

1. (a). The rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ is 2.

(b). The eigen values of A are 1, 2, 3.
 \Rightarrow The eigen values of A^2 are 1, 4, 9.

(c) Given differential equation is $\left[1 + \frac{d^2y}{dx^2}\right]^{3/2} = \frac{d^2y}{dx^2}$
 $\Rightarrow \left(1 + \frac{d^2y}{dx^2}\right)^3 = \left(\frac{d^2y}{dx^2}\right)^2$

\therefore order = 2, Degree = 3.

(d). The general solution of $\frac{dy}{dx} = x^{-3} e^{2y}$ is $y = -\frac{1}{2} \log\left(\frac{1}{x^2} - 2c\right)$.

(e). Newton's law of cooling: The temperature of a body changes at a rate which is proportional to the difference in temperature between that of the surrounding medium and that of the body itself.

(f). $(D^3 - 3D + 2)y = x$

The Auxiliary equation is $D^3 - 3D + 2 = 0 \Rightarrow D = 1, 1, -2$.

\therefore C.F. = $(c_1 + c_2 x)e^x + c_3 e^{-2x}$.

(g). The differential equation of L-C-R circuit with an emf: P cosnt is

$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = P \cos nt.$$

$$(h). L(t^{3/2}) = \frac{\Gamma(\frac{3}{2} + 1)}{s^{3/2 + 1}} = \frac{\frac{3}{2} \cdot \frac{1}{2} \cdot \sqrt{\pi}}{s^{5/2}} = \frac{3\sqrt{\pi}}{4s^{5/2}}$$

$$(i) L^{-1}\left(\frac{s-2}{s^2-4s+13}\right) = L^{-1}\left(\frac{s-2}{(s-2)^2+3^2}\right) = e^{2t} \cos 3t$$

$$(j) \text{ If } L[f(t)] = \bar{f}(s), \text{ then } L[f(at)] = \frac{1}{a} \bar{f}\left(\frac{s}{a}\right).$$

2(a). Given equations are $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$.
 The given system of equations can be represented in the matrix equation as $AX=B$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

consider $[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array} \right]$

$R_2 - R_1; R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{bmatrix} \xrightarrow{R_3 - R_2} \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix} \quad \text{--- 2M}$$

The given system of equations have

- (i) No solution when $\lambda=3$, $\mu \neq 10$.
- (ii) a unique solution when $\lambda \neq 3$, for any μ
- (iii) an infinite no. of solutions when $\lambda=3$, $\mu=10$. --- 3M

2(b) let $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

consider $[A : I_3] = \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array} \right]$ --- 2M

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 3/2 \\ 0 & 1 & 0 & -5/4 & -1/4 & -3/4 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix} = -\frac{1}{4} \begin{bmatrix} -12 & -4 & -6 \\ 5 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \quad \text{--- 3M}$$

(OR)

3(a). Let $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

(3)

The characteristic equation of A is $|A - \lambda I| = 0$.

$$\Rightarrow \begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - 8\lambda^2 + 15\lambda = 0$$

$$\Rightarrow \lambda = 0, 3, 5.$$

_____ 2M

\therefore The eigen values of A are $\lambda = 0, 3, 5$.

when $\lambda = 0$, the corresponding eigen vector is $(1, 2, 2)$.

when $\lambda = 3$, the corresponding eigen vector is $(2, 1, -2)$

when $\lambda = 5$, the corresponding eigen vector is $(2, -2, 1)$.

_____ 3M

3(b). Given $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 2 & 2 \end{bmatrix}$

The characteristic equation of the matrix A is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 1 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0. \text{ _____ 2M}$$

According to the Cayley-Hamilton theorem,

Every square matrix satisfies its own characteristic equation.

$$\therefore A^3 - 5A^2 + 7A - 3I = 0 \text{ _____ ①}$$

Consider $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

$$= A^5(A^3 - 5A^2 + 7A - 3I) + A(A^3 - 5A^2 + 7A - 3I) + A^2 + A + I$$

$$= A^5(0) + A(0) + A^2 + A + I \text{ (}\because \text{ By ①)}$$

$$= A^2 + A + I.$$

_____ 3M

7(a). $\frac{dy}{dx} = (4x+y+1)^2$, $y(0)=1$.

Let $4x+y+1 = t \Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 4$

$\therefore \frac{dt}{dx} - 4 = t^2$

$\Rightarrow \frac{dt}{dx} = t^2 + 4$ _____ 2M

$\Rightarrow \int \frac{dt}{t^2+4} = \int dx$

$\Rightarrow \frac{1}{2} \tan^{-1}\left(\frac{t}{2}\right) = x + C$

$\Rightarrow 4x+y+1 = 2 \tan(2x+2C)$

Since $y(0)=1 \Rightarrow C = \frac{\pi}{8}$.

Hence the solution is $4x+y+1 = 2 \tan\left(2x + \frac{\pi}{4}\right)$. _____ 3M

4(b). Given $xy(1+xy^2) \frac{dy}{dx} = 1$.

$\Rightarrow \frac{dx}{dy} - xy = x^2 y^3$

$\Rightarrow \frac{1}{x^2} \frac{dx}{dy} - \frac{y}{x} = y^3$ _____ ①

Let $\frac{1}{x} = z \Rightarrow -\frac{1}{x^2} \frac{dx}{dy} = \frac{dz}{dy}$

① $\Rightarrow \frac{dz}{dy} + yz = -y^3$, which is Leibnitz's linear in z . _____ 2M

IF = $e^{\int y dy} = e^{y^2/2}$

\therefore The solution is $z(\text{IF}) = \int (-y^3)(\text{IF}) dy + C$

$\Rightarrow z e^{y^2/2} = \int -y^3 e^{y^2/2} dy + C$

$= -2 \int t e^t dt + C$

$= -2 [t e^t - e^t] + C$

$= (2 - y^2) e^{y^2/2} + C$

$\Rightarrow z = (2 - y^2) + C e^{-y^2/2}$. _____ 3M

Let $\frac{y^2}{2} = t$

$\Rightarrow y dy = dt$

(OR)

5(a). $(x^2 - 4xy - 2y^2) dx + (y^2 - 4xy - 2x^2) dy = 0.$

Here $M = x^2 - 4xy - 2y^2$; $N = y^2 - 4xy - 2x^2$

$\frac{\partial M}{\partial y} = -4x - 4y$; $\frac{\partial N}{\partial x} = -4y - 4x.$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Thus the equation is exact and its solution is

$\int M dx + \int (\text{Terms of } N \text{ not containing } x) dy = c$ _____ 2M
(y constant)

$\Rightarrow \int (x^2 - 4xy - 2y^2) dx + \int y^2 dy = c$

$\Rightarrow \frac{x^3}{3} - 2x^2y - 2xy^2 + \frac{y^3}{3} = c$

$\Rightarrow x^3 - 6x^2y - 6xy^2 + y^3 = c.$ _____ 3M

5(b). Let u be the amount of bacteria at any time $t.$
and let u_0 be the initial amount of bacteria.

Given $\frac{du}{dt} \propto u$

$\Rightarrow \frac{du}{dt} = ku$, where k is constant.

$\Rightarrow u = ce^{kt}$

_____ 2M

Since $u = u_0$, when $t = 0.$

$\therefore u_0 = ce^{k(0)} \Rightarrow c = u_0.$

$\therefore u = u_0 e^{kt}$

Given that the original number doubles in 2 hours

i.e, $u = 2u_0$ when $t = 2$ hours.

$\therefore 2u_0 = u_0 e^{2k}$

$\Rightarrow k = \frac{1}{2} \log 2$

when $u = 3u_0$, $3u_0 = u_0 e^{(\frac{1}{2} \log 2)t}$

$\Rightarrow t = \frac{2 \log 3}{\log 2}.$

_____ 3M

6(a). $(D-2)^2 y = 8(e^{3x} + \sin 2x + x^2)$

To find C.F: Its A.E is $(D-2)^2 = 0 \Rightarrow D = 2, 2$

Thus C.F = $(C_1 + C_2 x) e^{2x}$

_____ 2M

To find P.I:

P.I = $\frac{1}{(D-2)^2} 8(e^{3x} + \sin 2x + x^2)$

= $8 \left[\frac{1}{(D-2)^2} e^{3x} + \frac{1}{(D-2)^2} \sin 2x + \frac{1}{(D-2)^2} x^2 \right]$

= $8 \left[\frac{8}{(D-2)^2} e^{3x} + \frac{1}{-4D} \sin 2x + \frac{1}{4} \left(1 - \frac{D}{2}\right)^{-2} x^2 \right]$

= $8 \left[\frac{8}{8} e^{3x} + \frac{1}{8} \cos 2x + \frac{1}{4} \left(1 + (-2) \frac{D}{2} + \frac{(-2)(-3)}{2!} \left(-\frac{D}{2}\right)^2 + \dots \right) x^2 \right]$

= $8 \left[\frac{e^{3x}}{1} + \frac{1}{8} \cos 2x + \frac{1}{4} \left(x^2 + 2x + \frac{3}{2}\right) \right]$

= $8 e^{3x} + \cos 2x + 2x^2 + 4x + 3.$

_____ 3M

Hence the complete solution is $y = (C_1 + C_2 x) e^{2x} + 8 e^{3x} + \cos 2x + 2x^2 + 4x + 3.$

6(b). $(D^2 + a^2) y = \operatorname{cosec} ax.$

To find C.F: Its A.E is $D^2 + a^2 = 0 \Rightarrow D = \pm ai$

Thus C.F = $C_1 \cos ax + C_2 \sin ax.$

_____ 2M

To find P.I: Here $y_1 = \cos ax$; $y_2 = \sin ax$ and $x = \operatorname{cosec} ax.$

$\therefore W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos ax & \sin ax \\ -a \sin ax & a \cos ax \end{vmatrix} = a.$

Thus P.I = $-y_1 \int \frac{y_2 x}{W} dx + y_2 \int \frac{y_1 x}{W} dx$

= $-\cos ax \int \frac{\sin ax \operatorname{cosec} ax}{a} dx + \sin ax \int \frac{\cos ax \operatorname{cosec} ax}{a} dx$

= $-\frac{\cos ax}{a} \int \tan ax dx + \frac{\sin ax}{a} \int \cot ax dx$

= $-\frac{x \cos ax}{a} + \frac{1}{a^2} \sin ax \log(\sin ax)$

_____ 3M

Hence the complete solution is $y = C_1 \cos ax + C_2 \sin ax - \frac{x \cos ax}{a} + \frac{1}{a^2} \sin ax \log(\sin ax)$

7(a) $(D^4 + 2D^2 + 1)y = x^2 \cos x$. (OR)

To find C.F: Let's A.E is $(D^2 + 1)^2 = 0 \Rightarrow D = \pm i, \pm i$

\therefore CF = $(c_1 + c_2 x) \cos x + (c_3 + c_4 x) \sin x$. 2M

To find P.I:

P.I = $\frac{1}{(D^2 + 1)^2} x^2 \cos x = \frac{1}{(D^2 + 1)^2} x^2 (\text{Rp. } e^{ix})$

= Rp $\left\{ \frac{1}{(D^2 + 1)^2} e^{ix} x^2 \right\} = \text{Rp} \left\{ e^{ix} \frac{1}{[(D+i)^2 + 1]^2} x^2 \right\}$

= Rp $\left\{ e^{ix} \frac{1}{(D^2 + 2iD)^2} x^2 \right\} = \text{Rp} \left\{ e^{ix} \left\{ -\frac{1}{4D^2} \left(1 - \frac{i}{2}D\right)^{-2} x^2 \right\} \right\}$

= Rp $\left[-\frac{1}{4} e^{ix} \cdot \frac{1}{D^2} \left\{ 1 + 2 \frac{iD}{2} + 3 \left(\frac{iD}{2}\right)^2 + \dots \right\} x^2 \right]$

= Rp $\left\{ -\frac{1}{4} e^{ix} \cdot \frac{1}{D^2} (x^2 + 2ix - \frac{3}{2}) \right\} = \text{Rp} \left\{ -\frac{1}{4} e^{ix} \cdot \frac{1}{D} \left(\frac{x^3}{3} + ix^2 - \frac{3}{2}x \right) \right\}$

= $-\frac{1}{4} \text{Rp} \left\{ e^{ix} \left(\frac{x^4}{12} + \frac{ix^3}{3} - \frac{3x^2}{4} \right) \right\} = -\frac{1}{48} \text{Rp} \left\{ (\cos x + i \sin x) (x^4 + 4ix^3 - 9x^2) \right\}$

= $-\frac{1}{48} [(x^4 - 9x^2) \cos x - 4x^3 \sin x]$. 3M

Hence the complete solution is $y = (c_1 + c_2 x) \cos x + (c_3 + c_4 x) \sin x + \frac{1}{48} [4x^3 \sin x - (x^4 - 9x^2) \cos x]$.

7(b). The differential equation is $L \frac{d^2 q}{dt^2} + \frac{q}{C} = E \sin\left(\frac{t}{LC}\right)$

Let's A.E is $LD^2 + \frac{1}{C} = 0 \Rightarrow D = \pm \frac{i}{\sqrt{LC}}$

\therefore CF = $c_1 \cos\left(\frac{t}{\sqrt{LC}}\right) + c_2 \sin\left(\frac{t}{\sqrt{LC}}\right)$. 2M

P.I = $\frac{1}{LD^2 + \frac{1}{C}} E \sin\left(\frac{t}{\sqrt{LC}}\right)$

= $Et \cdot \frac{1}{2LD} \sin\left(\frac{t}{\sqrt{LC}}\right)$

= $-\frac{Et}{2L} \cos\left(\frac{t}{\sqrt{LC}}\right) \sqrt{LC}$

= $-\frac{Et}{2} \sqrt{\frac{C}{L}} \cos\left(\frac{t}{\sqrt{LC}}\right)$

Hence the complete solution is

$q = c_1 \cos\left(\frac{t}{\sqrt{LC}}\right) + c_2 \sin\left(\frac{t}{\sqrt{LC}}\right) - \frac{Et}{2} \sqrt{\frac{C}{L}} \cos\left(\frac{t}{\sqrt{LC}}\right)$. 3M

8(a). $L[t \cos at]$

$$L(\cos at) = \frac{s}{s^2 + a^2}$$

$$\Rightarrow L(t \cos at) = (-1) \frac{d}{ds} \left[\frac{s}{s^2 + a^2} \right]$$

$$= - \frac{(s^2 + a^2 - 2s^2)}{(s^2 + a^2)^2} = \frac{s^2 - a^2}{(s^2 + a^2)^2}$$

$$\therefore L(t^n f(t)) = (-1)^n \frac{d^n}{ds^n} \bar{f}(s)$$

2M

2M

(b) Let $\frac{5s+3}{(s-1)(s^2+2s+5)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+2s+5}$

$$\Rightarrow \frac{5s+3}{(s-1)(s^2+2s+5)} = \frac{1}{s-1} + \frac{-s+2}{s^2+2s+5}$$

3M

$$\Rightarrow L^{-1} \left(\frac{5s+3}{(s-1)(s^2+2s+5)} \right) = L^{-1} \left(\frac{1}{s-1} \right) + L^{-1} \left(\frac{-s+2}{s^2+2s+5} \right)$$

$$= L^{-1} \left(\frac{1}{s-1} \right) + L^{-1} \left(\frac{-(s+1)+3}{(s+1)^2+2^2} \right)$$

$$= L^{-1} \left(\frac{1}{s-1} \right) - L^{-1} \left(\frac{s+1}{(s+1)^2+2^2} \right) + 3 L^{-1} \left(\frac{1}{(s+1)^2+2^2} \right)$$

$$= e^t - e^{-t} \cos 2t + \frac{3}{2} e^{-t} \sin 2t.$$

3M

(OR)

9(a) $y'' - 2y' + y = e^t$; $y=2, y'=-1$ at $t=0$.

Taking the Laplace transforms of both sides, we get

$$L(y'' - 2y' + y) = L(e^t)$$

$$\Rightarrow s^2 L(y) - sy(0) - y'(0) - 2(sL(y) - y(0)) + L(y) = \frac{1}{s-1}$$

$$\Rightarrow (s^2 - 2s + 1) L(y) = \frac{1}{s-1} + 2s - 5$$

$$\Rightarrow L(y) = \frac{1}{(s-1)^3} + \frac{2s-5}{(s-1)^2}$$

$$\Rightarrow L(y) = \frac{1}{(s-1)^3} - \frac{3}{(s-1)^2} + \frac{2}{s-1}$$

3M

$$\Rightarrow y = L^{-1} \left(\frac{1}{(s-1)^3} \right) - 3 L^{-1} \left(\frac{1}{(s-1)^2} \right) + 2 L^{-1} \left(\frac{1}{s-1} \right)$$

$$\Rightarrow y = e^t \cdot \frac{t^2}{2} - 3 e^t \cdot t + 2 e^t$$

$$\Rightarrow y = 2e^t - 3te^t + \frac{t^2}{2}e^t$$

_____ 2M

9(b) $L^{-1} \left[\frac{1}{(s^2+1)(s^2+9)} \right]$

Since $L^{-1} \left(\frac{1}{s^2+1} \right) = \sin t$; $L^{-1} \left(\frac{1}{s^2+9} \right) = \frac{\sin 3t}{3}$

∴ By convolution theorem,

$$L^{-1} \left[\frac{1}{s^2+1} \cdot \frac{1}{s^2+9} \right] = \int_0^t \sin u \cdot \frac{\sin(3t-3u)}{3} du$$

_____ 2M

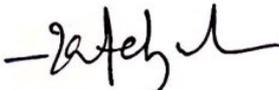
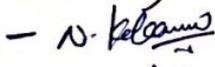
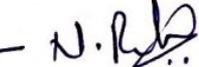
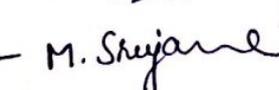
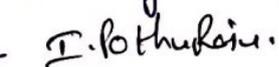
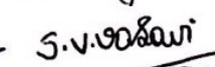
$$= \frac{1}{6} \int_0^t [\cos(4u-3t) - \cos(3t-2u)] du$$

$$= \frac{1}{6} \left[\frac{\sin(4u-3t)}{4} - \frac{\sin(3t-2u)}{-2} \right]_0^t$$

$$= \frac{1}{6} \left\{ \frac{1}{4} (\sin t + \sin 3t) + \frac{1}{2} (\sin t - \sin 3t) \right\}$$

$$= \frac{1}{8} (\sin t - \frac{1}{3} \sin 3t)$$

_____ 3M

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