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14. Developments of the theory of S−Normal Almost Distributive Lattices

By

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In 1972, Cornish presented the notion of a normal lattice, defining it as a bounded distributive lattice R where for s, t ∈ R with s ∧ t =0, the relation $(s)^* \vee (t)^* = R$ holds. Here $(s)^* = \{a \in R \mid a \wedge s = 0\}$. It was established that a lattice R for which the intersection of all its maximal filters is{1} is normal if and only if the set of its maximal filters, equipped with the hull-kernel topology, is normal. Later, in 1971, R. Cignoli generalized this concept by introducing S-normal distributive lattices.

In this context, a bounded distributive lattice R with a sublattice S, where S includes 0 and 1, is termed S-normal if, for every s, t ∈ R satisfying s ∧ t = 0, there are p, q ∈ S satisfies s ∧ p = 0 = t ∧ q and p ∨ q = 1 When S equals R, the S-normal lattice reduces to the earlier notion of a normal lattice.

Further, the structure of almost distributive lattices (ADLs) was introduced. These structures share most properties of distributive lattices but may not satisfy ∧, ∨ commutativity and right distributivity of ∨ over ∧. However, the lattice of principal ideals of R remains distributive, enabling the extension of many concepts from distributive lattices to ADLs. This led to the development of S-normal ADLs. An ADL R with maximal elements is S-normal if the principal ideal lattice PI(R) is PI(S)-normal, where S is a unisub-ADL of R containing 0.

An S-ideal (or S-filter) in R with a sub-ADL S was subsequently defined. An ideal I of R qualifies as an S-ideal if, for each x ∈ I there exists an element s ∈ I ∩ S such that s ∧ x = x. For any ideal I in S, the set $I^e = \{x \wedge a \mid x \in R$ and $a \in I\}$ is the smallest S-ideal in R containing I, and $I = I^e \cap S$. Additional constructs, including S-prime ideals, S-maximal ideals, and their duals, were introduced and studied. For example, it was demonstrated that R with maximal elements is S-normal iff every S-prime filter of R is contained in a unique maximal filter.

The Birkhoff center B of an ADL, as defined by U. M. Swamy and S. Ramesh in 2009, consists of $s \in R$ for this there is $t \in R$ such that $s \wedge t = 0$ and $s \vee t$ is maximal in R. This center forms a relatively complemented sub-ADL. It was shown that R is B-normal precisely when R satisfies conjunctive B-regularity.

The notion of S-relatively normal ADLs was also introduced and characterized by using S-relative annihilators and S-prime filters. The notions of dually S-normal ADLs and dually S-relatively normal ADLs were defined and examined using dual annihilators and S-prime ideals. Topological characterizations of normality in ADLs were also explored. For example, an ADL R in which each dense element is maximal was proven to be B-normal if and only if the collection of maximal filters of R, with the hull-kernel topology, forms a Boolean space.

Finally, dually normal ADLs were investigated through their maximal ideals, revealing that the mapping assigns each prime ideal to the unique maximal containing it is continuous.

This map serves as a connection between the maximal spectrum and the prime spectrum of R, with the maximal spectrum being a retract of the prime spectrum. Similarly, the characterization of dually B-normal ADLs assumed that every dual dense element of R equals 0.

- **Dr. Noorbhasha Rafi**