BAPATLA ENGINEERING COLLEGE, BAPATLA

DEPARTMENT OF MATHEMATICS

18MA001

I/IV B.Tech (Regular) DEGREE EXAMINATION

Linear Algebra and ODE Scheme of Evaluation

Dec, 2018

I Semester.

1.

(a). The rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ is 2.

(b). The eigen values of A are 1, 2, 3.

$\Rightarrow$ The eigen values of A^2 are 1, 4, 9.

(c) Given differential equation is $\left[1+\frac{d^2y}{dx^2}\right]^{3/2} = \frac{d^2y}{dx^2}$

$$\Rightarrow \left(1+\frac{d^2y}{dx^2}\right)^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

$\therefore$ order = 2, Degree = 3.

(d). The general solution of $\frac{dy}{dx} = x^{-3}e^{2y}$ is $y = -\frac{1}{2}\log\left(\frac{1}{x^2} - 2c\right)$.

(e). Newtons law of cooling: The temperature of a body changes at a rate which is proportional to the difference in temperature between that of the surrounding medium and that of the body itself.

(f). $(D^3 - 3D + 2)y = x$

The Auxiliary equation is $D^3 - 3D + 2 = 0 \Rightarrow D = 1, 1, -2$.

$\therefore C.F = (c_1 + c_2 x)e^x + c_3 e^{-2x}$.

(g). The differential equation of L-C-R circuit with an emf: $P\cos nt$ is

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = P\cos nt.$$

(h). $L(t^{3/2}) = \frac{\Gamma(\frac{3}{2}+1)}{s^{3/2+1}} = \frac{\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\pi}}{s^{5/2}} = \frac{3\sqrt{\pi}}{4s^{5/2}}$

(i) $L^{-1}\left(\frac{s-2}{s^2-4s+13}\right) = L^{-1}\left(\frac{s-2}{(s-2)^2+3^2}\right) = e^{2t}\cos 3t$

(j) If $L[f(t)] = \bar{f}(s)$, then $L[f(at)] = \frac{1}{a}\bar{f}\left(\frac{s}{a}\right)$.

2(a). Given equations are $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$.
The given system of equations can be represented in the matrix equation as $AX = B$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

consider $[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$

R_2-R_1 ; R_3-R_1

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right] \underset{R_3-R_2}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$ ——— 2M

The given system of equations have
(i) No solution when $\lambda=3$, $\mu \neq 10$.
(ii) a unique solution when $\lambda \neq 3$, for any μ
(iii) an infinite no. of solutions when $\lambda=3$, $\mu=10$. ——— 3M

2(b) Let $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

consider $[A : I_3] = \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array}\right]$ ——— 2M

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 3/2 \\ 0 & 1 & 0 & -5/4 & -1/4 & -3/4 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 3 & 1 & \frac{3}{2} \\ \frac{-5}{4} & \frac{-1}{4} & \frac{-3}{4} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} -12 & -4 & -6 \\ 5 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}.$$ ——— 3M

(OR)

3(a). Let $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

The characteristic equation of A is $|A - \lambda I| = 0$.

$$\Rightarrow \begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - 8\lambda^2 + 15\lambda = 0$$

$$\Rightarrow \lambda = 0, 3, 5.$$ —— 2M

$\therefore$ The eigen values of A are $\lambda = 0, 3, 5$.

when $\lambda = 0$, the corresponding eigen vector is $(1, 2, 2)$.

when $\lambda = 3$, the corresponding eigen vector is $(2, 1, -2)$

when $\lambda = 5$, the corresponding eigen vector is $(2, -2, 1)$. —— 3M

3(b). Given $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 2 & 2 \end{bmatrix}$

The characteristic equation of the matrix A is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 1 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0.$$ —— 2M

According to the cayley - Hamilton theorem,
Every square matrix satisfies its own characteristic equation.

$$\therefore A^3 - 5A^2 + 7A - 3I = 0 \quad \text{——} (1)$$

consider $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

$$= A^5(A^3 - 5A^2 + 7A - 3I) + A(A^3 - 5A^2 + 7A - 3I) + A^2 + A + I$$

$$= A^5(0) + A(0) + A^2 + A + I \quad (\because \text{By } (1))$$

$$= A^2 + A + I.$$ —— 3M

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4(a). $\dfrac{dy}{dx} = (4x+y+1)^2, \; y(0)=1.$

Let $4x+y+1 = t \Rightarrow \dfrac{dy}{dx} = \dfrac{dt}{dx} - 4$

$\therefore \dfrac{dt}{dx} - 4 = t^2$

$\Rightarrow \dfrac{dt}{dx} = t^2+4$ ———— 2M

$\Rightarrow \int \dfrac{dt}{t^2+4} = \int dx$

$\Rightarrow \dfrac{1}{2}\tan^{-1}\left(\dfrac{t}{2}\right) = x+c$

$\Rightarrow 4x+y+1 = 2\tan(2x+2c)$

Since $y(0)=1 \Rightarrow c = \dfrac{\pi}{8}.$

Hence the solution is $4x+y+1 = 2\tan\left(2x+\dfrac{\pi}{4}\right)$. ———— 3M

4(b). Given $xy(1+xy^2)\dfrac{dy}{dx} = 1.$

$\Rightarrow \dfrac{dx}{dy} - xy = x^2y^3$

$\Rightarrow \dfrac{1}{x^2}\dfrac{dx}{dy} - \dfrac{y}{x} = y^3$ ———— ①

Let $\dfrac{1}{x} = z \Rightarrow \dfrac{-1}{x^2}\dfrac{dx}{dy} = \dfrac{dz}{dy}$

① $\Rightarrow \dfrac{dz}{dy} + yz = -y^3$, which is Leibnitz's linear in z. ———— 2M

$IF = e^{\int y\,dy} = e^{y^2/2}$

$\therefore$ The solution is $z(IF) = \int(-y^3)(IF)\,dy + c$

$\Rightarrow z e^{y^2/2} = \int -y^3 e^{y^2/2}\,dy + c$

Let $\dfrac{y^2}{2} = t$

$\Rightarrow y\,dy = dt$

$= -2\int t e^t\,dt + c$

$= -2\left[t e^t - e^t\right] + c$

$= (2-y^2)e^{y^2/2} + c$

$\Rightarrow z = (2-y^2) + c\,e^{-y^2/2}.$ ———— 3M

(OR)

5(a). $(x^2-4xy-2y^2)\,dx + (y^2-4xy-2x^2)\,dy = 0.$

Here $M = x^2-4xy-2y^2$; $N = y^2-4xy-2x^2$

$$\frac{\partial M}{\partial y} = -4x-4y \quad ; \quad \frac{\partial N}{\partial x} = -4y-4x.$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Thus the equation is exact and its solution is

$$\int M\,dx + \int (\text{Terms of } N \text{ not containing } x)\,dy = c \quad \text{———— 2M}$$

(y constant)

$$\Rightarrow \int (x^2-4xy-2y^2)\,dx + \int y^2\,dy = c$$

$$\Rightarrow \frac{x^3}{3} - 2x^2y - 2xy^2 + \frac{y^3}{3} = c$$

$$\Rightarrow x^3 - 6x^2y - 6xy^2 + y^3 = c. \quad \text{———— 3M}$$

5(b). Let u be the amount of bacteria at any time t.

and let u_0 be the initial amount of bacteria.

Given $\frac{du}{dt} \propto u$

$\Rightarrow \frac{du}{dt} = ku$, where k is constant.

$$\Rightarrow u = ce^{kt} \quad \text{———— 2M}$$

Since $u = u_0$, when $t = 0$.

$$\therefore u_0 = ce^{k(0)} \Rightarrow c = u_0.$$

$$\therefore u = u_0 e^{kt}$$

Given that the original number doubles in 2 hours

i.e, $u = 2u_0$ when $t = 3$ hours.

$$\therefore 2u_0 = u_0 e^{3k}$$

$$\Rightarrow k = \frac{1}{3}\log 2$$

when $u = 3u_0$, $3u_0 = u_0\, e^{\left(\frac{1}{3}\log 2\right)t}$

$$\Rightarrow t = \frac{3\log 3}{\log 2}. \quad \text{———— 3M}$$

6(a). $(D-2)^2 y = 8\left(e^{3x} + \sin 2x + x^2\right)$

To find C.F: Its A.E is $(D-2)^2 = 0 \Rightarrow D = 2, 2$

Thus $C.F = (c_1 + c_2 x)\, e^{2x}$ ——— 2M

To find P.I:

$$P.I = \frac{1}{(D-2)^2}\, 8\left(e^{3x} + \sin 2x + x^2\right)$$

$$= 8\left[\frac{1}{(D-2)^2} e^{3x} + \frac{1}{(D-2)^2}\sin 2x + \frac{1}{(D-2)^2} x^2\right]$$

$$= 8\left[\frac{1}{(D-2)^2} e^{3x} + \frac{1}{-4D}\sin 2x + \frac{1}{4}\left(1 - \frac{D}{2}\right)^{-2} x^2\right]$$

$$= 8\left[\text{—}\, e^{2x} + \frac{1}{8}\cos 2x + \frac{1}{4}\left(1 + (-2)\frac{D}{2} + \frac{(-2)(-3)}{2!}\left(-\frac{D}{2}\right)^2 + \dots\right)(x^2)\right]$$

$$= 8\left[\frac{e^{2x}}{2} + \frac{1}{8}\cos 2x + \frac{1}{4}\left(x^2 + 2x + \frac{3}{2}\right)\right]$$

$$= 8\, e^{2x} + \cos 2x + 2x^2 + 4x + 3.$$ ——— 3M

Hence the complete solution is $y = (c_1 + c_2 x)\, e^{2x} + 8\, e^{2x} + \cos 2x + 2x^2 + 4x + 3$.

6(b). $(D^2 + a^2)\, y = \operatorname{cosec} ax$.

To find C.F: Its A.E. is $D^2 + a^2 = 0 \Rightarrow D = \pm ai$

Thus $C.F = c_1 \cos ax + c_2 \sin ax$. ——— 2M

To find P.I: Here $y_1 = \cos ax$; $y_2 = \sin ax$ and $X = \operatorname{cosec} ax$.

$$\therefore W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos ax & \sin ax \\ -a\sin ax & a\cos ax \end{vmatrix} = a.$$

Thus $P.I = -y_1 \int \frac{y_2 X}{W}\, dx + y_2 \int \frac{y_1 X}{W}\, dx$

$$= -\cos ax \int \frac{\sin ax \operatorname{cosec} ax}{a}\, dx + \sin ax \int \frac{\cos ax \operatorname{cosec} ax}{a}\, dx$$

$$= -\frac{\cos ax}{a} \int \overset{1.dx}{\cancel{\tan ax}} + \frac{\sin ax}{a} \int \cot ax\, dx$$

$$= -\frac{x\cos ax}{a} + \frac{1}{a^2} \sin ax \log(\sin ax)$$ ——— 3M

Hence the complete solution is $y = c_1 \cos ax + c_2 \sin ax - \frac{x\cos ax}{a} + \frac{1}{a^2}\sin ax \log(\sin ax)$

(CR)

7(a) $(D^4+2D^2+1)\,y = x^2\cos x$.

To find C.F: Its A.E is $(D^2+1)^2=0 \Rightarrow D=\pm i, \pm i$

$\therefore CF = (c_1+c_2x)\cos x + (c_3+c_4x)\sin x$. ——— 2M

To find P.I:

$$P.I = \frac{1}{(D^2+1)^2}\,x^2\cos x = \frac{1}{(D^2+1)^2}\,x^2\,(Rp.\ e^{ix})$$

$$= Rp\left\{\frac{1}{(D^2+1)^2}\,e^{ix}x^2\right\} = Rp\left\{e^{ix}\,\frac{1}{[(D+i)^2+1]^2}\,x^2\right\}$$

$$= Rp\left\{e^{ix}\,\frac{1}{(D^2+2iD)^2}\,x^2\right\} = Rp\left\{e^{ix}\left\{-\frac{1}{4D^2}\left(1-\frac{i}{2}D\right)^{-2}x^2\right\}\right\}$$

$$= Rp\left[-\frac{1}{4}e^{ix}\cdot\frac{1}{D^2}\left\{1+2\frac{iD}{2}+3\left(\frac{iD}{2}\right)^2+\cdots\right\}x^2\right]$$

$$= Rp\left\{-\frac{1}{4}e^{ix}\cdot\frac{1}{D^2}\left(x^2+2ix-\frac{3}{2}\right)\right\} = Rp\left\{-\frac{1}{4}e^{ix}\cdot\frac{1}{D}\left(\frac{x^3}{3}+ix^2-\frac{3}{2}x\right)\right\}$$

$$= -\frac{1}{4}Rp\left\{e^{ix}\left(\frac{x^4}{12}+\frac{ix^3}{3}-\frac{3x^2}{4}\right)\right\} = \frac{-1}{48}Rp\left\{(\cos x+i\sin x)(x^4+4ix^3-9x^2)\right\}$$ ——— 3M

$$= \frac{-1}{48}\left[(x^4-9x^2)\cos x - 4x^3\sin x\right].$$

Hence the complete solution is $y=(c_1+c_2x)\cos x+(c_3+c_4x)\sin x+\frac{1}{48}\left[4x^3\sin x-(x^4-9x^2)\cos x\right]$.

7(b). The differential equation is $L\frac{d^2q}{dt^2}+\frac{q}{C}=E\sin\left(\frac{t}{LC}\right)$

Its A.E is $LD^2+\frac{1}{C}=0 \Rightarrow D=\pm\frac{i}{\sqrt{LC}}$

$\therefore CF = c_1\cos\left(\frac{t}{\sqrt{LC}}\right)+c_2\sin\left(\frac{t}{\sqrt{LC}}\right)$. ——— 2M

$$P.I = \frac{1}{LD^2+\frac{1}{C}}\,E\sin\left(\frac{t}{\sqrt{LC}}\right)$$

$$= Et\cdot\frac{1}{2LD}\sin\left(\frac{t}{\sqrt{LC}}\right) \quad \left(\because \text{Here } D^2=-\frac{1}{LC}, \text{ the denominator is } 0\right)$$

$$= -\frac{Et}{2L}\cos\left(\frac{t}{\sqrt{LC}}\right)\sqrt{LC}$$

$$= -\frac{Et}{2}\sqrt{\frac{C}{L}}\cos\left(\frac{t}{\sqrt{LC}}\right)$$

Hence the complete solution is

$$q = c_1\cos\left(\frac{t}{\sqrt{LC}}\right)+c_2\sin\left(\frac{t}{\sqrt{LC}}\right)-\frac{Et}{2}\sqrt{\frac{C}{L}}\cos\left(\frac{t}{\sqrt{LC}}\right).$$ ——— 3M

8(a). $L\left[t\cos at\right]$

$$L(\cos at) = \frac{s}{s^2+a^2}$$

$$\Rightarrow L(t\cos at) = (-1)\frac{d}{ds}\left[\frac{s}{s^2+a^2}\right] \qquad \left(\because L(t^n f(t)) = (-1)^n \frac{d^n}{ds^n}\bar{f}(s)\right)$$ —— 2M

$$= -\frac{(s^2+a^2-2s^2)}{(s^2+a^2)^2} = \frac{s^2-a^2}{(s^2+a^2)^2}$$ —— 2M

(b) Let $\dfrac{5S+3}{(S-1)(S^2+2S+5)} = \dfrac{A}{S-1} + \dfrac{BS+C}{S^2+2S+5}$

$$\Rightarrow \frac{5S+3}{(S-1)(S^2+2S+5)} = \frac{1}{S-1} + \frac{-S+2}{S^2+2S+5}$$ —— 3M

$$\Rightarrow L^{-1}\left(\frac{5S+3}{(S-1)(S^2+2S+5)}\right) = L^{-1}\left(\frac{1}{S-1}\right) + L^{-1}\left(\frac{-S+2}{S^2+2S+5}\right)$$

$$= L^{-1}\left(\frac{1}{S-1}\right) + L^{-1}\left(\frac{-(S+1)+3}{(S+1)^2+2^2}\right)$$

$$= L^{-1}\left(\frac{1}{S-1}\right) - L^{-1}\left(\frac{S+1}{(S+1)^2+2^2}\right) + 3\,L^{-1}\left(\frac{1}{(S+1)^2+2^2}\right)$$

$$= e^t - e^{-t}\cos 2t + \frac{3}{2}e^{-t}\sin 2t.$$ —— 3M

(OR)

9(a) $y'' - 2y' + y = e^t$; $y=2,\ y'=-1$ at $t=0$.

Taking the Laplace transforms of both sides, we get

$$L(y''-2y'+y) = L(e^t)$$

$$\Rightarrow s^2 L(y) - s\,y(0) - y'(0) - 2\left(s\,L(y) - y(0)\right) + L(y) = \frac{1}{S-1}$$

$$\Rightarrow (S^2-2S+1)\,L(y) = \frac{1}{S-1} + 2S-5$$

$$\Rightarrow L(y) = \frac{1}{(S-1)^3} + \frac{2S-5}{(S-1)^2}$$

$$\Rightarrow L(y) = \frac{1}{(S-1)^3} - \frac{3}{(S-1)^2} + \frac{2}{S-1}$$ —— 3M

$$\Rightarrow y = L^{-1}\left(\frac{1}{(S-1)^3}\right) - 3\,L^{-1}\left(\frac{1}{(S-1)^2}\right) + 2\,L^{-1}\left(\frac{1}{S-1}\right)$$

$$\Rightarrow y = e^t \cdot \frac{t^2}{2} - 3\,e^t \cdot t + 2e^t$$

$$\Rightarrow y = 2e^t - 3t\,e^t + \frac{t^2}{2}e^t.$$ ———— 2M

9(b) $L^{-1}\left[\frac{1}{(s^2+1)(s^2+9)}\right]$

Since $L^{-1}\left(\frac{1}{s^2+1}\right) = \sin t$; $L^{-1}\left(\frac{1}{s^2+9}\right) = \frac{\sin 3t}{3}$

∴ By convolution theorem,

$$L^{-1}\left[\frac{1}{s^2+1}\cdot\frac{1}{s^2+9}\right] = \int_0^t \sin u \;\frac{\sin(3t-3u)}{3}\,du$$ ———— 2M

$$= \frac{1}{6}\int_0^t \left[\cos(4u-3t) - \cos(3t-2u)\right]du$$

$$= \frac{1}{6}\left[\frac{\sin(4u-3t)}{4} - \frac{\sin(3t-2u)}{-2}\right]_0^t$$

$$= \frac{1}{6}\left\{\frac{1}{4}(\sin t + \sin 3t) + \frac{1}{2}(\sin t - \sin 3t)\right\}$$

$$= \frac{1}{8}\left(\sin t - \frac{1}{3}\sin 3t\right).$$ ———— 3M

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