15. A Comprehensive Overview of Mathematical Modeling

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Mathematical modeling is an important interdisciplinary activity, which involves the study of some aspects of diverse disciplines. Biology, Epidemiology, Physiology, Ecology, Immunology, Bio-Economics, Genetics, Pharmacokinetics are some of those disciplines. Mathematical Modeling is a quest of methodical scientific computing of the interaction between the multifarious species. Ever since its inception it has been extending its yeoman services for the development of science and technology in general and engineering in particular. It has become the backbone of modern scientific development. It has extended its sphere with manifold dimensions and every branch of mathematics has its own importance. This mathematical modeling has raised to the zenith in recent years and spread to all branches of life and drew the attention of every one.

(A).Classifications of Mathematical Modeling:

A Mathematical model uses mathematical languages to describe a system. These models are applied not only in natural and technical sciences but also in social sciences. The whole process of formulating a mathematical model is named as "Mathematical Modeling". According to the definition of Eykhoff a mathematical model is "A Representation of the essential aspects of an existing system which presents knowledge of that system in a usable form". These Mathematical models take many forms. They include dynamical systems statistical models defines equations or game theoretic models but they are not limited to them. Along with other types of models these mathematical

models are prone to overlap with a given model involving a variety of abstract structures. This Mathematical model is used by Mathematicians to analyze a system that is to be controlled or optimized. To have a comprehensive understanding of Mathematical models they can be classified as follows.

(i). **Linear verses Non-Linear:** It is a known fact that mathematical models are generally composed by variables. These variables are abstractions of quantities of interest in described systems and operators which act on these variables. Those operators can be algebraic operators, functions, differential operators etc.If all the operators which are in a mathematical model exhibit linearity that Mathematical model is defined as linear. If it is not so that model is considered as a non-linear mathematical model.

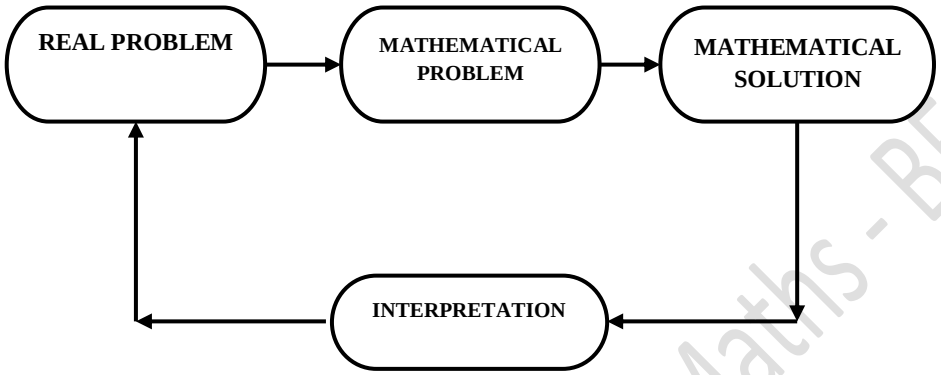
(ii). **Deterministic verses probabilistic (Stochastic):** In the Mathematical model where every set of variable states is uniquely determined by parameter and also by sets of previous states of these variables, that model is known as a deterministic Mathematical model. According to this definition deterministic models perform the same way for a given set of initial conditions. In a stochastic model the variable states are not described by unique values. They are described by probability distribution. Randomness is found in this stochastic model.

iii) **Static verses Dynamic:** The model which does not account for the element of time is known as static model. But the dynamic model accounts for the element of time. That is why dynamic models are represented with differential equations.

iv) **Lumped verses distributed parameters:** Lumped Mathematical model is homogeneous. Throughout the entire system its state will be consistent. Whereas the model of distributed parameters is heterogeneous. In this system the parameters are distributed. There will be varying state within the system. The distributed parameters are represented with partial differential equations.

Mathematical modeling transliterates real world problems into Mathematical problems using Mathematical alphabets. After that this modeling solves the mathematical problems and interprets the solutions in the languages of the real world. This system can be illustrated as follows.

Basic block diagram of real world problems can be shown as below:



It is evident that to get a suitable solution a real world problem is to be translated into a Mathematical problem. It is also necessary to simplify the problem by another problem which stands very close to the original problem. In this method of idealization we try our best to retain all the essential features of the problem. At the same time we give up those features which are not so relevant to our investigation.

In this process of Mathematical modeling if we do not get satisfactory comparisons between the conclusions that are based on the model and the real world problems then we modify the assumptions. We try for another structure for Mathematical model. In the field of analyzing the model equations, different types of mathematical techniques are put into practice. In order to understand the reality, these Mathematical and biological models help each other. If the Mathematical model is structured in biological model it will enable us in the quantitative estimation of population strength in the fourth coming moments of time. If we do not have a biological model the mathematical treatment becomes more hypothetical. In that situation it becomes difficult to draw the meaningful conclusions based on our analization.Kapoor mentioned this in his treatise on mathematical modeling exhaustively about certain principles that are to be followed to formulate the Mathematical model. He has also illustrated the diverse techniques that can be adopted in Mathematical model.Kapoor presented a full fledged monograph with different topics on mathematical modeling in biological and medical sciences.

More over, Kapoor gave us vast information along with the wonderful basic ideas of modeling in diverse areas with real life applications.

(B). Phases of Mathematical Modeling:

i). Identification of the problem in the scientific, in particular, biological / medical / social setting.

ii). Formulation of the mathematical model.

iii). Solution of the mathematical problems that would arise in the study of the model selected.

iv). Developments of algorithms and associated computer programs for the relevant computations.

v). Explanation and interpretation of the results in the context of the original problem and the communication of this information to all those interested. This amounts to the evaluation of the results.

- **Sri I.Pothuraju**