Scheme &Solutions of 14EI801 (MAR-2019)

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14EI801

Hall Ticket Number:

IV/IV B.Tech (Regular/Supplementary) DEGREE EXAMINATION

March, 2019 **Electronics & Instrumentation Engineering**

Eighth Semester **Computer Control of Process**

Time: Three Hours **Maximum :** 60 Marks

Answer Question No.1 compulsorily. (1X12 = 12 Marks)

Answer ONE question from each unit. (4X12=48 Marks)

1 Answer all questions (1X12=12 Marks)

a. List the applications of data acquisition system.
b. Distinguish between the data logging and supervisory control.
c. Define Observability.
d. Write the transfer function of zero order hold (ZOH).
e. Mention the applications of modified Z-transforms.
f. List the advantages of PID algorithm over other digital algorithms.
g. What are the steps involved in the configuration of DCS.
h. List the PLCs available in market by various venders.
i. What is meant by scan of PLC?
j. Define system identification.
k. List the steps involved in the design of an expert system.
l. Mention the applications of an adaptive controller.

UNIT I

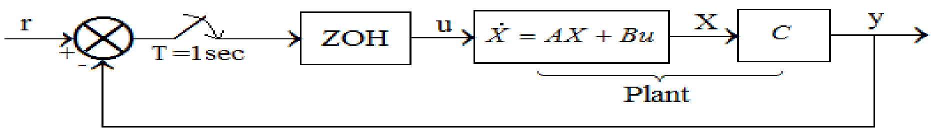
2 a) Illustrate the block diagram of direct digital control system with neat diagram. 8M

b) A computer must sequentially sample 100 process parameters. It requires 14 instructions at 5.3 µs / instruction for the computer to address and process one line of data. The multiplexer switching time is 2.3 µs, and the ADC Conversion time is 34 us. Find the maximum sampling rate for a line. 4M

(OR)

3 a) The block diagram of a computer control system is shown in fig. below. Obtain the discrete time state model of the system. 8M

$$\text{Given } A=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix};\ b=\begin{bmatrix}0\\1\end{bmatrix};\ c=\begin{bmatrix}1 & 0\end{bmatrix}$$



b) Convert the state space model of the system into transfer function model. 4M

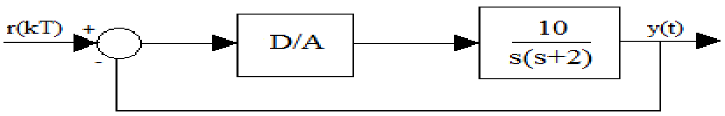
UNIT II

4 a) Formulate the Mathematical model of second order processes without and with pure delay in discrete time domain 6M

b) The characteristic equation of a linear digital system is $Z^3 - 0.1Z^2 + 0.2KZ - 0.1K = 0$ Using Jury stability criterion, solve the values of K > 0 for which the system is stable. 6M

(OR)

5 a) Consider the computer control system shown in fig. below. Determine unit step response of the system in discrete time domain. 6M



b) Design the dahlin's algorithm for the computer control system with plant transfer function $G_P(s) = e^{-0.8S}/(0.4s+1)$, sampling time T=0.5sec and time constant T_f = 0.15sec 6M

UNIT III

6 a) List any six applications of PLC in detail. 6M

b) What is meant by ladder diagram? Explain the basic operations of ladder diagram 6M

(OR)

7 a) Explain the structure of distributed control system in detail with neat diagram 8M

b) Write a short note on communication facilities of DCS 4M

UNIT IV

8 a) Identify the suitable adaptive control system used for non-deterministic systems work under varying operating conditions and illustrate its operation in detail. 8M

b) Explain the differences between AI and EDP machines 4M

(OR)

9 a) List the desirable characteristics of expert system. Explain the operation of an expert system with neat structure. 8M

b) Write a short notes about System identification 4M

Scheme (MAR-2019)

Q.No:1

(a) to (l) carries 12 marks, one mark for each question

Q.No:2

(a) Diagram -3M, Explanation-5M

(b) Calculation of maximum sampling rate for a line-4M

Q.No:3

(a) Determination of matrix ‘F’-5M, Determination of ‘G’-5M

(b) Conversion from state model to transfer function model-4M

Q.No:4

(a) Mathematical modelling of First order process without pure delay -3M, Mathematical modelling of First order process with pure delay -3M

(b) Formation of Jury’s table-3M, Calculation of K value-3M

Q.No:5

(a) Unit step response determination in Discrete-time domain-6M

(b) Calculation of $HG_P(z)$-3M, Dahlin’s controller transfer function-3M

Q.No:6

(a) Six PLC applications-2M, Explanation of each application-4M

(b) Ladder diagram-1M, Basic operations in ladder programming-4M

Q.No:7

(a) DCS architecture diagram-3M, Explanation of each block-5M

(b) Communication facilities of DCS-4M

Q.No:8

(a) Identification of suitable system-2M, Block diagram of MRAC -3M, Explanation-3M

(b) Differences between AI and EDP systems-4M

Q.No:9

(a) Expert system Characteristics-2M, Explanation with digrams-6M

(b) Short note on System identification-4M

Solutions (MAR-2019)

1. Answer all questions

(a) List the applications of data acquisition system:

- Data acquisition involves gathering signals from measurement sources and digitizing the signals for storage, analysis, and presentation on a PC.
- Data acquisition systems (i.e. DAS or DAQ) convert analog waveforms into digital values for processing.

(b) Distinguish between the data logging and supervisory control.

Data Logger:

- Data loggers are electronic devices which automatically monitor and record environmental parameters over time, allowing conditions to be measured, documented, analyzed and validated.
- The data logger contains a sensor to receive the information and a computer chip to store it. Then the information stored in the data logger is transferred to a computer for analysis.

Supervisory control:

- It is a natural extension of a computer data-logging system involves computer feedback on the process through automatic adjustment of loop set points.

(c) Observability:

Observability: The system is said to be observable if every initial state i.e. x (0)

can be determined from observations of y(k) over a finite number of sampling periods

The necessary and sufficient condition for the system to be completely observable is that

n x n observability matrix (V) has rank equal to n

i.e. $\rho(V) = n; \quad V = \begin{bmatrix} C \\ CF \\ CF^2 \\ \vdots \\ CF^{n-1} \end{bmatrix}$

(d) The transfer function of zero order hold (ZOH).

$$G_{ho}(s) = \frac{1-e^{-Ts}}{s}$$

(e) The applications of modified Z-transforms:

- To obtain the values of the response between sampling instants
- To analyze sampled data control systems containing transportation lag. i.e. dead-time

(f) Advantages of PID algorithm over other digital algorithms:

- Bump less transfer
- No integral wind up

(g) Steps involved in the configuration of DCS:

Configuration of DCS consists of two Steps:
Step-1: Configuration of operating system
Step-2: Configuration of controller functions

(h) PLCs available in market by various venders :

- Siemens
- GE
- ABB
- Festo… etc.

(i) Scan of PLC:

- The process of reading the inputs, executing the control application program and updating the output is known as PLC SCAN

(j) System identification:

- System identification is a methodology for building mathematical models of dynamic systems using measurements of the system's input and output signals.
- It is needed to study the behavior of the system mathematically i.e. quantitatively

(k) Steps involved in the design of an expert system:

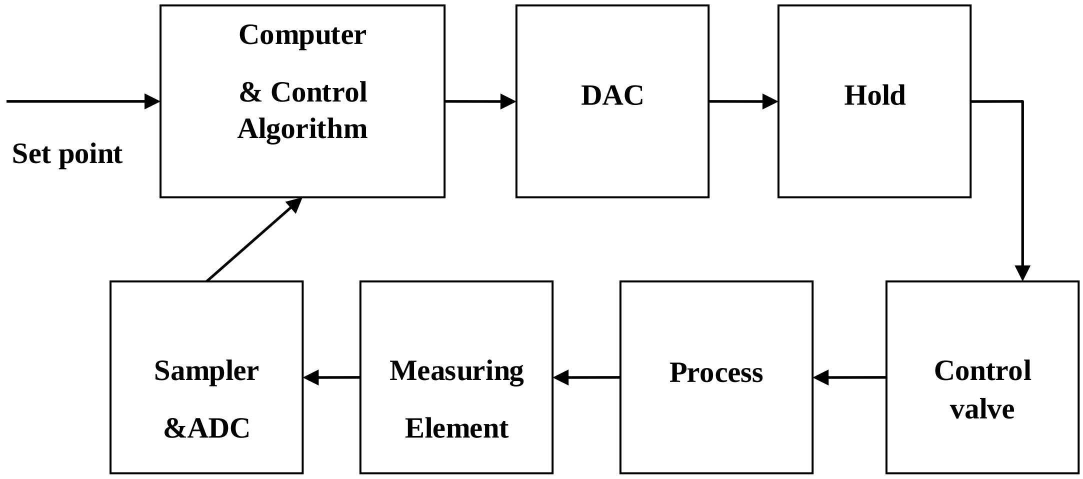
- Knowledge acquisition
- Knowledge representation
- Interface strategy

(l) Applications of an adaptive controller:

- Self-tuning of subsequently fixed linear controllers during the implementation phase for one operating point.
- Self-tuning of subsequently fixed robust controllers during the implementation phase for whole range of operating points.
- Self-tuning of fixed controllers on request if the process behavior changes due to ageing, drift, wear, etc.;
- Adaptive control of linear controllers for nonlinear or time-varying processes.
- Adaptive control or self-tuning control of nonlinear controllers for nonlinear processes.
- Adaptive control or self-tuning control of multivariable controllers for multivariable processes (MIMO systems).

UNIT-I

2. (a) Illustrate the block diagram of Computer control system with neat diagram. 8M



- The computer receives the measurement from the process at discrete times and Gives control signal value also at discrete –times
- We need a Hardware element DAC between digital computer and process
- Sampler produces a sequence of sampled values at particular time instants
- An ADC converts the value in to digital value
- The control commands produced by the computer program should be converted from discrete-time to continuous- time signals. This is achieved by ZOH

(b) A computer must sequentially sample 100 process parameters. It requires 14 instructions at 5.3 µs / instruction for the computer to address and process one line of data. The multiplexer switching time is 2.3 µs, and the ADC Conversion time is 34 us. Find the maximum sampling rate for a line. 4M

14 instructions require a time of 5.3µsecx14=74.2µsec and this must be done for 100 parameters.

Thus the total instruction time is 100(74.2) =7420 µsec

The ADC converts in 34 µsec, so that for 100 conversions we have (100) (34) =3400 µsec

The total time spent in MUX switching is (100) (2.3) = 230 µsec

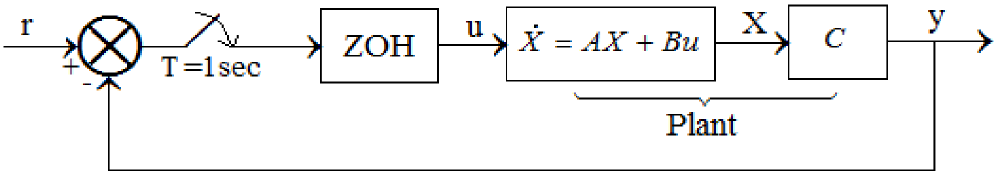
Therefore the total time (T) = 7420+3400+230= 11050 µsec

Now the sampling rate for line = $\frac{1}{T} = \frac{10^6}{11050} = 90.5$

(OR)

3. (a) The block diagram of a computer control system is shown in Fig.1.Obtain the discrete time state model of the system.

Given $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix};\ b = \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ c = \begin{bmatrix} 1 & 0 \end{bmatrix}$



8M

Fig.1

State Model is combination of State and output equations. For continuous-time system the state model is given by:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad ...... (1)$$
$$y(t) = Cx(t) + D.u(t) \ .... \ (2)$$

State Model is combination of State and output equations. For Discrete-time system the state model is given by:

$$x(k+1) = Fx(k) + gu(k) \ ...... \quad (3)$$
$$y(k) = Cx(k) + D.u(k) \ .... \quad (4)$$

$$F = e^{AT} \big|\ at\ T = 1\sec$$

$$g = \int_0^T e^{A\lambda} b d\lambda \ \big| at\ T = 1\sec$$

Here $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} b = \begin{bmatrix} 0 \\ 1 \end{bmatrix} C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

Computation of e^{At}

$$\lambda I - A = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} \lambda & -1 \\ 2 & \lambda+3 \end{bmatrix}$$

$$|\lambda I - A| = 0 \Rightarrow \lambda^2 + 3\lambda + 2 = 0$$

The roots of the above equation are called Eigen values, they are $\lambda_1 = -1\ and\ \lambda_2 = -2$

$$f(\lambda) = g(\lambda) = \beta_0 + \beta_1 \lambda$$

Here $f(\lambda) = e^{\lambda t}$

$$e^{-t} = \beta_0 - \beta_1$$
$$e^{-2t} = \beta_0 - 2\beta_1$$

Solving these two equations we get

$$\beta_0 = 2e^{-t} - e^{-2t}$$
$$\beta_1 = e^{-t} - e^{-2t}$$

Now $e^{At} = \beta_0 I + \beta_1 A = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ 2e^{-2t} - 2e^{-t} & 2e^{-2t} - e^{-t} \end{bmatrix}$

Computation of F:

$$F = e^{AT} = e^{At} \,|\, at\ t = T = 1\sec = \begin{bmatrix} 0.6 & 0.236 \\ -0.4651 & -0.09720 \end{bmatrix}$$

Here $e^{A\lambda} = e^{At}|at\ t = \lambda$

Therefore $e^{A\lambda} = \begin{bmatrix} 2e^{-\lambda} - e^{-2\lambda} & e^{-\lambda} - e^{-2\lambda} \\ 2e^{-2\lambda} - 2e^{-\lambda} & 2e^{-2\lambda} - e^{-\lambda} \end{bmatrix}$

Now $e^{A\lambda}b = \begin{bmatrix} 2e^{-\lambda} - e^{-2\lambda} & e^{-\lambda} - e^{-2\lambda} \\ 2e^{-2\lambda} - 2e^{-\lambda} & 2e^{-2\lambda} - e^{-\lambda} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2e^{-\lambda} - e^{-2\lambda} \\ 2e^{-2\lambda} - e^{-\lambda} \end{bmatrix}$

Computation of g:

$$g = \int_0^T e^{A\lambda} b\, d\lambda \,|at\ T = 1\sec$$

$$= \begin{bmatrix} \int_0^1 (2e^{-\lambda} - e^{-2\lambda})d\lambda \\ \int_0^1 (2e^{-2\lambda} - e^{-\lambda})d\lambda \end{bmatrix} = \begin{bmatrix} 0.2002 \\ 0.2325 \end{bmatrix}$$

(b) Convert the state space model into transfer function model 4M

State Model is combination of State and output equations. For continuous-time system the state model is given by:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad \ldots\ldots (1)$$
$$y(t) = Cx(t) + D.u(t) \ldots\ldots\ldots (2)$$

State Model is combination of State and output equations. For Discrete-time system the state model is given by:

$$x(k+1) = Fx(k) + gu(k) \ldots\ldots \quad (3)$$
$$y(k) = Cx(k) + D.u(k) \ldots\ldots\ldots\ldots \quad (4)$$

For continuous-time system we use Laplace transforms and for discrete-time system we use Z-transforms

For Continuous-time system state model:

Take Laplace transform on both sides for equations (1) & (2) we get the following expressions:

$$LT\{\dot{x}(t)\} = LT\{Ax(t) + Bu(t)\} \Rightarrow sX(s) = AX(s) + BU(s) \Rightarrow X(s) = (sI - A)^{-1}BU(s)$$
$$LT\{y(t)\} = LT\{Cx(t) + D.u(t)\} \Rightarrow Y(s) = CX(s) + DU(s) = C(sI - A)^{-1}BU(s) \quad + DU(s)$$
$$\Rightarrow \frac{Y(s)}{U(s)} = transfer\ \ function = C(sI - A)^{-1}B \quad + D$$

(or)

For Discrete-time system state model:

Take the Z-transform on both sides for equations (3) & (4) we get the following expressions:

$$Z\{x(k+1)\} = Z\{Fx(k) + gu(k)\} \Rightarrow zX(z) = FX(z) + gU(z) \Rightarrow X(z) = (zI - F)^{-1} gU(z)$$
$$Z\{y(k)\} = Z\{Cx(k) + D.u(k)\} \Rightarrow CX(z) + DU(z) \Rightarrow C(zI - F)^{-1} gU(z) + DU(z)$$
$$\Rightarrow \frac{Y(z)}{U(z)} = C(zI - F)^{-1} g + D$$

UNIT-II

4. (a) Formulate the Mathematical model of first order processes without and with pure delay in discrete time domain **6M**

The transfer function for the **first order with delay** continuous-time system is given by the expression

$$\frac{Y(s)}{M(s)} = \frac{ke^{-t_d s}}{\tau s + 1} \ldots\ldots(1)$$

The equation (1) is rearranged as follows

$$\frac{Y(s)}{M(s)} = \frac{ke^{-t_d s}}{\tau s + 1} \Rightarrow Y(s) + \tau s Y(s) = ke^{-t_d s} M(s) \ldots\ldots(2)$$

Take the inverse Laplace transform for the equation (2)

$$L^{-1}\{Y(s) + \tau s Y(s)\} = L^{-1}\{ke^{-t_d s} M(s)\} \Rightarrow y(t) + \tau \frac{dy}{dt} = km(t - t_d) \ldots\ldots(3)$$

Equation (3) is the differential equation that describes the continuous-time I order system with Delay

Conversion of Continuous-time system into Discrete-time system:

$$y(t) \to y(n)$$
$$\frac{dy(t)}{dt} \to \frac{1}{T}[y(n) - y(n-1)]$$
$$m(t - t_d) \to m(n - n_o)$$

With the above replacements equation (3) becomes:

$$y(n) + \tau \frac{1}{T}[y(n) - y(n-1)] = km(n - n_o) \Rightarrow y(n) = \frac{\tau}{T + \tau} y(n-1) + \frac{kT}{T + \tau} m(n - n_o)$$
$$\Rightarrow y(n) = y(n-1) + \frac{kT}{\tau} m(n - n_o); \quad T <<<< \tau$$

The transfer function for the **first order without delay** continuous-time system is given by

the expression

$$\frac{Y(s)}{M(s)} = \frac{k}{\tau s + 1} \ldots\ldots(1)$$

The equation (1) is rearranged as follows

$$\frac{Y(s)}{M(s)} = \frac{k}{\tau s+1} \Rightarrow Y(s)+\tau sY(s) = kM(s)\ldots\ldots(2)$$

Take the inverse Laplace transform for the equation (2)

$$L^{-1}\{Y(s)+\tau sY(s)\} = L^{-1}\{kM(s)\} \Rightarrow y(t)+\tau\frac{dy}{dt} = km(t)\ldots\ldots(3)$$

Equation (3) is the differential equation that describes the continuous-time I order system

Conversion of Continuous-time system into Discrete-time system:

$$y(t) \to y(n)$$
$$\frac{dy(t)}{dt} \to \frac{1}{T}\left[y(n)-y(n-1)\right]$$
$$m(t) \to m(n)$$

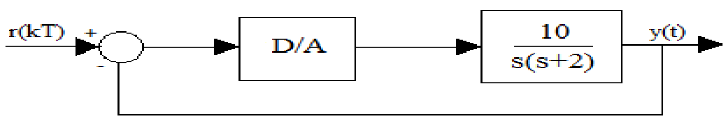
With the above replacements equation (3) becomes:

$$y(n)+\tau\frac{1}{T}\left[y(n)-y(n-1)\right] = km(n) \Rightarrow y(n) = \frac{\tau}{T+\tau}y(n-1)+\frac{kT}{T+\tau}m(n)$$
$$\Rightarrow y(n) = y(n-1)+\frac{kT}{\tau}m(n); \quad T <<<< \tau$$

(b) The characteristic equation of a linear digital system is $Z^3-0.1Z^2+0.2KZ-0.1K=0$ Using Jury stability criterion, solve the values of K > 0 for which the system is stable. 6M

(OR)

5. (a) Consider the computer control system shown in fig. below. Determine unit step response of the system in discrete time domain.



6M

5. (b) Design Dahlin's algorithm for the computer control system with plant transfer function

$G_P(s) = \frac{e^{-0.8S}}{(0.4s+1)}$ **and sampling time T=0.5sec, Time constant T_f = 0.15sec**

UNIT-III

6. (a) List any six applications of PLC in detail. **6M**

- ✓ PLC's are used in many industries for one or more of the functional areas.
- ✓ Sequence control, timing, counting, and data calculation.
- ✓ Quick change of machine or process logic, to manufacture different items using the same machine or process equipment.
- ✓ Auto compilation of production/consumption/Down time/Maintenance data.
- ✓ Batch or Continuous process control.
- ✓ Open loop or feedback control, process data acquisition and display
- ✓ Precise motion/Position control

- ❖ **Tyre manufacture:** Controlling sequence of events such as:
  - Serving plies in the right order
  - Rotary movements of drum
  - Multiplanary movements in the spindles
- ❖ **Tyre Curing Press:**
  - PLC's provides correct sequencing of the curing cycle.
  - Time measurement&control in each cycle.
  - Monitor parameters such as temperature, pressure during a curing cycle.
  - Used for controlling the processes like mixing of the raw rubber with carbon black, oil and other chemicals.
- ❖ **Plastic Injection moulding:**
  - Control of variables such as temperature and pressure.
  - Control the velocity levels of injection to maintain consistent filling thus reducing surface defects shortening cycle time.
- ❖ **Chemical Batching:**
  - PLC's are used to control the batching ratios of various ingredients used in a process
  - To determine the rate of discharge of each ingredient.
- ❖ **Ammonia&Ethylene Processing:**
  - PLC's are used to control and monitor compressors which are used in manufacturing process of Ammonia and ethylene, and other chemical
  - To measure qualitative parameters like temperature, power consumption, suction flow…etc
- ❖ **Pulp batch blending:**
  - For sequence control and quantity measurement of ingredients, and storage of recipes for the blending process
- ❖ **Paper mill digester:**
  - Provides control of pulp digester for the process of making pulp from wood chips.
  - To calculate and control the amount of chips, based on density and the digester volume
  - To control temperature till cooking is completed
- ❖ **Gypsum Board plant:**
  - Used for controlling three main sections of a gypsum board plant
  - Furnace section: Controls the furnace temperature for preparation of wet board
  - Wet & Transfer section: To control for partial drying of board & cutting to preset length.
  - Take off section: The board is dried completely in a hot air furnace controlled by a PLC.

(b) What is meant by ladder diagram? Explain the basic operations of ladder diagram **6M**

Ladder diagrams are specialized schematics commonly used to document industrial control logic systems. They are called "ladder" diagrams because they resemble a ladder, with two vertical rails (supply power) and as many "rungs" (horizontal lines) as there are control circuits to represent.

PLC Ladder programming

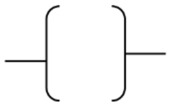
- It is one of the popular programming languages for PLC's.
- Ladder logic was based on the circuit diagrams used to run relay logic hardware. The term stems from the look of the programming language and its resemblance to ladders, with two vertical rails and series of horizontal rungs between them.
- It consists of symbols for specific functions.
- This approach consists several Ladder instructions for various operations

Basic Ladder Instructions:

1. Normally open contact:

2. Normally closed contact:

3. Turn on output:



4. Turn off output

(OR)

7. (a) Explain the structure of distributed control system in detail with neat diagram 8M

DCS is Layered structure. Here each layer consists of group of tasks to be performed on the lower layer by getting instructions from the higher level.

Level-1:

- It is known as Field level
- It consists of specially designed dedicated controllers.
- The functions of this level are: Emergency shutdown, Provides manual backup when other layers are failed

Level-2:

It is area control level. It consists of a direct digital control link. Functions of this Level are:

- Process data collection
- Message for alarm signals
- Open loop and Closed loop control
- Hardware test

Level-3:

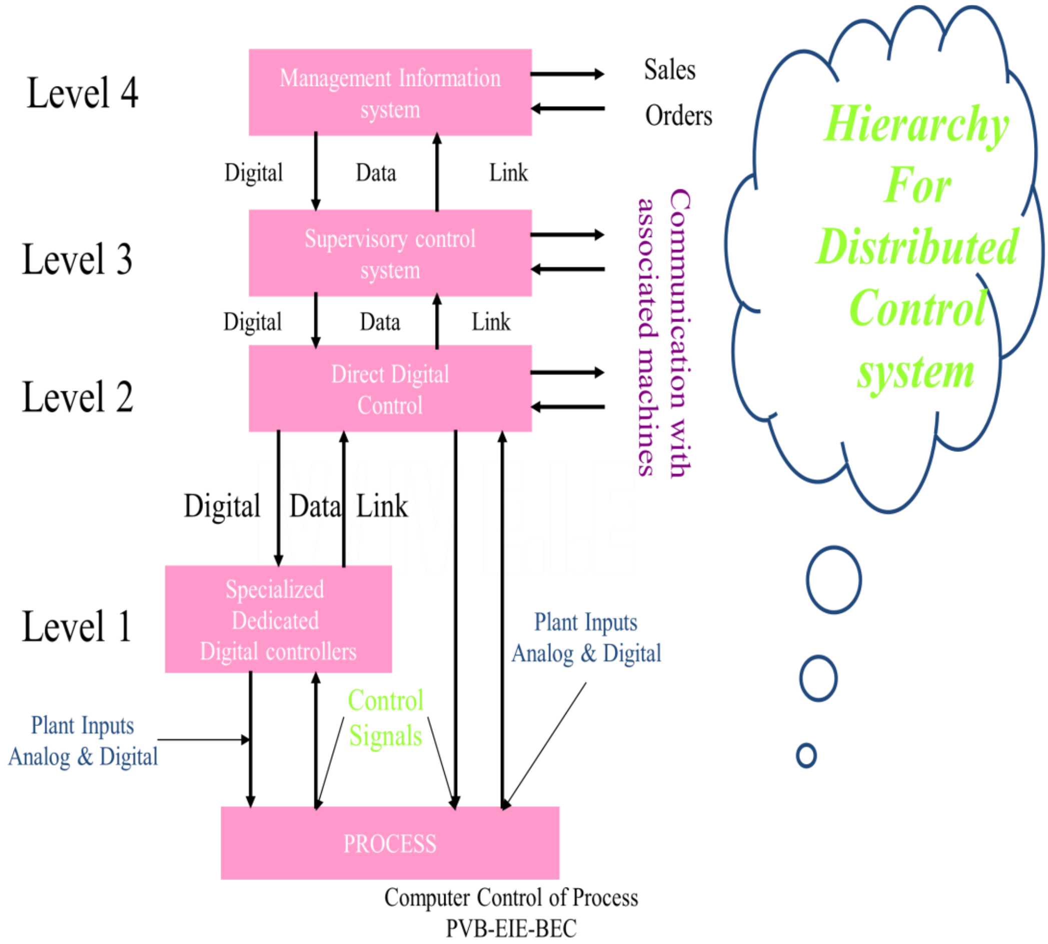
It is known as plant control level. It consists of supervisory control link. Functions of this level are:

- Optimal control
- Adaptive control

Level-4:

It is known as plant management level. Functions of this level are:

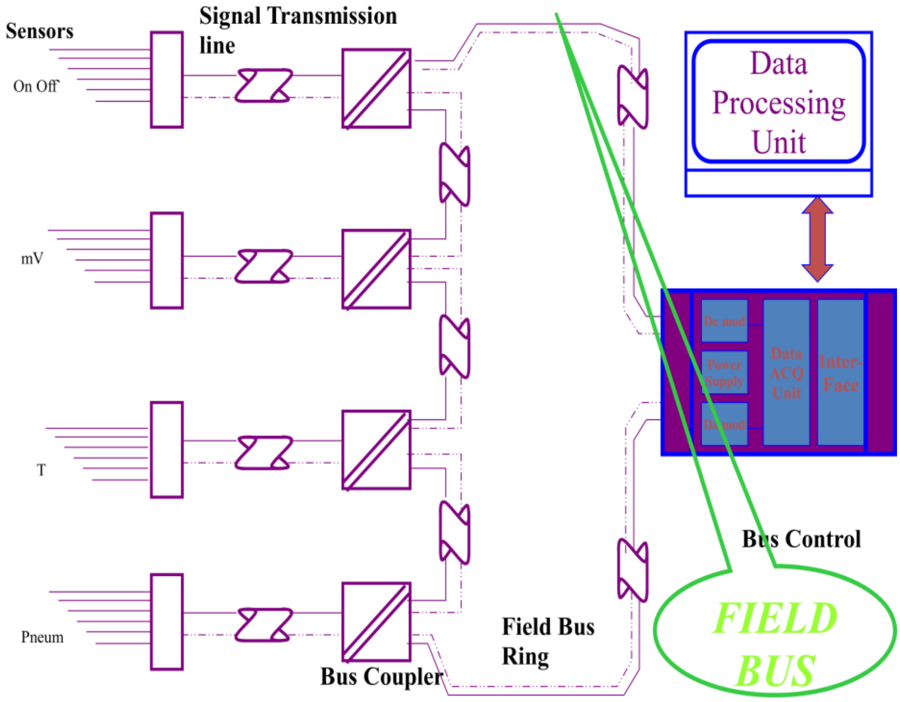
- Resource scheduling
- Production planning
- Management functions related to process or production management.



(b) Write a short note on communication facilities of DCS **4M**

Communication facilities:

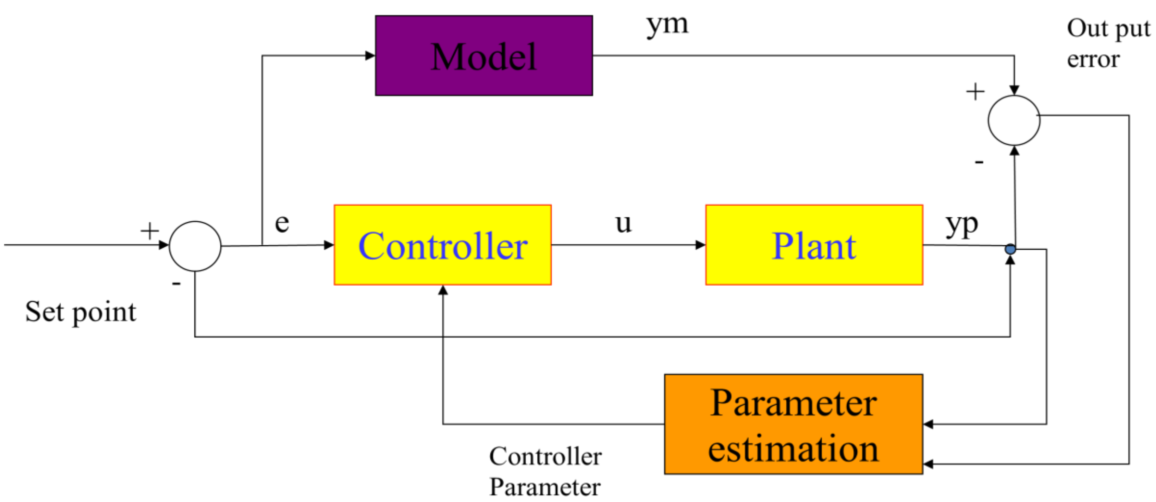
- The communication paths are essential for DCS. The path must be implemented from top to the bottom of the system, to transfer any necessary control action down to the individual final elements or to collect the actual information from them
- Short and long distance buses will be used in a combination to cover a big area of distributed plants like steel, chemical, mining industry
- For short distances, the star connection is applicable which usually combined with bus and point-point connection will result in to a mesh network.
- At level-1 usually point-point connection will be applied to connect individual sensors and actuators to analog and digital inputs and outputs of the local field station.
- Recent advance for this is PROFIBUS (process field bus).
- For shorter distances an instrumentation bus IEEE-488 can be applied.



UNIT-IV

8. (a) Identify the suitable adaptive control system used for deterministic systems work under varying operating conditions and illustrate its operation in detail **8M**

- The system suitable for deterministic systems work under operating conditions is 'Model Reference adaptive control system (MRAC)
- Block diagram of MRAC is shown in figure below:



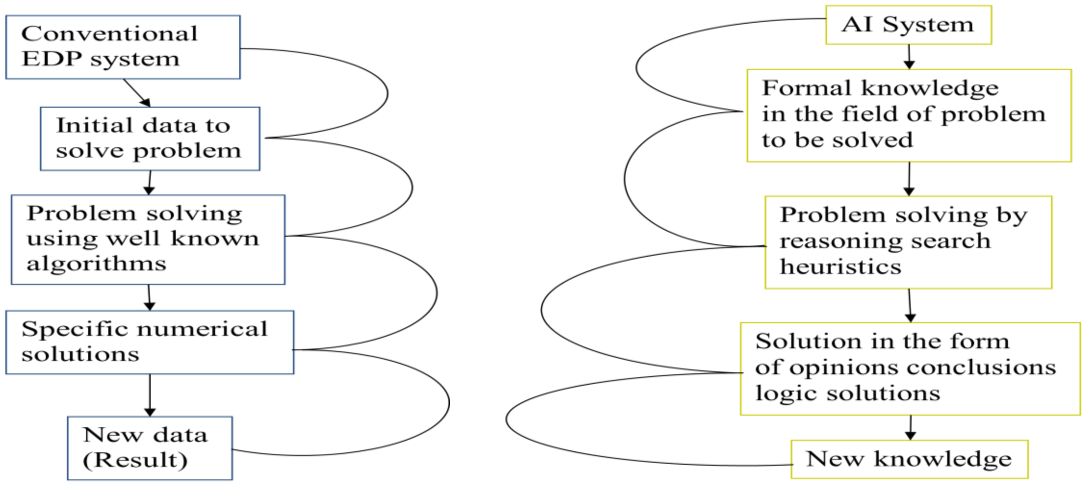
- Model reference adaptive controller has a system model to which all inputs and known perturbations are applied in parallel to the actual system.
- The output of reference model (y_m) and output of system (y_n) are compared.
- The adaptation mechanism then calculates the controller parameters to maximize the performance of index.
- The following method are used for parameter estimation
    - ➢ Least square method
    - ➢ Method of instrumental variables
    - ➢ Maximum likelihood method
    - ➢ Stochastic approximation method

(b) Explain the differences between AI and EDP machines **4M**

Artificial intelligence:

May be defined as the branch of computer science which deals with the software and hardware techniques to solve symbolic problems as against the problems solved by EDP machines.

Difference in the working of conventional EDP system and AI system:

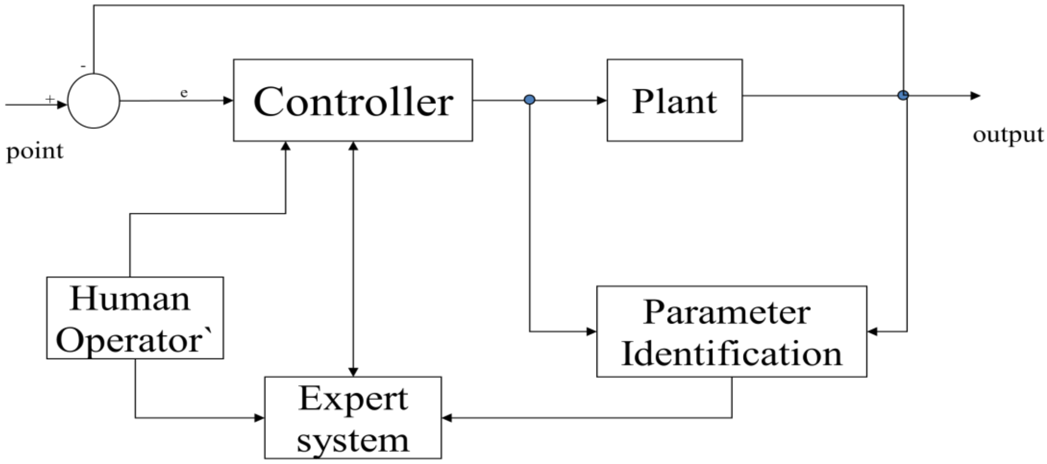


9. (a) List the desirable characteristics of expert system. Explain the operation of an expert control system with neat block diagram to control time varying processes with variety of disturbances **8M**

Desirable characteristics of expert system:

- Knowledge update capability
- Flexible problem solving
- Reasoning and explanation capability

Expert system Controller:



Expert system should satisfy the following to use it in process control:

- Ability to control a large class of processes which may be time varying, non-linear, with variety of disturbances.
- Requirement of minimum prior knowledge about the process.
- Ability to improve its performance with time as it is required more and more knowledge
- Ability to monitor the performance of the system and detect problems with sensors actuators, and other components
- In addition to main feedback loop, there is an outer loop involving parameter identification and expert system which performs the tasks of supervision and controller design
- The controller may be simple PID controller or one of the advanced controllers

(b) Write a short notes about System identification **4M**

System Identification:

The determination of a mathematical model of a system is known as system identification.Mathematical models deals with Linear Time invariant Single input Single output discrete time System is described by difference equation.

The proposed method is for estimating he coefficients of difference equations

- A set of “N” linear algebraic equations are formulated “N” is the number of measurements.
- Derive the canonical equation whose solution yields the Parameter estimate of θ (N) ϴ is the vector parameter under identification
- Estimate of initial condition of the dynamic equation is required.
- (N+n measurements are taken and N+n equations are formed where n is the order of the difference equation).

Example:

Consider the simple first order system

$y(k) + a_1 y(k-1) = b_1 u(k-1)$

Definition of the problem:

For the given input sequence u(-1) , u(0) , u(1), u(2) , -----, u(n-1)

The output is y(0), y(1), y(2), …. , y(n)

Find out the parameter a_1 and b_1 and the initial condition y(-1)

Step: 1

$y(0) + a_1 y(-1)= b_1 u(-1)$

$y(1) + a_1 y(0) = b_1 u(0)$

$Y(2) + a_1 y(1) = b_1 u(1)$

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$y(N) + a_1 y(N-1) = b_1 u(N-1)$

The equation 1 is used to determine the initial condition and remaining equations are used to estimate the unknown a_1 and b_1.

Representing the above equations in matrix form

$$y = \Phi\theta ----3$$

$$y = \begin{bmatrix} y(1) \\ y(2) \\ . \\ . \\ y(N) \end{bmatrix} \& \theta = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix},,,,\Phi = \begin{bmatrix} -y(0) & u(0) \\ -y(1) & u(1) \\ & \\ & \\ -y(N+1) & u(N-1) \end{bmatrix}$$

The equation represents the set of N linear algebraic equations for N measurements with unknowns a_1 and b_1 .

Suppose an error occurs while taking the measurements, then input and output relation becomes
Y = Φθ+e   error = e=y- Φθ
Cost function (J):
$J = e^T e$
Where e is the N dimensional error vector

$$e = \begin{bmatrix} e(1) \\ e(2) \\ e(3) \\ . \\ e(N) \end{bmatrix} \qquad e^T = [e(1) \quad e(2) \quad . \quad . \quad e(N)]$$

$$J = [e^2(1) + e^2(2) + e^2(3) + ...+ e^2(N)] \qquad J = \sum_{k=1}^{N} e^2(k)$$

Step: 2

Determination of canonical form of the equation:

We know that e = y-Φθ
$J = (y - \Phi\theta)^T (y-\Phi\theta)$
$(y - \Phi\theta)^T = (y-\Phi\theta)$ (y-Φθ) is a similar matrix
$J = (y - \Phi\theta)^2$

$$\frac{\partial J}{\partial \theta} = 2(y - \Phi\theta)(-\Phi)$$
$$= -2\Phi^T (y - \Phi\theta)$$
$$\frac{\partial}{\partial \theta}(A\theta) = \frac{\partial}{\partial \theta}(\theta^T \Phi^T) = A^T$$
$$\frac{\partial J}{\partial \theta} = 0 \Rightarrow \Phi^T y = \Phi^T \Phi\theta (canonocal \quad form)$$

Step: 3

To determine the solution of canonical form of equation:

$$\theta = (\theta^T \Phi)^{-1} \Phi^T y$$

$$\Phi^T \Phi = \begin{bmatrix} \sum_{k=0}^{N-1} y^2(k) & -\sum_{k=0}^{N-1} y(k)u(k) \\ -\sum_{k=0}^{N-1} y(k)u(k) & \sum_{k=0}^{N-1} u^2(k) \end{bmatrix}$$

$$\Phi^T y = \begin{bmatrix} -\sum_{k=1}^{N} y(k)y(k-1) \\ \sum_{k=1}^{N} y(k)u(k-1) \end{bmatrix}$$

Step: 4

Initial condition estimation:

From (1) we have got

$y(0) + a_1 y(-1) = b_1 u(-1)$

$y(-1) = [b_1 u(-1) – y(0)] / a_1$

Where it is assumed that condition $a_1 \neq 0$ & u(-1) is a known