3/4 B.Tech I Sem Regular examination Nov-2019

Electromagnetic waves and transmission lines-14EC504

Scheme of valuation

14EC504

Hall Ticket Number:

III/IV B.Tech (Regular\Supplementary) DEGREE EXAMINATION

November, 2019 — **Electronics and Communication Engineering**

Fifth Semester — **EM Waves & Transmission lines**

Time: Three Hours — **Maximum:** 60 Marks

Answer Question No.1 compulsorily. (1X12 = 12 Marks)

Answer ONE question from each unit. (4X12=48 Marks)

1. Answer all questions (1X12=12 Marks)
   a) Define transmission coefficient and write an expression for it.
   b) Define Brewester's angle.
   c) Define Snell's law for reflection.
   d) List the applications of transmission line.
   e) Define and Express the term input impedance.
   f) Define standing wave ratio.
   g) What is meant by impedance mismatch?
   h) List out all the parameters which we can acquire from Smith chart?
   i) What is the difference between short and long pulses?
   j) What are the possible modes for TM in circular wave guide?
   k) Why circular wave guides are not preferred over rectangular wave guides?
   l) Give the characteristics of TEM waves.

UNIT I

2. a) Define and write a short note on plane of incidence, perpendicular polarization and parallel polarization. 6M
   b) What is reflection and transmission .Explain with layered materials at normal incidence? 6M

(OR)

3. a) What is normal incidence? Obtain the expressions for reflection coefficient & transmission coefficient for dielectric interface under Normal Incidence. 6M
   b) Derive an expression for brewester's angle 6M

UNIT II

4. a) Briefly explain: i) Standing wave ratio ii) loss-less terminated transmission line iii) loss-less resistively loaded transmission line 6M
   b) A lossless transmission line of length 100m has an inductance of 28 μH and a capacitance of 20 nF. Find propagation velocity, phase constant at an operating frequency of 100 kHz and characteristic impedance of the line. 6M

(OR)

5. a) Derive a relation between reflection coefficient and characteristic impedance. 6M
   b) The velocity of propagation on a certain lossless transmission line is 25 m/μs. If C= 30 pF/m, find inductance, characteristic Impedance, phase constant at 100 MHz and reflection co-efficient if line is terminated by 50Ω resistor. 6M

UNIT III

6. a) Explain the principle of Impedance matching with quarter wave transformer. 6M
   b) Explain the constructional features of the Smith Chart. 6M

(OR)

7. a) Distinguish between short and long pulses. What is the important measure in discontinuities? 6M
   b) Explain the effect of propagating the narrow pulses on finite distortion less transmission lines. 6M

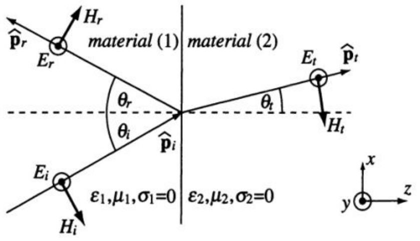
UNIT IV

8. a) Derive the expression for guided wavelength of TM_{mn} mode in rectangular wave guide. 6M
   b) An air-filled rectangular wave guide has cross section 6 cm X 4 cm calculate its cutoff frequency of the dominant mode, wave length, intrinsic Impedance. 6M

(OR)

9. a) Derive the expressions for field components of TM waves between parallel plates. 6M
   b) Compare TE and TM mode of Propagation inside a rectangular Waveguides. 6M

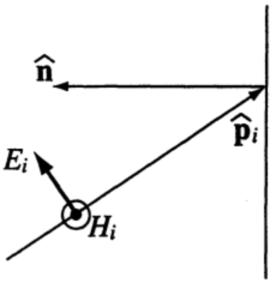
1.

a)	Transmission Coefficient: Ratio of amplitudes of the transmitted and incident waves. $$T = \frac{E_t}{E_i}$$
b)	Brewester's angle: the angle of incidence for which the reflection coefficient is zero.
c)	Snell's law for reflection:  $$\frac{sin\theta_t}{sin\theta_i} = \frac{n_1}{n_2}$$
d)	Applications of transmission line: 1. Low frequency radio wave transmission. 2. Antenna matching to the source. 3. Transmission of analog and digital information.
e)	Input Impedance: the impedance at the input of the transmission line is known as input impedance. $$Z_{in} = Z(l) at\ l = 0$$
f)	Standing Wave Ratio: ratio between the maximum and minimum voltage is called standing wave ratio.
g)	The load impedance is not equal to line characteristic impedance is known as impedance mismatch.
h)	1. Reflection coefficient. 2. Line impedance. 3. Standing Wave Ration. 4. Maximum and minimum voltage locations
i)	The pulse whose width is far less than the propagation time is known as short pulse and the pulse whose width is far greater than the propagation time is known as long pulse.
j)	Possible modes for TM in circular waveguide are $TM_{11}, TM_{21}, TM_{02}$
k)	In circular waveguide the frequency difference between the lowest frequency on the dominant mode and next mode is smaller than in a rectangular waveguide.
l)	Transverse Electro Magnetic waves are those in which the electric and magnetic fields are transverse to the wave propagation that is $E_z = H_z = 0$ for wave propagating in Z-direction.

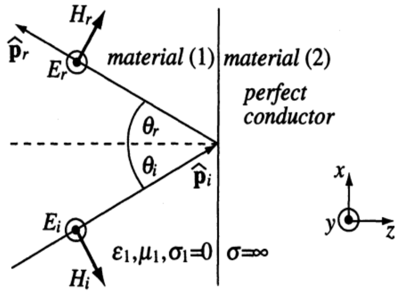
UNIT-1

2.

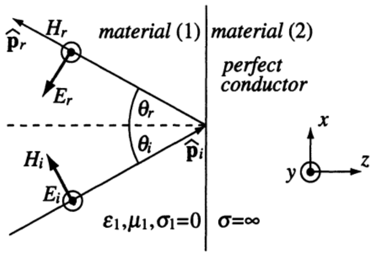
a)



Plane of incidence



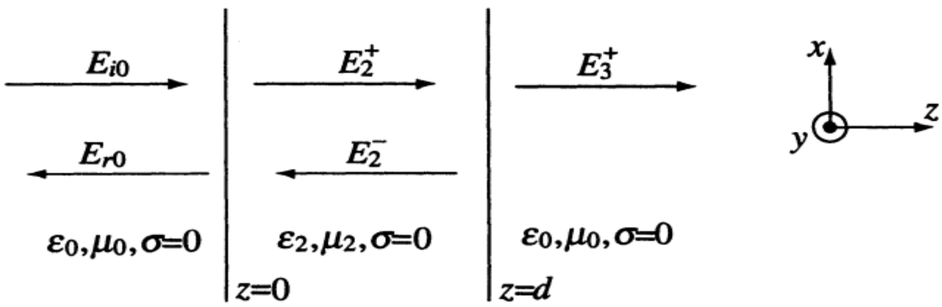
Perpendicular polarization



Parallel Polarization

Plane of Incidence: the plane described by the direction of propagation of the incident wave and normal to the surface of interface.
Perpendicular Polarization: for an incident EM wave if electric field is perpendicular to the plane of incidence is known as perpendicular polarization.
Parallel Polarization: for an incident EM wave if electric field is parallel to the plane of incidence is known as parallel polarization.

b)



Electric and field components in material1 are

$$E_1 = \hat{x}\left[E_{i0}e^{-j\beta_0 z} + E_{r0}e^{j\beta_0 z}\right]$$
$$H_1 = \frac{\hat{y}}{\eta_0}\left[E_{i0}e^{-j\beta_0 z} - E_{r0}e^{j\beta_0 z}\right]$$

Electric and field components in material2 are

$$E_2 = \hat{x}\left[E_2^+ e^{-(\alpha_2+j\beta_2)z} + E_2^- e^{(\alpha_2+j\beta_2)z}\right]$$
$$H_2 = \frac{\hat{y}}{\eta_0}\left[E_2^+ e^{-(\alpha_2+j\beta_2)z} - E_2^- e^{(\alpha_2+j\beta_2)z}\right]$$

Electric and field components in material3 are

$$E_3 = \hat{x}\left[E_3^+ e^{-j\beta_0 z}\right]$$
$$H_3 = \frac{\hat{y}}{\eta_0}\left[E_3^+ e^{-j\beta_0 z}\right]$$

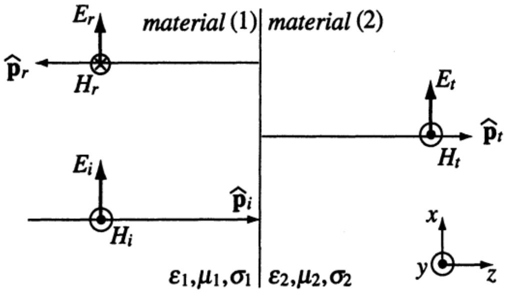
Total reflection coefficient is

$$\boxed{\Gamma_{slab} = \frac{E_{r0}}{E_{i0}} = -\frac{j\left[\eta_0^2 - \eta_2^2\right]\tan(\beta_2 - j\alpha_2)d}{2\eta_0\eta_2 + j\left[\eta_0^2 + \eta_2^2\right]\tan(\beta_2 - j\alpha_2)d}}$$

Total transmission coefficient is

$$\boxed{T_{slab} = \frac{E_3^+}{E_{i0}} = \frac{2\eta_0\eta_2 e^{\alpha_2 d}e^{j\beta_0 d}}{2\eta_0\eta_2\cos(\beta_2 - j\alpha_2)d + j\left[\eta_0^2 + \eta_2^2\right]\sin(\beta_2 - j\alpha_2)d}}$$

3.

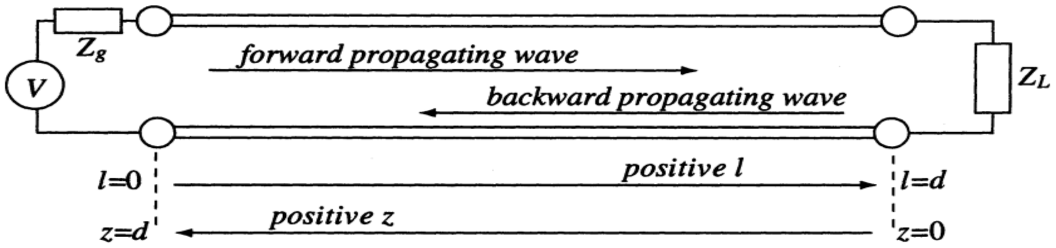
a)	When an EM wave propagating in a dielectric medium incident on interference normal is known as normal incidence. The propagation of EM wave is in normal to the interference.  Incident electric field is $$E_i = \hat{\boldsymbol{x}} E_{i1} e^{-\gamma_1 z}$$ The reflected electric field in material 1 is $$E_r = \hat{\boldsymbol{x}} E_{r1} e^{+\gamma_1 z}$$ Total electric field is medium 1 is $$\boldsymbol{E}_1 = \hat{\boldsymbol{x}} E_{i1} e^{-\gamma_1 z} + \hat{\boldsymbol{x}} E_{r1} e^{+\gamma_1 z}$$ Total magnetic field in medium 1 is $$\boldsymbol{H}_1 = \hat{\boldsymbol{y}} \frac{E_{i1}}{\eta_1} e^{-\gamma_1 z} - \hat{\boldsymbol{y}} \frac{E_{r1}}{\eta_1} e^{+\gamma_1 z}$$ Total electric and magnetic field in medium 2 is $$\boldsymbol{E}_t = \boldsymbol{E}_2 = \hat{\boldsymbol{x}} E_2 e^{-\gamma_2 z}$$ $$\boldsymbol{H}_t = \boldsymbol{H}_2 = \hat{\boldsymbol{y}} \frac{E_2}{\eta_2} e^{-\gamma_2 z}$$ Reflection coefficient $$\Gamma = \frac{E_{r1}}{E_{i1}}$$ $$\Gamma = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}$$ Transmission coefficient $$\mathrm{T} = \frac{E_{t2}}{E_{i1}}$$ $$\mathrm{T} = \frac{2\eta_2}{\eta_1 + \eta_2}$$
b)	Brewester's angle: the angle of incidence for which the reflection coefficient is zero. $$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}, \qquad \Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$ Brewester's angle for parallel polarization: $$\boxed{\theta_b = \sin^{-1}\left[\sqrt{\frac{\varepsilon_2}{\mu_1}\frac{(\mu_2\varepsilon_1 - \mu_1\varepsilon_2)}{(\varepsilon_1^2 - \varepsilon_2^2)}}\right]}$$ $$\boxed{\sin\theta_b = \sqrt{\frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}} \quad \text{or} \quad \theta_b = \sin^{-1}\left[\sqrt{\frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}}\right] \quad \text{if } \mu_1 = \mu_2}$$ Brewester's angle for perpendicular polarization: $$\boxed{\theta_b = \sin^{-1}\left[\sqrt{\frac{\mu_2(\varepsilon_1\mu_2 - \varepsilon_2\mu_1)}{\varepsilon_1(\mu_2^2 - \mu_1^2)}}\right]}$$ $$\boxed{\theta_b = \sin^{-1}\left[\frac{\mu_2}{\mu_2 + \mu_1}\right] \quad \text{if } \varepsilon_1 = \varepsilon_2}$$

4.

a)	i) Standing Wave ratio: ratio between the maximum and minimum voltage is called standing wave ratio. ii) Loss-less terminated transmission line: • R=0 and G=0 we leads to α=0. • Line is made of pure conductor. • Practically not existing only approximated line exist. • The field components propagate along line with speed dictated by L and C. $\gamma = j\beta = jw\sqrt{LC}[rad/m]$ $Z_0 = \sqrt{\frac{L}{C}}[\Omega]$ $\lambda = \frac{2\pi}{w\sqrt{LC}}[m]$ $v_p = \frac{1}{\sqrt{LC}}[m/s]$ iii) Loss-less resistively loaded transmission line: • R=0. • These lines are made of pure conductors. • The conducting nature of the line guides the wave but all the propagation parameters are effected by dielectric alone. • These equations can holds for any line therefore by knowing one parameters remaining can be measured. $\gamma = jw\sqrt{LC}\sqrt{1+\frac{G}{jwC}}$ $Z_0 = \sqrt{\frac{jwL}{G+jwC}}$ $LC = \mu\varepsilon, \frac{G}{C} = \frac{\sigma}{\varepsilon}$
b)	Given $l = 100m$ $L = 28\mu H$ $C = 20nF$ $f = 100M\ Hz$ Propagation velocity $v_p = \frac{1}{\sqrt{LC}} = 1.33 * 10^6\ m/s$ Phase constant $\beta = w\sqrt{LC} = 470\ rad/s$ Characteristic impedance $Z_0 = \sqrt{\frac{L}{C}} = 37.41\Omega$

5.

a)



$$\boxed{V(z) = V^+ e^{\gamma z} + V^- e^{-\gamma z}} \quad \text{and} \quad \boxed{I(z) = I^+ e^{\gamma z} + I^- e^{-\gamma z}}$$

The characteristic impedance of the line is:

$$Z_0 = \frac{V^+ e^{\gamma z}}{I^+ e^{\gamma z}} = \frac{V^+}{I^+} = -\frac{V^-}{I^-}$$

The load impedance of the line is:

$$Z_L = \frac{V(0)}{I(0)} = \frac{V^+ + V^-}{I^+ + I^-} = \frac{V^+ + V^-}{V^+/Z_0 - V^-/Z_0} = Z_0 \frac{V^+ + V^-}{V^+ - V^-}$$

Backward propagating wave amplitude is:

$$V^- = V^+ \frac{Z_L - Z_0}{Z_L + Z_0}$$

Load reflection coefficient in terms of characteristic impedance:

$$\boxed{\Gamma_L = \frac{V^-}{V^+} = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

b) Given

Propagation velocity $v_p = 25 m/\mu s$

$$C = 30 \ pF/m$$
$$f = 100M\ Hz$$
$$Z_L = 50\Omega$$

Inductance $L = {}^{1}\!\big/_{(v_p^2 C)} = 53\mu H$

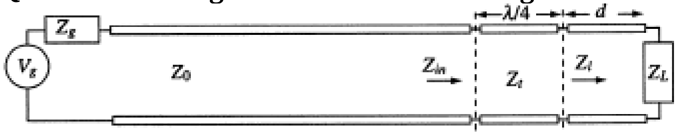
Characteristic impedance $Z_0 = \sqrt{\frac{L}{C}} = 1.3K\Omega$

Phase constant $\beta = w\sqrt{LC} = 25 \ \frac{rad}{s}$

Reflection coefficient $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -0.925$

6.

a) **Quarter wavelength transformer matching:**



Stub matching, in effect, is capable of removing a mismatch for any load (except a purely reactive load), but it is not an impedance transformer. If different lines must be matched, a transmission line transformer can be used, as in **Figure 15.27**.

From **Eq. (14.94)**, the line impedance Z_{in} of a lossless transmission line of characteristic impedance Z_0, at a distance z_0, from the load may be viewed as the input impedance of the line section between z_0 and the load:

$$Z_{in} = Z_0 \frac{[Z_L \cos \beta z_0 + jZ_0 \sin \beta z_0]}{[Z_0 \cos \beta z_0 + jZ_L \sin \beta z_0]} \tag{15.15}$$

Now, suppose we chose a transmission line section, with characteristic impedance Z_t, cut it so it is $\lambda/4$ long, and connect it to a load impedance Z_l. Setting $z_0 = \lambda/4$ and $\beta z_0 = \beta\lambda/4 = (2\pi/\lambda)(\lambda/4) = \pi/2$ and replacing Z_L by Z_l and Z_0 by Z_t in **Eq. (15.16)**, we get for the input impedance of the $\lambda/4$ section

$$Z_{in} = Z_t \frac{[Z_l \cos \frac{\pi}{2} + jZ_t \sin \frac{\pi}{2}]}{[Z_t \cos \frac{\pi}{2} + jZ_l \sin \frac{\pi}{2}]} = \frac{Z_t^2}{Z_l} \tag{15.16}$$

Referring now to **Figure 15.27**, where Z_l is the line impedance at a distance d from the load, we get the condition for matching using the quarter-wavelength transformer shown:

$$\boxed{Z_t = \sqrt{Z_{in} Z_l}} \tag{15.17}$$

Thus, two different transmission lines or any two impedances may be matched, provided a transformer of proper characteristic impedance Z_t can be found. The quarter-wavelength transformer is normally connected at a point of maximum or minimum voltage since the line impedance is real at that point. The line impedance at a point of minimum voltage is

$$Z_l = \frac{Z_0}{\text{SWR}} \tag{15.18}$$

where Z_0 is the characteristic impedance of the line, and the standing wave ratio on the line is given as

$$\text{SWR} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} \tag{15.19}$$

The location of the minimum voltage on the line for a general load is at a distance (see **Section 14.7.3**)

$$d_{\min} = \frac{\lambda}{4\pi}(\theta_L + \pi) + n\frac{\lambda}{2} \tag{15.20}$$

from the load, where n is any integer, including zero. For a resistively loaded line, the location of minimum voltage is either at the load (if $R_L < Z_0$) or at a distance $\lambda/4$ (if $R_L > Z_0$). Thus, the transformer can be located at any of the points in **Eq. (15.20)**. If the characteristic line impedance is Z_0, the characteristic impedance of a transformer located at a point of minimum voltage must be

$$\boxed{Z_t = Z_0 \sqrt{\frac{1}{\text{SWR}}}} \tag{15.21}$$

Similarly, if the transformer is located at a point of maximum voltage (by moving it a quarter-wavelength in either direction of any of the points in **Eq. (15.20)**), the characteristic impedance of the transformer for matching is

$$\boxed{Z_t = Z_0 \sqrt{\text{SWR}}} \tag{15.22}$$

b) Smith chart:

$$\Gamma_L = \frac{(Z_L - Z_0)/Z_0}{(Z_L + Z_0)/Z_0} = \frac{(R_L/Z_0 - 1) + jX_L/Z_0}{(R_L/Z_0 + 1) + jX_L/Z_0} = \frac{(r-1) + jx}{(r+1) + jx} = \Gamma_r + j\Gamma_i$$

$$\Gamma_r + j\Gamma_i = \frac{(r-1)+jx}{(r+1)+jx} \tag{15.4}$$

Cross-multiplying gives

$$(r+1)\Gamma_r - \Gamma_i x + j\Gamma_i(r+1) + jx\Gamma_r = (r-1)+jx \tag{15.5}$$

Separating the real and imaginary parts and rearranging terms, we get two equations:

$$(\Gamma_r - 1)r + \Gamma_i x = -(\Gamma_r + 1) \tag{15.6}$$

$$\Gamma_i r + (\Gamma_r - 1)x = -\Gamma_i \tag{15.7}$$

We now write two equations: one for r and one for x, by first eliminating x and then, separately, r. First, we eliminate x by substituting from **Eq. (15.7)** into **Eq. (15.5)**.

After rearranging terms, this gives

$$\Gamma_r^2(r+1) - 2\Gamma_r r + \Gamma_i^2(r+1) = 1 - r \tag{15.8}$$

Dividing by the common term $(r+1)$,

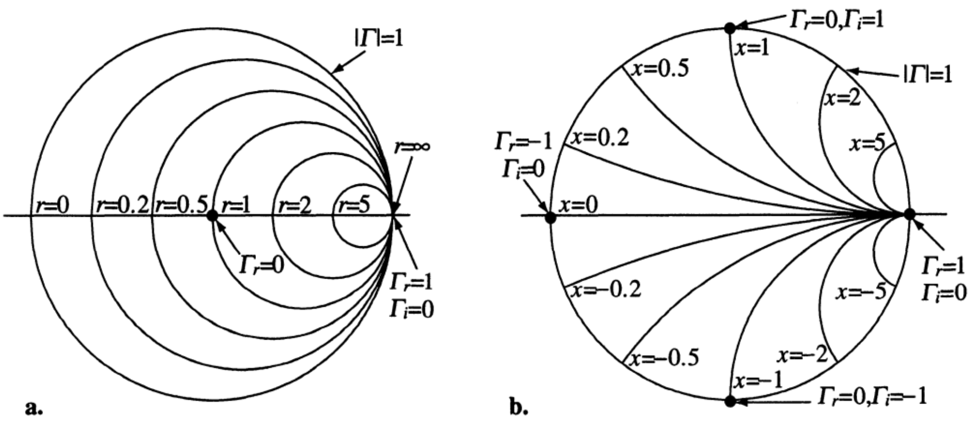
$$\Gamma_r^2 - \frac{2\Gamma_r r}{(r+1)} + \Gamma_i^2 = \frac{1-r}{(r+1)} \tag{15.9}$$

Adding $r^2/(r+1)^2$ to both sides of the equation and rearranging terms, we get

$$\boxed{\left(\Gamma_r - \frac{r}{(r+1)}\right)^2 + \Gamma_i^2 = \frac{1}{(r+1)^2}} \tag{15.10}$$

Repeating the process starting with **Eqs. (15.6)** and **(15.5)** and eliminating r from both equations, we get an equation in terms of x alone:

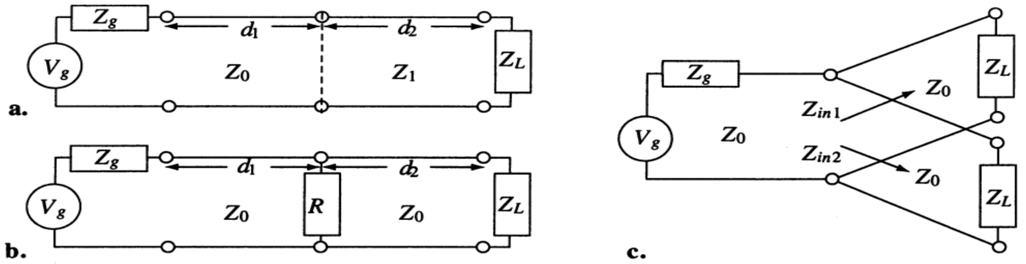
$$\boxed{(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2} \tag{15.11}$$

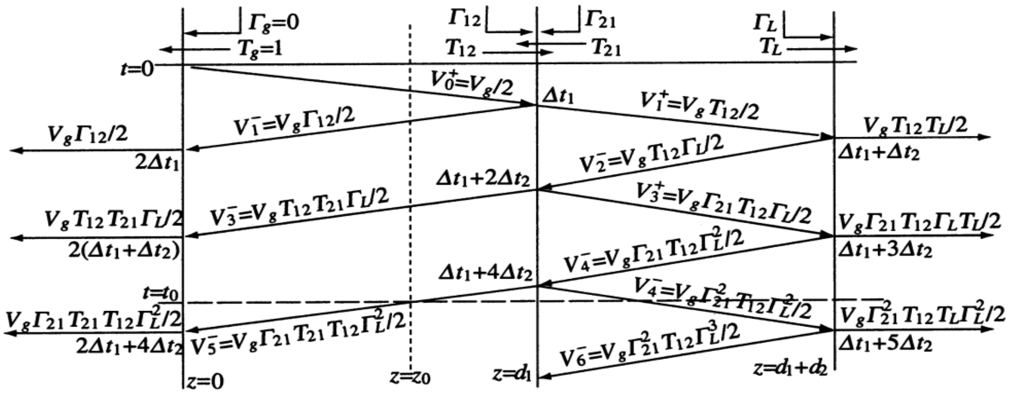


7.

a) The pulse whose width is far less than the propagation time is known as short pulse and the pulse whose width is far greater than the propagation time is known as long pulse.

Discontinuities:





To understand how the waves behave, we will follow the propagation of waves in **Figure 16.24a** and draw the reflection diagram as we go along. For simplicity, we assume that the generator is matched ($Z_g = Z_0$). Therefore, the forward-propagating wave launched by the generator at time $t = 0$ is

$$V_0^+ = \frac{V_g Z_0}{Z_0 + Z_0} = \frac{V_g}{2} \tag{16.36}$$

This wave propagates on line 1 at a speed of propagation v_{p1}. After a time $\Delta t_1 = d_1/v_{p1}$, the wave reaches the discontinuity. Part of the wave is reflected and part of it is transmitted with the reflection and transmission coefficients Γ_{12} and T_{12}, respectively:

$$\Gamma_{12} = \frac{Z_1 - Z_0}{Z_1 + Z_0}, \qquad T_{12} = \frac{2Z_1}{Z_1 + Z_0} \tag{16.37}$$

The reflection coefficient Γ_{12} is the reflection coefficient at the interface between line 1 and line 2 and the transmission coefficient indicates the transmission from line 1 to line 2. These two coefficients are shown in **Figure 16.25**, where the arrows indicate the direction of the waves being reflected and transmitted. The reflected and transmitted voltage waves at d_1 are

$$V_1^- = V_0^+ \Gamma_{12}, \qquad V_1^+ = V_0^+ T_{12} \tag{16.38}$$

The reflected wave V_1^- propagates back to the generator and reaches the generator after a time Δt_1. Since the reflection coefficient at the generator is zero, no additional reflections occur at this point. The wave transmitted across the discontinuity, V_1^+ propagates toward the load at a speed of propagation v_{p2}, and reaches the load after an additional time $\Delta t_2 = d_2/v_{p2}$. At the load, the wave is partly reflected and partly transmitted into the load (where it is dissipated or, in the case of an antenna, radiated). The reflection and transmission coefficients at the load are

$$\Gamma_L = \frac{Z_L - Z_1}{Z_L + Z_1}, \qquad T_L = \frac{2Z_L}{Z_L + Z_1} \tag{16.39}$$

Thus, the reflected and transmitted waves are

$$V_2^- = V_1^+ \Gamma_L = V_0^+ T_{12} \Gamma_L, \qquad V_{L1}^+ = V_0^+ T_{12} T_L \tag{16.40}$$

$$\Gamma_{21} = \frac{Z_0 - Z_1}{Z_1 + Z_0}, \qquad T_{21} = \frac{2Z_0}{Z_1 + Z_0} \tag{16.41}$$

The transmitted wave (from line 2 into line 1) and the reflected wave (into line 2) are

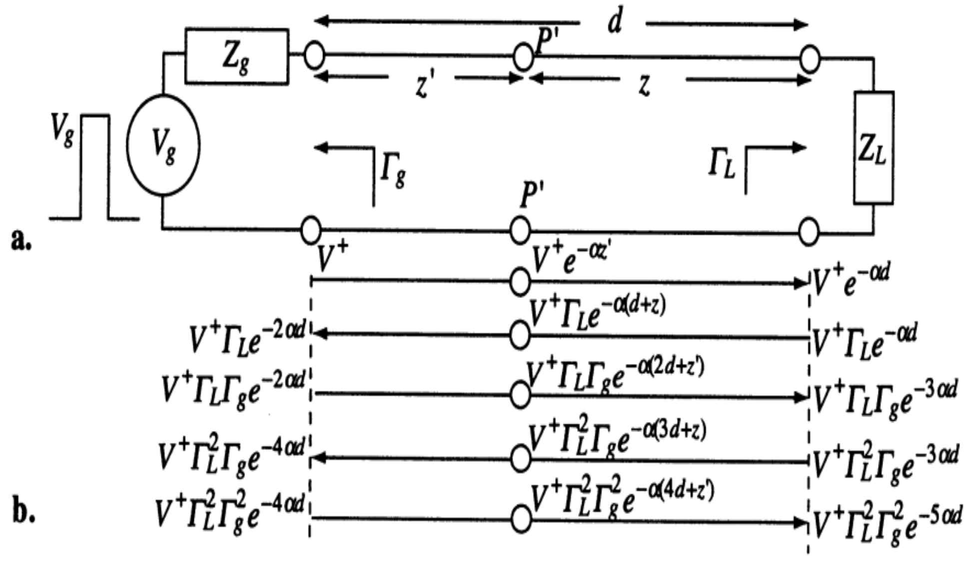
$$V_3^+ = V_2^- \Gamma_{21} = V_0^+ T_{12} \Gamma_L \Gamma_{21}, \qquad V_3^- = V_0^+ T_{12} \Gamma_L \Gamma_{21} \tag{16.42}$$

Now, these two waves propagate in opposite directions. V_3^+ propagates toward the load while V_3^- propagates toward the generator. The sequence repeats itself indefinitely. A few reflections are shown in **Figure 16.25**, together with the definitions of reflection and transmission coefficients at the various locations.

All other aspects of propagation remain as discussed in **Section 16.4**. Note, in particular, the times at which the waves reach various locations on the line. The main difficulty in treating discontinuities is in keeping track of the increasing number of reflections and transmissions and the associated times. We note also that the reflection and transmission coefficients at the discontinuity depend on the direction of propagation. The following relations hold:

$$\Gamma_{21} = -\Gamma_{12}, \qquad T_{21} = 1 - \Gamma_{12} \tag{16.43}$$

b) Propagation of Narrow pulse on finite distortion less transmission line:



$$V(z) = V^+ e^{-\alpha z'} = V_g \frac{Z_0}{Z_0 + Z_g} e^{-\alpha z'} \tag{16.12}$$

where z' is the distance from generator to point P' in **Figure 16.7a**. At the load, the forward-propagating wave is

$$V_L^+ = V^+ e^{-\alpha d} \tag{16.13}$$

The reflected wave is

$$V_1^- = \Gamma_L V^+ e^{-\alpha d} \tag{16.14}$$

At the load, the total voltage is the sum of this and the reflected voltage. This gives

$$V_L = V^+ e^{-\alpha d}(1 + \Gamma_L) \tag{16.15}$$

	However, this sum only exists for a time equal to the pulse width Δt. The reflected wave in **Eq. (16.14)** propagates back and is attenuated. The expression for the reflected wave anywhere on the line between load and generator is $$V_1^-(z) = \Gamma_L V^+ e^{-ad} e^{-az} \quad (16.16)$$ This reflected wave reaches the generator and is reflected at the generator unless the generator is matched. At the generator, the first reflection is $$V_1^-(z = d) = V^+ e^{-2ad} \Gamma_L \quad (16.17)$$ Taking into account the generator reflection coefficient Γ_g, the total voltage at the generator is $$V_{g1} = V^+ \Gamma_L e^{-2ad} (1 + \Gamma_g) \quad (16.18)$$ This sum also exists for a period Δt. The new forward-propagating wave after the first reflection at the generator is $$V_1^+(z') = V^+ e^{-2ad} e^{-az'} \Gamma_L \Gamma_g \quad (16.19)$$

8.

a)	We know the phase constant of a waveguide is $$\beta = \sqrt{w^2\mu\epsilon - w_c^2\mu\epsilon} = \frac{2\pi}{\lambda_g}$$ $$\frac{1}{\lambda_g} = \sqrt{f^2\mu\epsilon - f_c^2\mu\epsilon}$$ $$\frac{1}{\lambda_g} = \sqrt{\frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}}$$ $$\frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}$$
b)	Given a=6cm, b=4cm Dominant mode is TE_{10} Cutoff frequency $f_c = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = 2.5G\ Hz$ Wavelength $\lambda = {}^{v_p}/_{f_c} = 0.12m$ Intrinsic impedance $\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\Omega$

9.

a)	**TM Propagation in Parallel Plate Waveguide:** Maxwell equations $$\nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$ $$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$ $$\nabla \cdot \mathbf{E} = 0$$ $$\nabla \cdot \mathbf{H} = 0$$ Expanding the curl equations $$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x$$ $$-\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$ $$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$$ $$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x$$ $$-\gamma E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$ $$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$ Helmholtz equation $$\boxed{\frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0}$$ By substituting H_z=0 $$\boxed{E_x = \frac{-\gamma}{\gamma^2 + k^2} \frac{\partial E_z}{\partial_x}}$$ $$\boxed{E_y = \frac{-\gamma}{\gamma^2 + k^2} \frac{\partial E_z}{\partial_y}}$$ $$\boxed{H_x = \frac{j\omega\varepsilon}{\gamma^2 + k^2} \frac{\partial E_z}{\partial_y}}$$ $$\boxed{H_y = \frac{-j\omega\varepsilon}{\gamma^2 + k^2} \frac{\partial E_z}{\partial x}}$$ By substituting boundary conditions $$E_x(x,z) = \frac{\beta_z}{\omega\varepsilon} e^{-j\beta_z z}\left[Ae^{-j\beta_x x} + Be^{+j\beta_x x}\right]$$ $$E_z(x,z) = \frac{\beta_x}{\omega\varepsilon} e^{-j\beta_z z}\left[-Ae^{-j\beta_x x} + Be^{+j\beta_x x}\right]$$ $$H_y(x,z) = H_o e^{-j\beta_z z} \cos \beta_x x$$ $$E_x(x,z) = \frac{\beta_z}{\omega\varepsilon} H_o e^{-j\beta_z z} \cos \beta_x x$$ $$E_z(x,z) = \frac{j\beta_x}{\omega\varepsilon} H_o e^{-j\beta_z z} \sin \beta_x x$$

b)	**Rectangular waveguide:** Maxwell equations $$\nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$ $$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$ $$\nabla \cdot \mathbf{E} = 0$$ $$\nabla \cdot \mathbf{H} = 0$$ Expanding the curl equations $$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x$$ $$-\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$ $$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$$ $$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x$$ $$-\gamma E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$ $$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$ Helmholtz equation $$\boxed{\frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0}$$ By substituting H_z=0 $$\boxed{E_x = \frac{-\gamma}{\gamma^2 + k^2} \frac{\partial E_z}{\partial_x}}$$ $$\boxed{E_y = \frac{-\gamma}{\gamma^2 + k^2} \frac{\partial E_z}{\partial_y}}$$ $$\boxed{H_x = \frac{j\omega\varepsilon}{\gamma^2 + k^2} \frac{\partial E_z}{\partial_y}}$$ $$\boxed{H_y = \frac{-j\omega\varepsilon}{\gamma^2 + k^2} \frac{\partial E_z}{\partial x}}$$ **TM mode of propagation:** By substituting boundary conditions $$\boxed{E_z(x,y,z) = E_0 \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z}}$$ $$E_x(x,y,z) = \frac{-\gamma}{\gamma^2 + k^2} E_0 \frac{m\pi}{a} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$ $$E_y(x,y,z) = \frac{-\gamma}{\gamma^2 + k^2} E_0 \frac{n\pi}{b} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$ $$H_x(x,y,z) = \frac{j\omega\varepsilon}{\gamma^2 + k^2} E_0 \frac{n\pi}{b} \sin\left(\frac{m\pi x}{b}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$ $$H_y(x,y,z) = -\frac{j\omega\varepsilon}{\gamma^2 + k^2} E_0 \frac{m\pi}{a} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

TE mode of propagation:
By substituting boundary conditions

$$\boxed{H_z(x,y,z) = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}}$$

$$E_x(x,y,z) = \frac{j\omega\mu}{\gamma^2 + k^2} H_0 \frac{n\pi}{b} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$E_y(x,y,z) = \frac{-j\omega\mu}{\gamma^2 + k^2} H_0 \frac{m\pi}{a} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_x(x,y,z) = \frac{\gamma}{\gamma^2 + k^2} H_0 \frac{m\pi}{a} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_y(x,y,z) = \frac{\gamma}{\gamma^2 + k^2} H_0 \frac{n\pi}{b} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

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