15. A Comprehensive Overview of Mathematical Modeling

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One significant interdisciplinary activity that incorporates the study of some facets of various fields is mathematical modeling. Among those fields are biology, epidemiology, physiology, ecology, immunology, bio-economics, genetics, and pharmacokinetics. The pursuit of systematic scientific computation of the interactions between the many species is known as mathematical modeling. It has been providing its yeoman services for the advancement of engineering in particular and science and technology in general since its founding. It is now the foundation of contemporary scientific advancement. Every area of mathematics has its own significance, and its domain has expanded to many dimensions. In recent years, this mathematical modeling has reached its pinnacle, permeated every aspect of existence, and captured everyone's interest.

(A) Mathematical Modeling Classifications:

A mathematical model describes a system using mathematical languages. These models are used in the social sciences as well as the natural and technological sciences. "Mathematical Modeling" refers to the entire process of creating a mathematical model. "A Representation of the essential aspects of an existing system which presents knowledge of that system in a usable form" is how Eykhoff defines a mathematical model. These mathematical models come in a variety of shapes. Among these are dynamical systems. Although they are not restricted to them, statistical models specify equations or game theoretic models. These mathematical models frequently overlap with other kinds of models, including models that involve a range of abstract structures. Mathematicians utilize this mathematical model to examine a system that needs to be optimized or regulated. They can be categorized in the following ways to provide a thorough grasp of mathematical models.

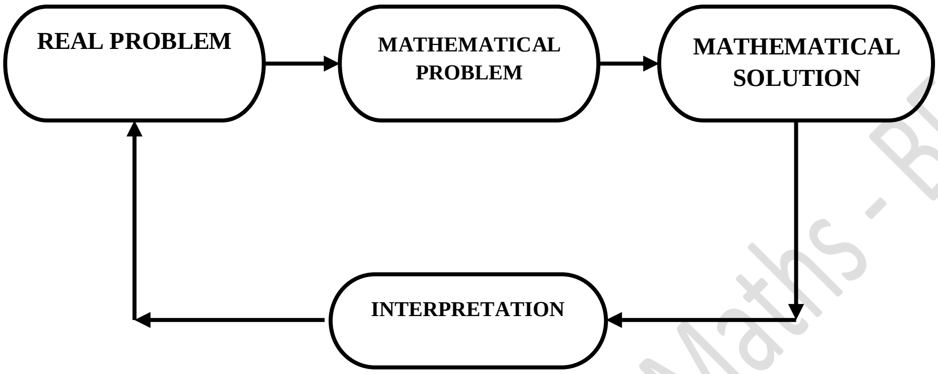
(i) Linear verses Non-Linear: It is well known that variables make up the majority of mathematical models. These variables are abstractions of the quantities of interest in the systems that are discussed, as well as the operators that affect them. These operators can be functions, differential operators, algebraic operators, and more.A mathematical model is said to be linear if every operator within it demonstrates linearity. If not, that model is regarded as a mathematical model that is non-linear.

(ii). Deterministic versus probabilistic (stochastic): A mathematical model is referred to as deterministic if each set of variable states is specifically specified by a parameter and by sets of these variables' prior states. This definition states that deterministic models function similarly for a specific set of initial circumstances. The variable states in a stochastic model are not characterized by distinct values. A probability distribution is used to characterize them. This stochastic model is characterized by randomness.

iii) Static verses Dynamic: A static model is one that does not take time into consideration. However, the dynamic model takes time into consideration. For this reason, differential equations are used to represent dynamic models.

iv) Lumped verses distributed parameters: The lumped mathematical model is homogeneous in comparison to the scattered parameters. Its status will remain constant throughout the system. On the other hand, the distributed parameters model is diverse. The parameters in this system are dispersed. The system will be in different states. Partial differential equations are used to represent the dispersed parameters. Mathematical modeling uses mathematical alphabets to translate real-world situations into mathematical problems. The mathematical difficulties are then resolved by this modeling, which also interprets the results in real-world languages. The following is an illustration of this system.

Basic block diagram:



To develop a workable solution, a real-world problem must first be translated into a mathematical problem. Additionally, an analogous problem, closely resembling the original, is often used to simplify the process. This idealization technique aims to retain all essential characteristics of the problem while discarding aspects that are less critical to the research focus.

We adjust the assumptions in this mathematical modeling procedure if we are unable to obtain sufficient comparisons between the model-based results and the real-world issues. We attempt a different mathematical model structure. Various kinds of mathematical techniques are applied in the subject of model equation analysis. These mathematical and biological models support one another in understanding reality. We shall be able to quantitatively estimate population strength in the fourth coming seconds of time if the mathematical model is constructed in a biological manner. The mathematical treatment becomes more speculative in the absence of a biological model. It becomes challenging to derive significant conclusions from our analysis in that scenario.In his comprehensive book on mathematical modeling, Kapoor discussed the concepts that must be adhered to in order to formulate the mathematical model. Additionally, he has provided examples of the various approaches that can be used in mathematical models.A comprehensive monograph covering a variety of subjects related to mathematical modeling in the biological and medical sciences was presented by Kapoor.

Furthermore, Kapoor provided us with a wealth of knowledge in addition to the fantastic fundamental concepts of modeling in a variety of fields with practical applications.

(B) Steps in Mathematical Modeling:

1. Understanding the Problem: Analyze the problem within its real-world context, especially in fields like biology, medicine, or social sciences.
2. Building the Model: Develop a mathematical representation that simplifies and structures the problem logically.
3. Addressing Challenges: Solve the mathematical equations and obstacles that arise during the model's application.
4. Creating Computational Tools: Design algorithms and develop software solutions to handle calculations efficiently.
5. Evaluating and Communicating Findings: Relate the results back to the original problem, interpret their significance, and share the insights with all relevant groups.

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