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14. Developments of the theory of S−Normal Almost Distributive Lattices

By

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In 1972, Cornish. W. H., introduced the concept of a normal lattice as a bounded distributive lattice in which $(x)^* \vee (y)^* = L$ for any $x, y \in L$ with $x \wedge y = 0$, where $(x)^* = \{t \in L \mid t \wedge x = 0\}$. This concept has a topological origin. It is well known that a topological space is called normal if any two disjoint closed sets can be separated by disjoint open sets and it was proved that a lattice L in which intersection of all maximal filters is $\{1\}$ is normal if and only if the set of all its maximal filters with hull-kernel topology is normal. Generalizing this concept of normal lattices later in 1971, Cignoli. R. introduced the concept of an $S-$normal distributive lattice. If $(L, \vee, \wedge, 0, 1)$ is a bounded distributive lattice and S is a sublattice of L containing 0 and 1, then L is called $S-$normal, if for any $x, y \in R$ with $x \wedge y = 0$, there exist $a, b \in S$ such that $x \wedge a = 0 = y \wedge b$ and $a \vee b = 1$. In case $S = L$, this concept coincides with that of a normal lattice.

An Almost Distributive Lattice (ADL) satisfies almost all the properties of a distributive lattice except possibly commutativity of $\wedge$, commutativity of $\vee$ and right distributivity of $\vee$ over $\wedge$. Also, its principal ideals form a distributive lattice through which we can extend many concepts from the class of distributive lattices to the class of ADLs. With this motivation, introduced the concept of an $S-$normal ADL as an ADL $(R, \vee, \wedge, 0)$ with maximal elements for which the principal ideal lattice $PI(R)$ is a $PI(S)-$normal lattice where S is a unisub ADL of R.

The main aim of this paper is to study and explore the properties of $S-$normal ADLs. If R is an ADL with 0 and S is a subADL of R containing 0, then defined an $S-$ ideal ($S-$filter) of R and observed that an ideal I of R is an $S-$ideal of R if and only if, for any $x \in I$, there exists an element $s \in I \cap S$ such that $s \wedge x = x$.

For any ideal I of S, noted that the set $I^e = \{x \wedge a \mid x \in R \text{ and } a \in I\}$ is the smallest S–ideal of R containing I and that $I = I^e \cap S$. Also, defined the concepts of an S–prime ideal (S–prime filter), an S–maximal ideal (S–maximal filter) and proved many results on the properties of S–prime ideals(S–prime filters), S–maximal ideals(S–maximal filters). Clearly, an ADL R with maximal elements is S–normal if and only if every S–prime filter of R is contained in a unique maximal filter of R.

In 2009, U.M. Swamy and S. Ramesh defined the concept of the Birkhoff centre of an ADL as $B = \{a \in R \mid$ there exists an element $b \in R$ such that $a \wedge b = 0$ and $a \vee b$ is a maximal element in $R\}$ and noted that it is a relatively complemented subADL of R. We proved that an ADL R with maximal elements is B–normal if and only if R is conjunctively B–regular. Introduced the concept of an S–relatively normal ADL and characterized an S–relatively normal ADL in terms of its S–relative annihilators and S–prime filters. The lattice theoretic duality principle does not hold good in case of an ADL for this reason, introduced the concept of a dually S–normal ADL and characterized it in terms of its S– prime ideals. In a similar way, introduced the concept of a dually S–relatively normal ADL and characterized it in terms of S–relative dual annihilators and its S–prime ideals. Characterized the normal ADL R in terms of its maximal filters topologically. Also, by assuming that every dense element of R is maximal, characterized a B–normal ADL R topologically in terms of its maximal filters with hull-kernel topology. In fact, observed that an ADL in which every dense element is maximal is B–normal if and only if the space of all maximal filters of R with hull-kernel topology is a Boolean space. Also, characterized the dually normal ADL R in terms of maximal ideals of R topologically and also in a dually normal ADL, the map which sends every prime ideal to the unique maximal ideal containing it, noted that to be continuous and it was proved that this map is the map from the prime spectrum of R to the maximal spectrum of R such that maximal spectrum of R is a retract of prime spectrum of R. Finally, characterized a dually B–normal ADL R in terms of its maximal ideals by assuming that every dual dense element of R is zero.

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