algebra, numerical analysis, probability, topology, and combinatorics. Any system involving binary relations can be modeled as a graph. Several research articles have been published in graph theory over the last three decades. In these articles, it was observed that certain areas of graph theory have received good attention from researchers in graph theory. A few of them are graph labeling, Coloring of graphs, domination in graphs and algebraic graphs. In recent years, among these areas, the **theory of domination** has become one of the potential research areas in graph theory.

The concept **of domination** was introduced by Claude Berge in 1958. A set $S \subseteq V$ is said to be a **dominating set** in a graph G = (V, E) if every vertex in V\S is adjacent to at least one vertex of S and the **domination number of G, denoted by,** is defined to be the minimum cardinality of all dominating sets in G. During the last 3 to 4 decades there has been a tremendous research activity in this area, and this was attributed mainly due to the following reasons:

1. The wide range of applications of domination to real-life situations as well as mathematical modeling.
2. A wide variety of domination parameters can be defined.
3. The NP-completeness of the basic domination problem, its close and natural relationship to other NP-completeness problems and the interest in finding the polynomial time solutions to domination problems in special class graphs.

The origin of domination problems came from the Queen's problem in the chess game in the 1850s. We know that a queen can move any number of squares vertically, horizontally or diagonally on the chessboard (assuming that no other chess figure is on its way). The chessboard problem is placing a minimum number of Queens on a chessboard so that at least one queen controls each square. This problem was modeled using graph theory by representing each square on the chess board as a vertex, and two vertices are adjacent in G if each square can be reached by a queen on the other square in a single move. The problem we are now looking for is the minimum number of queens that control or dominate all squares of the chessboard, which is nothing but the domination number of the graph. Many variations on this problem are also formed by choosing different sizes and pieces of chess boards.

The study of domination in graphs was further developed in the late 1950s or 1960s, with books published by Claude Berge, Ore and Harary. One interesting survey paper on domination was published by Cockayne and Hedetniemi in 1977. In 1963, Vizing Conjectured that the domination number of the Cartesian product of two graphs is at least the product of their domination number. So many partial results have been proved on this conjecture, but the conjecture has not yet been generally proved. This is one of the major open problems in the study of domination in graphs. Another good survey on domination articles was conducted by Hedetniemi and Laskar in 1990, containing 400 articles. At the end of 1997, this bibliography grew to cover 1200 articles, indicating the growth of research in domination. In 1995, Hartnell and Rall established **Vizing's Conjecture** for large graphs. One more approach to Vizing's conjecture is to find a constant such that $\gamma(GXH) \geq \gamma(G)\gamma(H)$. In 2000, Clark and Suen proved this inequality for c = ½. One of the most recent results related to Vizing's conjecture deals with the new concept of fair domination, first defined by Bresar and Rall in 2009. They have defined the fair domination number graph denoted by $\gamma_F(G)$ and proved that $\gamma(GXH) \geq \min\{\gamma(G)\gamma_F(H), \gamma_F(G)\gamma(H)\}$. Thus for any graph G having $\gamma(G) = \gamma_F(G)$, vizing's conjecture holds.

After the invention of the dominating set and domination number, several types of domination parameters were introduced by researchers, which are mainly formed by imposing additional conditions on dominating set S, V\S or on the vertex set V(G). There are more than 75 such domination linked parameters. Some of these types of domination are independent domination, locating domination, distance domination, connected domination, multiple domination, strong domination, directed domination, global domination and power domination etc.

In addition to the domination problems on the chessboard, domination in graphs has applications in several fields. Domination occurs in facility location problems, problems in finding sets of representatives, monitoring communication or electrical networks, school bus routing problems in transportation networks, land surveying problems, etc. We particularly explain a few of these applications.

1. Facility location problems: The dominating sets in graphs are direct or natural models for facility location problems in operations research. These are concerned with the location of one or more facilities (e.g., hospitals, fire stations) in such a way that optimizes a specific objective like minimizing transportation costs, providing equitable service to customers and capturing the largest market share.

2. Locating radar stations problem: The problem is fixing several strategic locations that need to be kept under surveillance. The aim is to locate the radar for surveillance at as few locations as possible. We can determine a set of locations where the radar stations are to be placed, similar to find dominating sets.

3. Land surveying: In land surveying, we can use dominating sets to minimize the number of places a surveyor must stand to take height measurements for an entire region.

Conclusion: The main purpose of this article is to convey theoretical ideas related to domination, particularly in various fields of science and engineering. Young researchers may focus on it in their research with these ideas in graph theoretical concepts and their applications.

- **Dr.P. Vijaya Saradhi**