Hall Ticket Number:
14EC505 /EC322

III/IV B.Tech Regular /Supplementary DEGREE EXAMINATION

November, 2019 **Electronics and Communication Engineering**

Fifth Semester **Digital Communications**

Time: Three Hours **Maximum:** 60 Marks

Answer Question No.1 compulsorily. (1X12 = 12 Marks)
Answer ONE question from each unit. (4X12=48 Marks)

1. Answer all questions (1X12=12 Marks)

a) Define processing gain in DPCM.

$$G_p = \frac{\sigma_M^2}{\sigma_E^2}$$

b) Calculate the Bandwidth need needed for transmission of 4 KHz signal using PCM with 128 Quantization levels.

Solution

Given $n = 24$

and $M = 128$

Therefore, $2^N = 128$

or $N = \log_2 128 = 7$

By putting $2f_m = 8000$ Hz is Eq. 8.8.3, we get

$$\text{BW} = [(24 \times 7) + 1]\ 8000 \text{ Hz}$$
$$= 1.352 \text{ MHz}$$

On the other hand, the approximate value of BW, as given by Eq. 8.8.4, is

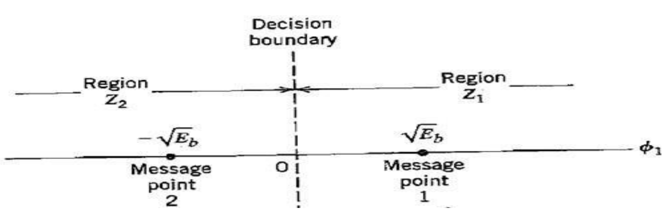
$$\text{BW} \approx 24 \times 7 \times 8000 \text{ Hz}$$
$$= 1.344 \text{ MHz}$$

Note: For comparison, if the same number of channels are frequency division multiplexed by using an SSB modulation, the required bandwidth, assuming 4 kHz per channel, will be

$$\text{BW} = 24 \times 4 = 96 \text{ kHz}$$

c) For the binary sequence 101101 draw the line code using ON-OFF Signalling.

d) Draw the constellation diagram for coherent BPSK.



e) Give the expression for Average Probability of Symbol Error for Non Coherent Binary FSK.

		The average probability of error for noncoherent binary FSK is given by $$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right)$$	
	f)	What is Euclidean distance? The distance between received signal point and message point. $\lVert \mathbf{x} - \mathbf{s}_k \rVert$	
	g)	Define Information Rate **Information Rate** : R = rH. Here R is the **information rate**. H is the Entropy or average **information**. And r is the **rate** at which messages are generated. **Information rate** R is represented in average number of bits of **information** per second.	
	h)	State Information capacity theorem. The *information capacity theorem* states that ideally these two parameters are related $$C = B\log_2(1 + \text{SNR})\ \text{b/s}$$ where C is the information capacity of the channel. The *information capacity* is defined as the maximum rate at which information can be transmitted across the channel without error; it is measured in *bits per second* (b/s).	
	i)	Give the expression for average information of a Discrete Memory less source generating K symbols. *average information rate* of the source is $H(\mathscr{S})/T$, bits per second.	
	j)	What do you mean by constraint length in Convolution codes? Constraint length is defined as the number of shifts over which a single message bit influence the encoder output.	
	k)	Define cyclic codes. An (n, k) linear code C is called a cyclic code if every cyclic shift of a code vector in C is also a code vector in C.	
	l)	Define Hamming weight. The *Hamming weight* $w(\mathbf{c})$ of a code vector **c** is defined as the number of nonzero elements in the code vector.	
UNIT I			
2.	a)	Explain in detail the transmitter and decoder of PCM with a neat sketch. Also list the merits of PCM in comparison to analog modulation techniques. Three steps involved in conversion of analog signal to digital signal • Sampling • Quantization • Binary encoding	8M

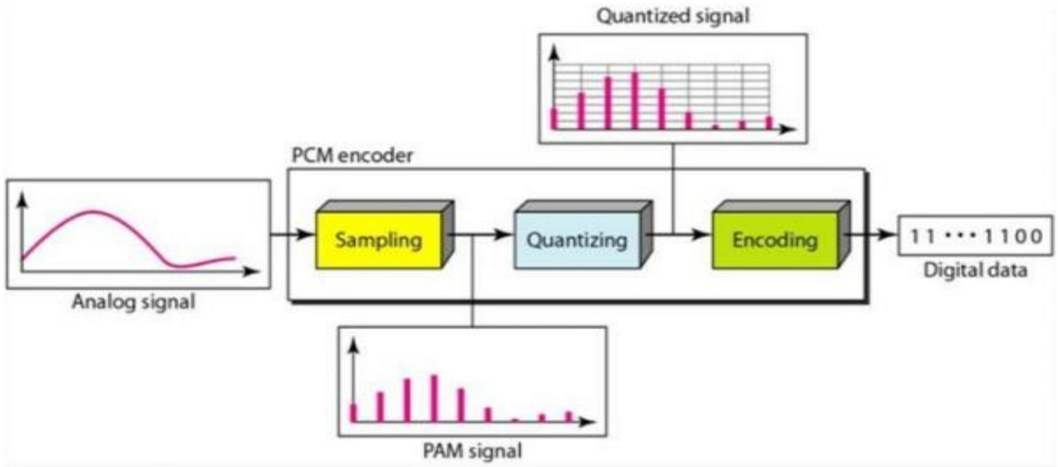


Fig. 2 Conversion of Analog Signal to Digital Signal

Elements of PCM System:

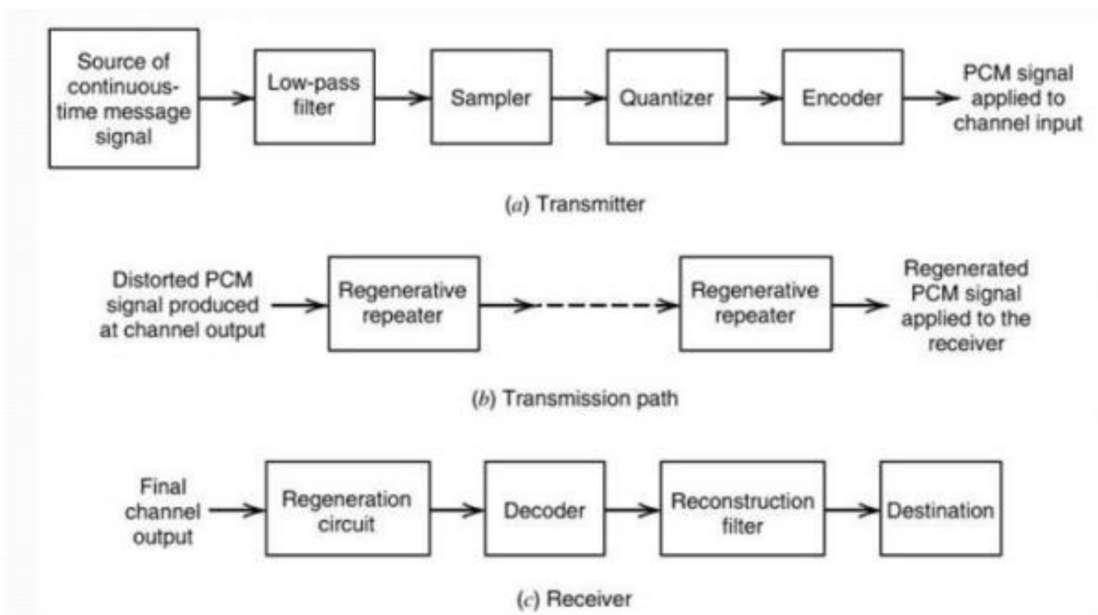


Fig. 3 Elements of PCM System

Sampling:

- Process of converting analog signal into discrete signal.
- Sampling is common in all pulse modulation techniques

- The signal is sampled at regular intervals such that each sample is proportional to amplitude of signal at that instant
- Analog signal is sampled every T_s $Secs$, called sampling interval. f_s=1/T_S is called sampling rate or sampling frequency.
- f_s=2f_m is Min. sampling rate called **Nyquist rate.** Sampled spectrum (ω) is repeating periodically without overlapping.
- Original spectrum is centered at ω=0 and having bandwidth of ω_m. Spectrum can be recovered by passing through low pass filter with cut-off ω_m.
- For f_s<2f_m sampled spectrum will overlap and cannot be recovered back. This is called **aliasing.**

* Advantages of PCM:-

(1) Uniform transmission quality.

(2) Compatibility of different classes of Traffic in the network.

(3) Integrated Digital network.

(4) Low manufacturing cost

(5) Good performance over very poor Transmission paths

* Disadvantages:-

(1) Large Bandwidth require for transmission

(2) Noise and Cross talk leaves low but, rises attenuation. (More no. of channels it increase attenuation & noise also increase).

* Applications:-

(1) In compact disk

(2) Digital telephony

(3) Digital audio applications.

This is known as "PCM"

<table>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>b)</td><td>

A speech signal has a total duration of 10 S. It is sampled at the rate of 8 KHz and then encoded. The signal to quantization noise ratio is required to be 40 dB. Calculate the minimum storage capacity needed to accommodate this digitized speech signal.

Assume Sinusoidal modulating signal. The Signal-to-Quantization Noise Ratio, in dB, is given by:

(??N?)) ?? = 1.8 + 6?

Where ? represents number of bits per sample.

If we choose ? =6, (??N?)) ?? = 37.8 and if we choose ? =7, (??N?)) ?? = 43.8

Therefore, the minimum number of bits per sample is 7 for a signal-to-quantization noise ratio of 40dB. Hence,

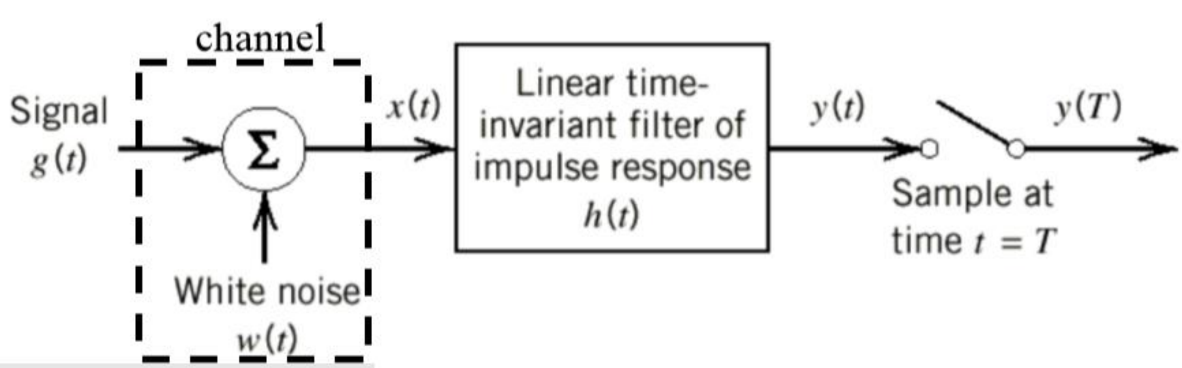
The number of samples in a duration of 10 sec.=8000×10 samples. The minimum storage is therefore equal to $7 \times 8 \times 10^4$ =560 Kbits.

</td><td>4M</td></tr>
<tr><td colspan="4">(OR)</td></tr>
<tr><td>3.</td><td>a)</td><td>

Define matched filter and derive an expression for impulse response of an optimum filter.

Matched filter is a device for the optimal detection of a digital pulse. It is so named because the *impulse response* of the matched filter matches the *pulse shape*.

System model without ISI



</td><td>8M</td></tr>
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To find $h(t)$ such that the output signal-to-noise ratio SNR_O is maximized.

$$x(t) = g(t) + w(t) \text{ for } 0 \le t < T$$

$$\begin{aligned} y(t) &= [g(t) + w(t)] * h(t) \\ &= g(t) * h(t) + w(t) * h(t) \\ &= g_o(t) + n(t) \end{aligned}$$

$$SNR_O = \frac{|g_o(T)|^2}{E[n^2(T)]}$$

$$g_o(t) = \int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi ft)df$$

$$\Rightarrow |g_o(T)|^2 = \left|\int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi fT)df\right|^2$$

With $w(t)$ being white with PSD $N_0/2$,

$$S_N(f) = S_W(f)|H(f)|^2 = \frac{N_0}{2}|H(f)|^2$$

$$\Rightarrow E[n^2(T)] = \int_{-\infty}^{\infty} S_N(f)df = \frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2 df$$

$$\Rightarrow \eta = \frac{\left|\int_{-\infty}^{\infty} G(f)H(f)\exp(j2\pi fT)df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2 df}$$

Cauchy-Schwarz inequality

$$\left|\int_{-\infty}^{\infty}\varphi_1(x)\varphi_2(x)dx\right|^2 \le \left(\int_{-\infty}^{\infty}|\varphi_1(x)|^2 dx\right)\left(\int_{-\infty}^{\infty}|\varphi_2(x)|^2 dx\right)$$

with equality holding if, and only if, $\varphi_1(x) = k \cdot \varphi_2^*(x)$ for some constant k.

By Cauchy-Schwarz inequality,

$$\left|\int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi fT)df\right|^2 \le \int_{-\infty}^{\infty} |H(f)|^2\, df \cdot \int_{-\infty}^{\infty} |G(f)\exp(j2\pi fT)|^2\, df$$

$$\Rightarrow \eta \le \frac{\int_{-\infty}^{\infty} |H(f)|^2\, df \cdot \int_{-\infty}^{\infty} |G(f)|^2\, df}{\frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\, df} = \frac{2}{N_0}\int_{-\infty}^{\infty} |G(f)|^2\, df$$

This is a constant bound, independent of the choice of $h(t)$.
Hence, the optimal η is achieved by:

$$\boxed{H(f) = k \cdot G^*(f)\exp(-j2\pi fT)}$$

$$\begin{aligned} h_{\text{opt}}(t) &= \int_{-\infty}^{\infty} k \cdot G^*(f)\exp(-j2\pi fT)\exp(j2\pi ft)df \\ &= k\left(\int_{-\infty}^{\infty} G(f)\exp(j2\pi f(T-t))df\right)^* \\ &= kg^*(T-t). \end{aligned}$$

Hence, under additive white noise, the *optimal received filter* matches the input signal in the sense that it is a time-inversed and delayed version of the complex-conjugated input signal $g(t)$.

b) A binary data stream 001010010 is applied to the input of a Duo-Binary System

1) Construct the Duo-Binary coder output and corresponding receiver output without precode.
2) Suppose that owing to error during transmission, the level at the receiver input produced by second digit is reduced to zero construct the new receiver output.

4M

1)

$\{b_k\}$ 0 0 1 0 1 0 0 1 0

PAM $\{a_k\}$ -1 -1 -1 1 -1 1 -1 -1 1 -1

$a_k+a_{k-1}=\{c_k\}$ -2 -2 0 0 0 0 -2 0 0

2)

$\{c_k\}$ - -4 -2 0 0 0 0 0 -2 0 0

-1

$\{a_k\}=\{c_k\}-\{a_{k-1}\}$ -1 1 -1 1 -1 1 -3 3 -3

$\{\hat{b}_k\} \Rightarrow$ 0 1 0 1 0 1 0 1 0

		UNIT II	
4.	a)	Discuss in detail the Gram-Schmidt orthogonalization procedure for a set of 'M' Real valued energy signals.	6M

The principle of Gram-Schmidt Orthogonalization (GSO) states that, any set of M energy signals, $\{s_i(t)\}$, $1 \le i \le M$ can be expressed as linear combinations of N orthonormal basis functions,

where $N \le M$.

If $s_1(t), s_2(t), \ldots, s_M(t)$ are real valued energy signals, each of duration 'T' sec,

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\Phi_j(t) ; \quad \begin{cases} 0 \le t \le T \\ i = 1, 2, \ldots, M \ge N \end{cases}$$

where,

$$s_{ij} = \int_0^T s_i(t)\varphi_j(t)dt \quad ; \begin{cases} i = 1, 2, \ldots\ldots, M \\ j = 1, 2, \ldots\ldots, N \end{cases}$$

The $\phi_j(t)$-s are the basis functions and 's_{ij}'-s are scalar coefficients. We will consider real-valued basis functions $\phi_j(t)$ - s which are orthonormal to each other, i.e.,

$$\int_0^T \varphi_i(t).\varphi_j(t)dt = \begin{cases} 1, \text{ if } i = j \\ 0, \text{ if } i \ne j \end{cases}$$

Note that each basis function has unit energy over the symbol duration 'T'. Now, if the basis functions are known and the scalars are given, we can generate the energy signals

G-S-O procedure

Part – I: We show that any given set of energy signals, $\{s_i(t)\}$, $1 \le i \le M$ over $0 \le t < T$, can be completely described by a subset of energy signals whose elements are linearly independent.

To start with, let us assume that all $s_i(t)$ -s are not linearly independent. Then, there must exist a set of coefficients $\{a_i\}$, $1 < i \le M$, not all of which are zero, such that,

$a_1s_1(t) + a_2s_2(t) + \ldots\ldots + a_Ms_M(t) = 0$, $0 \le t < T$

Verify that even if two coefficients are not zero, e.g. $a_1 \ne 0$ and $a_3 \ne 0$, then $s_1(t)$ and $s_3(t)$ are dependent signals.

Let us arbitrarily set, $a_M \ne 0$. Then,

$$s_M(t) = -\frac{1}{a_M}[a_1 s_1(t) + a_2 s_2(t) + \ldots\ldots + a_{M-1} s_{M-1}(t)]$$

$$= -\frac{1}{a_M}\sum_{i=1}^{M-1} a_i s_i(t)$$

Consider a reduced set with (M-1) signals $\{s_i(t)\}$, i = 1,2,....., (M – 1).

This set may be either linearly independent or not. If not, there exists a set of $\{b_i\}$, i = 1,2..., (M – 1), not all equal to zero such that,

$$\sum_{i=1}^{M-1} b_i s_i(t) = 0,\ \ 0 \leq t < T$$

Arbitrarily assuming that $b_{M-1} \neq 0$, we may express $s_{M-1}(t)$ as

$$s_{M-1}(t) = -\frac{1}{b_{M-1}}\sum_{i=1}^{M-2} b_i s_i(t)$$

Now, following the above procedure for testing linear independence of the remaining signals, eventually we will end up with a subset of linearly independent signals. Let $\{s_i(t)\}$, i = 1, 2, ..., $N \leq M$ denote this subset.

Part – II : We now show that it is possible to construct a set of 'N' orthonormal basis functions $\phi_1(t)$, $\phi_2(t)$, ..., $\phi_N(t)$ from $\{s_i(t)\}$, i = 1, 2, ..., N. Let us choose the first basis function as,

first signal $s_1(t)$, i.e., $E_1 = \int_0^T s_1^2(t)dt$:

$$\therefore s_1(t) = \sqrt{E_1}.\varphi_1(t) = s_{11}\varphi_1(t)$$

Where, $s_{11} = \sqrt{E_1}$

$$s_{21} = \int_0^T s_2(t)\phi_1(t)dt$$

$$g_2(t) = s_2(t) - s_{21}\varphi_1(t);\ \ 0 \leq t < T$$

$$\int_0^T g_2(t)\varphi_1(t)dt = \int_0^T s_2(t)\varphi_1(t)dt - s_{21}\int_0^T \varphi_1(t)\varphi_1(t)dt$$

$= s_{21} - s_{21} \quad = 0 \rightarrow g_2(t)$ Orthogonal to $\varphi_1(t)$; So, we verified that the function $g_2(t)$ is orthogonal to the first basis function. This gives us a clue to determine the second basis function.

Now, energy of $g_2(t)$

$$= \int_0^T \left[s_2(t) - s_{21}\varphi_1(t) \right]^2 dt$$

$$= \int_0^T s_2{}^2(t)dt - 2.s_{21}\int_0^T s_2(t)\varphi_1(t)dt + s_{21}{}^2\int_0^T \varphi_1{}^2(t)c$$

$$= E_2 - 2.s_{21}.s_{21} + s_{21}{}^2 = E_2 - s_{21}{}^2$$

$$= \int_0^T g_2{}^2(t)dt$$

So, we now set,

$$\varphi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2{}^2(t)dt}} = \frac{s_2(t) - s_{21}\varphi_1(t)}{\sqrt{E_2 - s_{21}{}^2}}$$

and $E_2 = \int_0^T s_2{}^2(t)dt$: Energy of $s_2(t)$

Verify that:

$\int_0^T \varphi_2{}^2(t)dt = 1$, i.e. $\varphi_2(t)$ is a time limited energy signal of unit energy.

and $\int_0^T \varphi_1(t).\varphi_2(t)dt = 0$, i.e. $\varphi_1(t)$ and $\varphi_2(t)$ are orthonormal to each other.

Proceeding in a similar manner, we can determine the third basis function, $\varphi_3(t)$. For i=3,

$$g_3(t) = s_3(t) - \sum_{j=1}^{2} s_{3j}\varphi_j(t); \quad 0 \le t < T$$

$$= s_3(t) - \left[s_{31}\varphi_1(t) + s_{32}\varphi_2(t) \right]$$

where,

$$s_{31} = \int_0^T s_3(t)\varphi_1(t)dt \quad \text{and} \quad s_{32} = \int_0^T s_3(t)\varphi_2(t)dt$$

$$\varphi_3(t) = \frac{g_3(t)}{\sqrt{\int_0^T g_3{}^2(t)dt}}$$

Indeed, in general,

$$\varphi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i{}^2(t)dt}} = \frac{g_i(t)}{\sqrt{Eg_i}}$$

for i = 1, 2,….., N, where

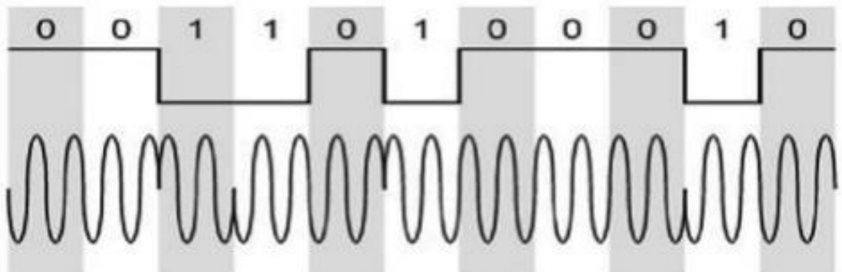
$$g_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij}\varphi_j(t)$$

		and $$s_{ij} = \int_0^T s_i(t).\varphi_j(t)dt$$ for i = 1, 2,….., N and j = 1, 2, …, M **Gram-Schmidt Orthogonalization procedure:** If the signal set {$s_j(t)$} is known for j = 1, 2,….., M, 0 ≤ t <T, • Derive a subset of linearly independent energy signals, {$s_i(t)$},i = 1, 2,….., N ≤ M. • Find the energy of $s_1(t)$ as this energy helps in determining the first basis function $\phi_1(t)$, which is a normalized form of the first signal. Note that the choice of this 'first' signal is arbitrary. • Find the scalar 's_{21}', energy of the second signal (E_2), a special function '$g_2(t)$' which is orthogonal to the first basis function and then finally the second orthonormal basis function $\phi_2(t)$ • Follow the same procedure as that of finding the second basis function to obtain the other basis functions.	
	b)	An FSK system transmits binary data at a rate of 2.5 X 10^6 bits per second. During the course of transmission, White Gaussian noise of zero mean and power spectral density10-20 W/Hz is added to the signal. In the absence of noise, the amplitude of the received sinusoidal wave for digit 1 or 0 is 1 mV. Determine the average probability of symbol error for the following configurations: 1) Coherent Binary FSK 2)Non coherent Binary FSK $T_b = 1/2.5\times10^6 = 0.4\mu s$ $E_b = \frac{1}{2}A_c^2T_b = \frac{1}{2}(10^{-3})^2 0.4\times10^{-6} = 2\times10^{-13}$ J $P_e = \frac{1}{2}\text{erfc}\left(\sqrt{E_b/2N_o}\right) = \frac{1}{2}\text{erfc}\left(\sqrt{2\times10^{-13}/2\times10^{-20}}\right) = \ldots = 0.85\times10^{-3}$	6M
(OR)			
5.	a)	Explain DPSK Modulation Scheme with a neat sketch.	6M

DIFFERENTIAL PHASE SHIFT KEYING (DPSK):

In DPSK (Differential Phase Shift Keying) the phase of the modulated signal is shifted relative to the previous signal element. No reference signal is considered here. The signal phase follows the high or low state of the previous element. This DPSK technique doesn't need a reference oscillator.

The following figure represents the model waveform of DPSK.



It is seen from the above figure that, if the data bit is LOW i.e., 0, then the phase of the signal is not reversed, but is continued as it was. If the data is HIGH i.e., 1, then the phase of the signal is reversed, as with NRZI, invert on 1 (a form of differential encoding).

If we observe the above waveform, we can say that the HIGH state represents an **M** in the modulating signal and the LOW state represents a **W** in the modulating signal.

The word binary represents two-bits. **M** simply represents a digit that corresponds to the number of conditions, levels, or combinations possible for a given number of binary variables.

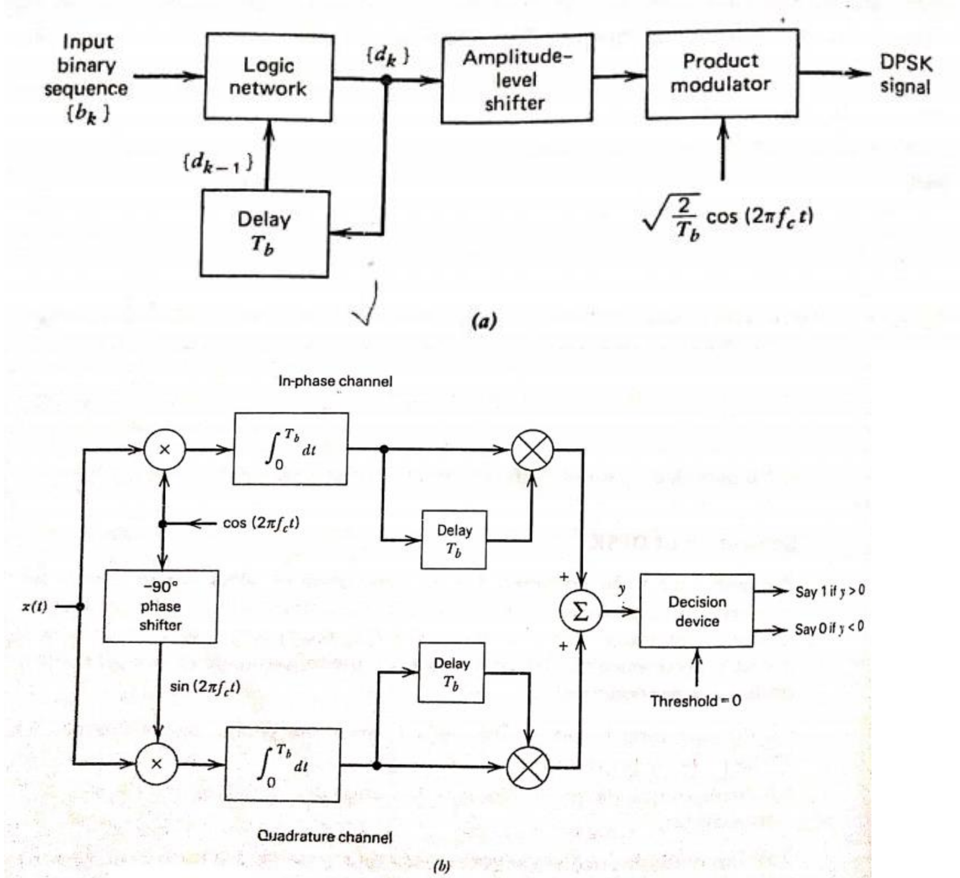
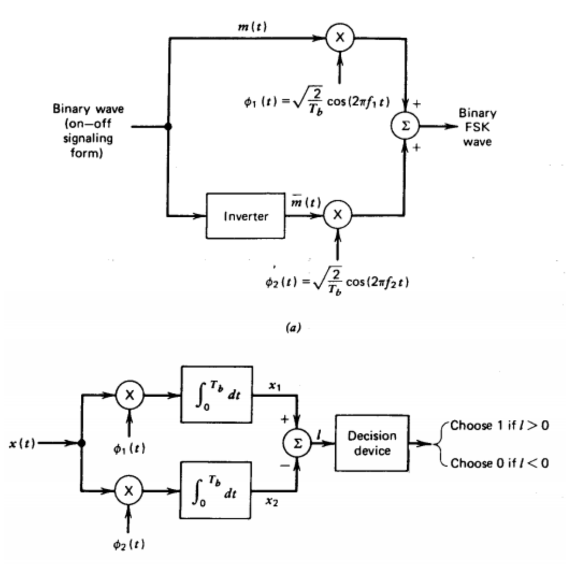
This is the type of digital modulation technique used for data transmission in which instead of one-bit, two or **more bits are transmitted at a time**. As a single signal is used for multiple bit transmission, the channel bandwidth is reduced.

DBPSK TRANSMITTER.:

Figure 2-37a shows a simplified block diagram of a *differential binary phase-shift keying* (DBPSK) transmitter. An incoming information bit is XNORed with the preceding bit prior to entering the BPSK modulator (balanced modulator).

For the first data bit, there is no preceding bit with which to compare it. Therefore, an initial reference bit is assumed. Figure 2-37b shows the relationship between the input data, the XNOR output data, and the phase at the output of the balanced modulator. If the initial reference bit is assumed a logic 1, the output from the XNOR circuit is simply the complement of that shown.

In Figure 2-37b, the first data bit is XNORed with the reference bit. If they are the same, the XNOR output is a logic 1; if they are different, the XNOR output is a logic 0. The balanced modulator operates the same as a conventional BPSK modulator; a logic I produces $+\sin \omega_c t$ at the output, and A logic 0 produces $-\sin \omega_c t$ at the output.

		 Block diagrams for (a) DPSK transmitter and (b) DPSK receiver.	
	b)	Derive the expression for probability of error in Coherent BFSK.  Block diagrams for (a) binary FSK transmitter, and (b) coherent binary FSK receiver.	6M

In a binary FSK system, symbols 1 and 0 are distinguished from each other by transmitting one of two sinusoidal waves that differ in frequency by a fixed amount. **A** typical pair of sinusoidal waves is described by

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_i t) & 0 \leqslant t \leqslant T_b \\ 0 & \text{elsewhere} \end{cases}$$

where i = 1, 2, and E_b is the transmitted signal energy per bit, and the transmitted frequency equals

$$f_i = \frac{n_c + i}{T_b} \quad \text{for some fixed integer } n_c \text{ and}$$

Thus symbol 1 is represented by $s_1(t)$, and symbol 0 by $s_2(t)$.

From Eq 4.9 we observe directly that the signals $s_1(t)$ and $s_2(t)$ are orthogonal, but not normalized to have unit energy. We therefore deduce that the moss useful form for the set of orthonormal basis

functions is

$$\phi_i(t) = \begin{cases} \sqrt{\dfrac{2}{T_b}} \cos(2\pi f_i t) & 0 \leqslant t \leqslant T_b \\ 0 & \text{elsewhere} \end{cases}$$

where i = 1, 2. Correspondingly, the coefficient s_{ij} for i = 1, 2, and j = 1, 2. is definers by

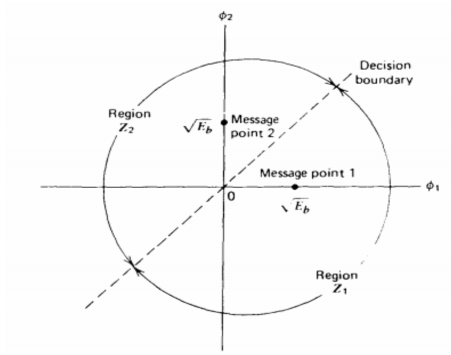


FIGURE 4-3 Signal space diagram for coherent binary FSK system.

$$\mathbf{s}_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix}$$

$$\mathbf{s}_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix}$$

- Note that the distance between the two message points is equal to $\sqrt{2E_b}$ The observation vector x has two elements, x_1 and x_2, that are defined by, respectively

$$x_1 = \int_0^{T_b} x(t)\phi_1(t)dt$$

$$x_2 = \int_0^{T_b} x(t)\phi_2(t)dt$$

The average probability of symbol error for coherent binary FSK is

$$P_e = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

UNIT III

6.	a)	State and prove Shannon's Second theorem.	6M

		Channel Coding Theorem (Shannon's Second Theorem) We have seen that the information is transmitted through the channel with rate 'R' called information rate. Shannon's theorem says that it is possible to transmit information with an arbitrarily small probability of error provided that information rate 'R' is less than or equal to a rate 'C', called channel capacity. Thus *channel capacity* is the maximum information rate with which the error probability is within the tolerable limits. **Statement of the theorem :** *Given a source of M equally likely messages, with $M >> 1$, which is generating information at a rate R. Given channel with channel capacity C. Then if,* $R \leq C,$ *there exists a coding technique such that the output of the source may be transmitted over the channel with a probability of error in the received message which may be made arbitrarily small.* **Explanation :** This theorem says that if $R \leq C$, it is possible to transmit information without any error even if noise is present. Coding techniques are used to detect and correct the errors.	
	b)	Consider the discrete memory less source, the source alphabets S={s1, s2, s3} with probabilities P = {0.25, 0.25, 0.5} respectively. Find the entropy of the 1st order source and the 2nd order extension source. The entropy of the source becomes – $H(\xi) = p_0 \log_2(\frac{1}{p_0}) + p_1 \log_2(\frac{1}{p_1}) + p_2 \log_2(\frac{1}{p_2})$ $= \frac{1}{4}\log_2(4) + \frac{1}{4}\log_2(4) + \frac{1}{2}\log_2(2)$ $= \frac{3}{2}$ For the second order extension of the source with the original source consisting 3 symbols the extended second order source will consists of 9 symbols as given by $\sigma_0 = s_0 s_0; \sigma_1 = s_0 s_1; \sigma_2 = s_0 s_2; \sigma_3 = s_1 s_0; \sigma_4 = s_1 s_1; \sigma_5 = s_1 s_2;$ $\sigma_6 = s_2 s_0; \sigma_7 = s_2 s_1; \sigma_8 = s_2 s_2;$ with their probabilities given as $\sigma_0 = \frac{1}{16}; \sigma_1 = \frac{1}{16}; \sigma_2 = \frac{1}{8}; \sigma_3 = \frac{1}{16}; \sigma_4 = \frac{1}{16}$ $\sigma_5 = \frac{1}{8}; \sigma_6 = \frac{1}{8}; \sigma_7 = \frac{1}{8}; \sigma_8 = \frac{1}{4}$	6M

		So the entropy of the extended source is $$H(\xi^2) = \sum_{i=0}^{8} p(\sigma_i) \log_2 \frac{1}{p(\sigma_i)}$$ $$= \frac{1}{16}\log_2(16) + \frac{1}{16}\log_2(16) + \frac{1}{8}\log_2(8) + \frac{1}{16}\log_2(16) + \frac{1}{16}\log_2(16) +$$ $$\frac{1}{8}\log_2(8) + \frac{1}{8}\log_2(8) + \frac{1}{8}\log_2(8) + \frac{1}{4}\log_2(4)$$ $$= 3$$ $so,$ $$H(\xi^2) = 2H(\xi).$$	
(OR)			
7.	a)	What is Entropy? State and prove the properties of Entropy. The *amount of potential information contained is a signal is termed the entropy*, usually denoted by H, which is defined as follows: $$H(X) = -\sum_X P(X) \log P(X)$$ We can essentially think of this as a weighted average of log 1/P(X) . The quantity 1/P(X) expresses the amount of "surprise" in an event X—i.e., if the probability is low, then there is a lot of surprise, and consequently a lot of information is conveyed by telling you that X happened. It's a measure of the ***average information content per source symbol***. The reason we are taking the log of the surprise is so that the total amount of surprise from independent events is additive. Logs convert multiplication to addition, so $$\log \frac{1}{P(X_1\ X_2)} = \log \frac{1}{P(X_1)} + \log \frac{1}{P(X_2)}.$$ Thus, entropy essentially measures the "average surprise" or "average uncertainty" of a random variable. If the distribution P(X) is highly peaked around one value, then we will rarely be surprised by this variable, hence it would be incapable of conveying much information. If on the other hand P(X) is uniformly distributed, then we will be most surprised on average by this variable, hence it could potentially convey a lot of information.	6M

Properties of Entropy

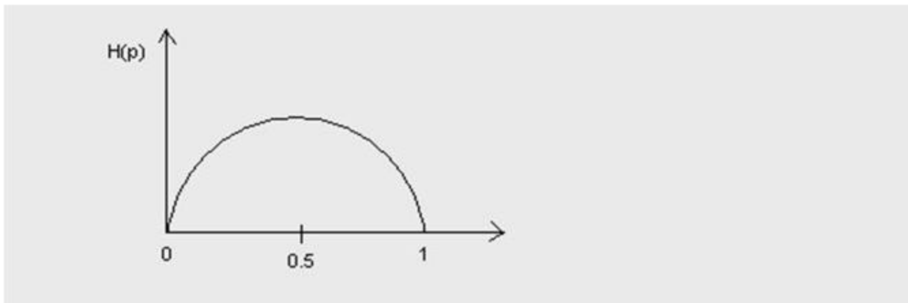
(P1) $H(x) \geq 0$

$0 \leq p(x) \leq 1 \rightarrow \log [1/p(x)] \geq 0$

[E] $x = \begin{cases} 1 & p \\ 0 & 1-p \end{cases}$

$H(x) = - p \log p - (1-p) \log(1-p)$

$= H(p)$ – the entropy function.



Limiting factor:

- ***Entropy is 0***, if and only if p_k=1 for some k and remaining probabilities in the set are all 0 – this means there is ***no uncertainty*** in the information.

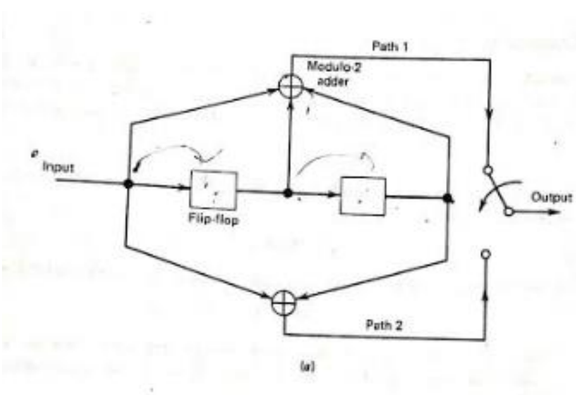
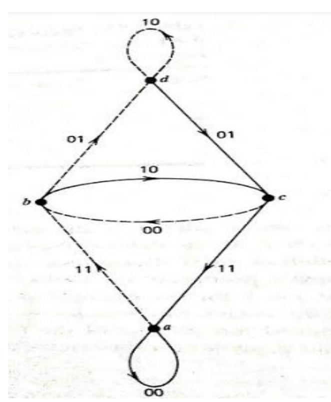
- ***Entropy is log$_2$ K***, if and only if p_k=1/K for all k i.e., all the symbols in the alphabet are equi-probable; this upper bound on entropy corresponds to ***maximum information***.

b) Consider the continuous random variable Y defined by Y=X+N, where X and N are statistically independent. Show that the conditional differential entropy of Y given X equals h(Y/X) = h(N) where h(N) is the differential entropy of N. 6M

By definition, we have

$$h(Y|X) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\log_2\left[\frac{1}{f_Y(y|x)}\right]dx\,dy$$

$$f_{X,Y}(x,y) = f_Y(y|x)f_X(x)$$

		$$h(Y\|X) = \int_{-\infty}^{\infty} f_X(x)dx \int_{-\infty}^{\infty} f_Y(y\|x)\log_2\left[\frac{1}{f_Y(y\|x)}\right]dy \quad (1)$$ The variable Y is related to X as $$Y = X + N$$ Hence, the conditional probability density function $f_Y(y\|x)$ is identical to that of N except for a translation of x, the given value of X. Let $f_N(n)$ denote the probability density function of N. Then $$f_Y(y\|x) = f_N(y-x)$$ Correspondingly, we may write $$\int_{-\infty}^{\infty} f_Y(y\|x)\log_2\left[\frac{1}{f_Y(y\|x)}\right]dy = \int_{-\infty}^{\infty} f_N(y-x)\log_2\left[\frac{1}{f_N(y-x)}\right]dy$$ $$= \int_{-\infty}^{\infty} f_N(n)\log_2\left[\frac{1}{f_N(n)}\right]dn \quad (2)$$ $$= h(N)$$	
8.	a)	Explain the decoding procedure of convolution codes using an example.  **Figure 11.13** *(a)* Constraint length-3, rate-$\frac{1}{2}$ convolutional encoder.  	6M

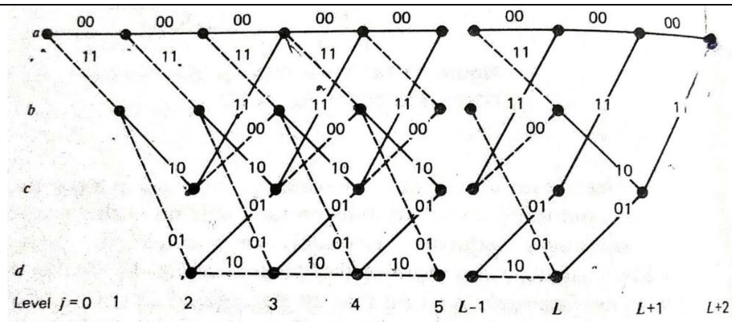


Figure 11.15 Trellis for the convolutional encoder of Fig. 11.13*a*.

Table 11.7 State Table for the Convolutional Encoder of Fig. 11.13*a*

State	Binary Description
a	00
b	10
c	01
d	11

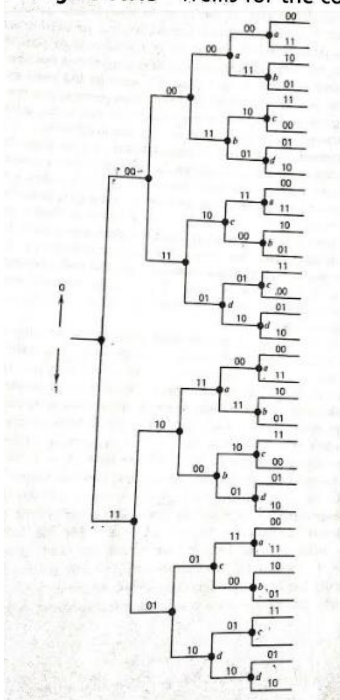


Figure 11.14 Code tree for the convolutional encoder of Fig. 11.13*a*.

b) Consider a (7, 3) cyclic code generated by G(p) = p4 + p3 + p2 + p1 . Find various code words of this code in systematic and non-systematic forms. 6M

Given that the generator polynomial for a (7,3) cyclic code is

$$g(p) = p^4 + p^3 + p^2 + 1$$

The output code words are given by

$$c(p) = M(p)g(p)$$

Tabulating the results

Input	$M(p)$	$c(p) = M(p)g(p)$	Code word	Weight of code word
000	0	0	0000000	0
001	1	$p^4 + p^3 + p^2 + 1$	0011101	4
010	p	$p^5 + p^4 + p^3 + p$	0111010	4
011	$p+1$	$p^5 + p^2 + p + 1$	0100111	4
100	p^2	$p^6 + p^5 + p^4 + p^3$	1111000	4
101	p^2+1	$p^6 + p^5 + p^3 + 1$	1101001	4
110	p^2+p	$p^6 + p^3 + p^2 + p$	1001110	4
111	p^2+p+1	$p^6 + p^4 + p + 1$	1010011	4

Note: XOR addition is used here, e.g.

$$\begin{aligned}(p^2+1)(p^4+p^3+p^2+1) &= p^6+p^5+p^4+p^2+p^4+p^3+p^2+1 \\ &= p^6+p^5+(1+1)p^4+p^3+(1+1)p^2+1 \\ &= p^6+p^5+(0)p^4+p^3+(0)p^2+1 \\ &= p^6+p^5+p^3+1\end{aligned}$$

The systematic form of the block code is,

$$\begin{aligned} X &= (\text{k message bits} : (n-k) \text{ check bits}) \\ &= (m_{k-1}\, m_{k-2} \ldots m_1\, m_0 : c_{q-1}\, c_{q-2} \ldots c_1\, c_0) \end{aligned}$$

Here the check bits form a polynomial as,

$$C(p) = c_{q-1} p^{q-1} + c_{q-2} p^{q-2} + \ldots c_1 p + c_0$$

The check bit polynomial is obtained by,

$$\boxed{C(p) = rem\left[\frac{p^q M(p)}{G(p)}\right]}$$

(OR)

9.	a)	Explain about PN Sequences and their characteristics.	6M

•Feedback shift register is said to be linear when…Feedback logic contains…Modulo 2 adders…

•Zero state is not permitted

•Period of PN Sequence cannot exceed

•When the period is exactly equal to It is called Maximal Length Sequence…or m-sequence

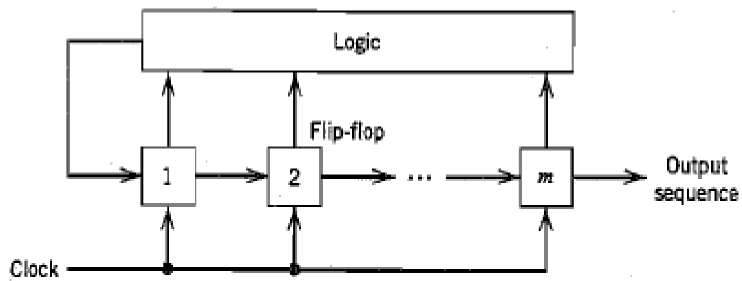


FIGURE 7.1 Feedback shift register.

100, 110, 111, 011, 101, 010, 001, 100, ...

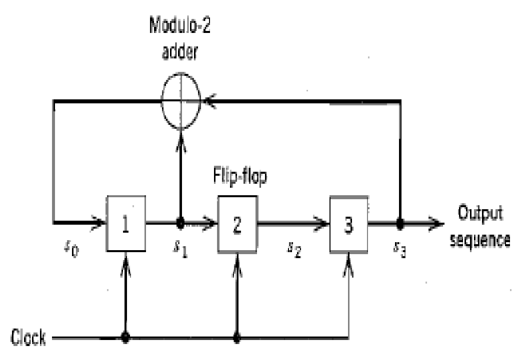


FIGURE 7.2 Maximal-length sequence generator for $m = 3$.

<table>
<tr><td></td><td></td><td>1. In each period of a maximal-length sequence, the number of 1s is always one more than the number of 0s. This property is called the *balance property*.

2. Among the runs of 1s and of 0s in each period of a maximal-length sequence, one-half the runs of each kind are of length one, one-fourth are of length two, one-eighth are of length three, and so on as long as these fractions represent meaningful numbers of runs. This property is called the *run property*. By a "run" we mean a subsequence of identical symbols (1s or 0s) within one period of the sequence. The length of this subsequence is the length of the run. For a maximal-length sequence generated by a linear feedback shift register of length m, the total number of runs is $(N + 1)/2$, where $N = 2^m - 1$.

3. The autocorrelation function of a maximal-length sequence is periodic and binary-valued. This property is called the *correlation property*.
The period of a maximum-length sequence is defined by

$$N = 2^m - 1 \qquad (7.2)$$</td><td></td></tr>
<tr><td></td><td>b)</td><td>Describe the concept of DSSS.

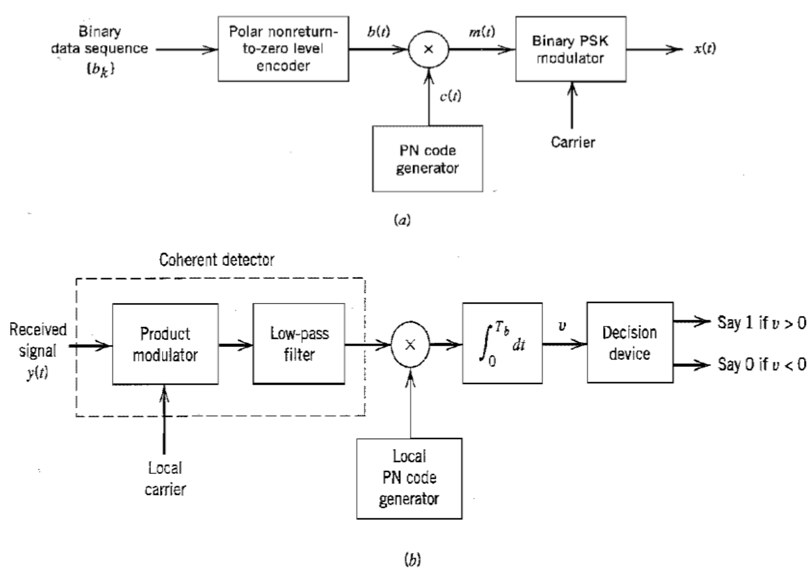
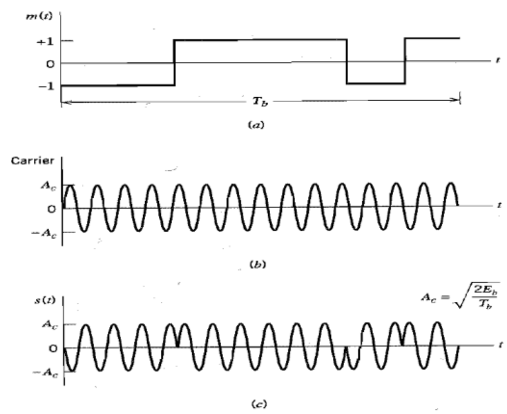


FIGURE 7.7 Direct-sequence spread coherent phase-shift keying. (*a*) Transmitter. (*b*) Receiver.</td><td>6M</td></tr>
</table>

TABLE 7.3 *Truth table for phase modulation $\theta(t)$, radians*

		Polarity of Data Sequence b(t) at Time t	
		+	−
Polarity of PN sequence c(t) at time t	+	0	π
	−	π	0



(*a*) Product signal $m(t) = c(t)b(t)$. (*b*) Sinusoidal carrier. (*c*) DS/BPSK signal.

$$y(t) = x(t) + j(t) = c(t)s(t) + j(t)$$

$$u(t) = c(t)y(t) = c^2(t)s(t) + c(t)j(t) = s(t) + c(t)j(t)$$

$$c^2(t) = 1 \qquad \text{for all } t$$

a)	Define processing gain in DPCM.
Ans	