14EE502

Hall Ticket Number:

III/IV B.Tech (Regular/Supplementary) DEGREE EXAMINATION

November, 2019 | **Electrical and Electronics Engineering**

Fifth Semester | **Control Systems**

Time: Three Hours | **Maximum:** 60 Marks

Answer Question No.1 compulsorily. (1X12 = 12 Marks)

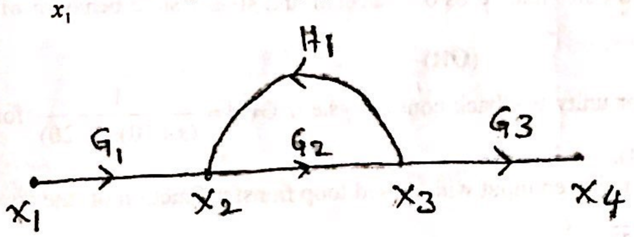
Answer ONE question from each unit. (4X12=48 Marks)

1. Answer all questions (1X12=12 Marks)

a) Name three applications for feedback control system.

b) Write the order , type whose transfer function is $\frac{C(s)}{R(s)} = \frac{1}{s^2+2s+5}$.

c) Find $\frac{x_4}{x_1}$ for the signal flow graph shown below.



d) Distinguish between type and order of a system

e) Write analogous electrical elements in torque-current analogy for the elements of rotational mechanical systems.

f) What is the effect of addition pole on transient response of the system?

g) Comment on system stability whose characteristic equation is (s+1)(s-2)(s+3)=0?

h) Define the terms "conditional stability" and "relative stability"?

i) What is the breakaway points of the root locus plot for a given open loop transfer function $G(s)H(s) = \frac{K}{s(s+2)}$

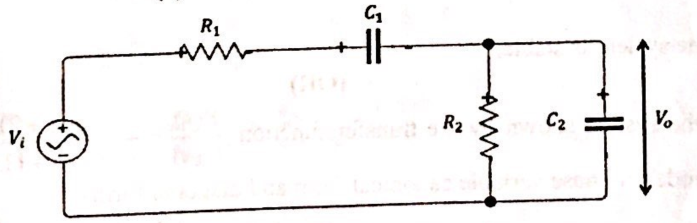
j) State Nyquist stability criterion?.

k) Define state of a system.

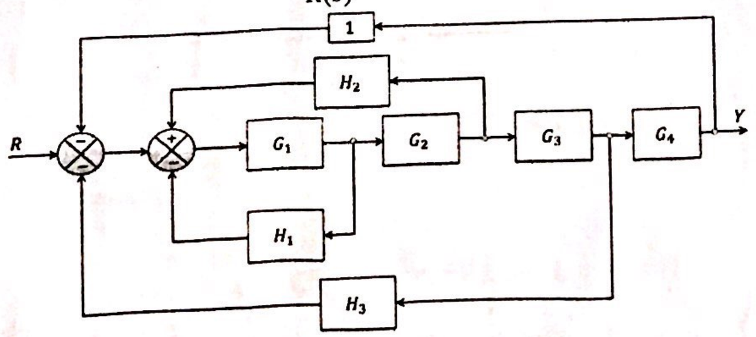
l) What is meant by observability of a system?

UNIT I

2. a) Find the transfer function $\frac{Vo(s)}{Vi(s)}$ for the circuit shown below? 6M



b) Determine the overall transfer function $\frac{Y(s)}{R(s)}$ of the following block diagram. 6M

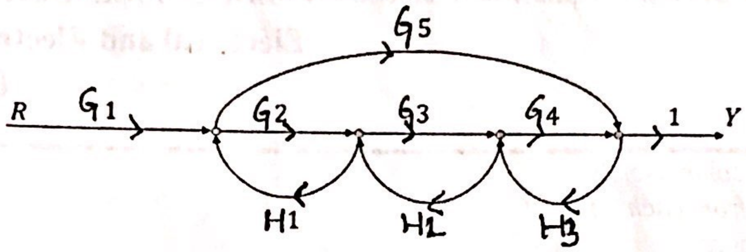


Scanned with CamScanner

(OR)

3. a) Find the transfer function of armature controlled D.C. Servomotor

   b) Find the gain $\frac{Y(s)}{R(s)}$ for the system with the following signal flow graph using mason's gain formula



UNIT II

4. a) What are the rise time, peak time, percentage peak overshoot and settling time (2% tolerance) for a system with the transfer function $\frac{Y(s)}{U(s)} = \frac{8}{s^2 + 4s + 8}$ 6M

   b) Explain the effect of addition poles and zeros on transient and steady state behavior of the system? 6M

(OR)

5. a) Find the steady state error for unity feedback control system $G(s) = \frac{1}{(s+10)(s+20)}$ for an input signal $(10+10t+10t^2)u(t)$. 6M

   b) Find the output signal for an unit step input with closed loop transfer function of the control system is $\frac{Y(s)}{U(s)} = \frac{2(s-1)}{(s+1)(s+2)}$. 6M

UNIT III

6. a) Determine the stability of the system represented by the following characteristic equation using RH criterion. $S^5 + 4S^4 + 3S^3 + 5S^2 + 2S + 5 = 0$. And find the number of roots of the characteristic equation in the RHS plane 6M

   b) Draw the polar plot for $G(s)H(s) = \frac{k}{s(s+1)(s+2)}$ with unity feedback and determine the range of 'k' for stability 6M

(OR)

7. a) Sketch the Bode plot for unity feedback control system whose open loop transfer function $G(s) = \frac{10}{s(1+0.1s)(1+0.05s)}$ and hence find Gain margin & Phase margin 6M

   b) What are the difficulties in RH criterion and explain the remedial methods? 6M

UNIT IV

8. The unity feedback control system is $\frac{k(S+9)}{S(S^2+4S+11)}$, sketch the root locus plot. Determine the 'k' for which the system is stable. 12M

(OR)

9. a) Consider the system shown by the transfer function $\frac{Y(s)}{U(s)} = \frac{20(s+2)}{s^3+9s^2+11s+3}$ and obtain the state model in Phase variable canonical form and diagonal form 6M

   b) Obtain the controllability of the given state equation $\dot{X} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$ with u(t) as unit step function and assume zero initial conditions? 6M

III/IV B.Tech Regular Degree Examination.

November - 2019.
5^{th} Semester.

Electrical and Electronics Engineering.
Control Systems - 14EE502.
Maximum: 60 Marks.

1) Answer all questions:- 1×12 = 12 Marks.

(a) amplifiers; oscillators; Counters; Flipflops to Control Mechanical, Thermal Elements.

(b) Order 1, Type 0.

(c) $\frac{G_1 G_2 G_3}{1 + G_2 H_1}$

(d) Type: No. of poles at origin
order: The Highest power of T/F Denominator value.

(e) Rotational Frictional Coefficient (B) — Conductance $\frac{1}{R}$.
Moment of inertia J — Capacitance C
Spring Constant K — inverse of Inductance $\frac{1}{L}$.

(f) pulling the Root locus to RHS of S-plane. Making the system less stable

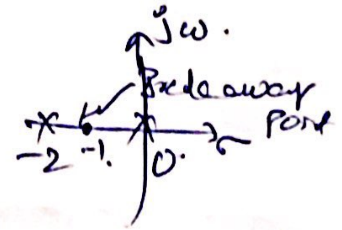
(g) system is unstable
one pole at RHS of S-plane

(h) If system o/p is stable for a limited range of variations of its parameters, then the system is called Conditionally stable.

a system Having poles away from the LHS s-plane Imaginary axis is Concidento be relative Stable.

(a)

(i) $G(s)\cdot H(s) = \dfrac{K}{s(s+2)}$. ; -1.



(j) Is a graphical Technique used in Control Engineering for determining the stability of a dynamical systems.

(K) A state is one of the set of variables that are used to describe the mathematical state of a dynamical system.

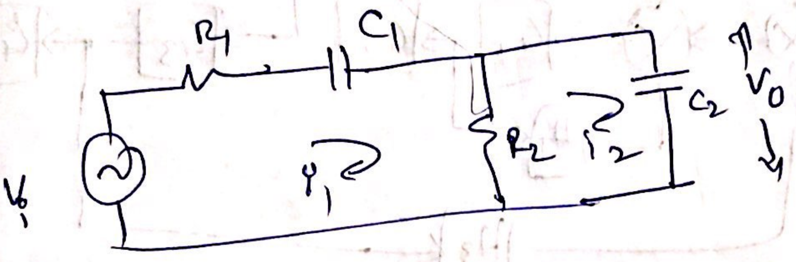
(l) observability is a measure of How well internal states of a system can be inferred from knowledge of its external outputs.

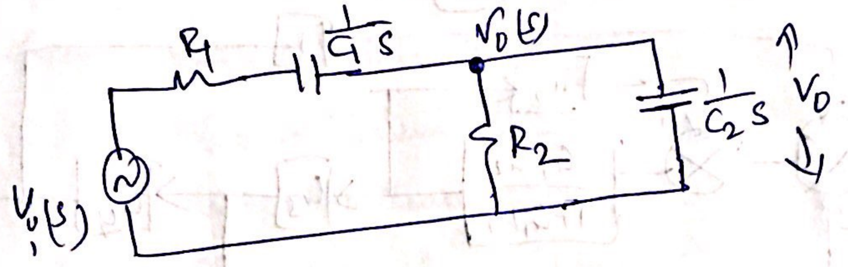
$$\left[C^T \quad A^T C^T \quad \ldots \quad (A^T)^{n-1} C^T\right].$$

LHS
2 de

UNIT- I

(2) (a)





2M

$$\frac{V_0(s)-V_i(s)}{R_1+\frac{1}{C_1 s}}+\frac{V_0}{R_2}+\frac{V_0}{\frac{1}{C_2 s}}=0 \quad \text{—(1)} \quad 1M$$

$$V_0(s)\left[\frac{1}{R_1+\frac{1}{C_1 s}}+\frac{1}{R_2}+C_2 s\right]=\left[\frac{V_i(s)}{R_1+\frac{1}{C_1 s}}\right]$$

$$V_0(s)\left[1+\frac{R_1+\frac{1}{C_1 s}}{R_2}+C_2 s\left[R_1+\frac{1}{C_1 s}\right]\right]=V_i(s) \quad 1M$$

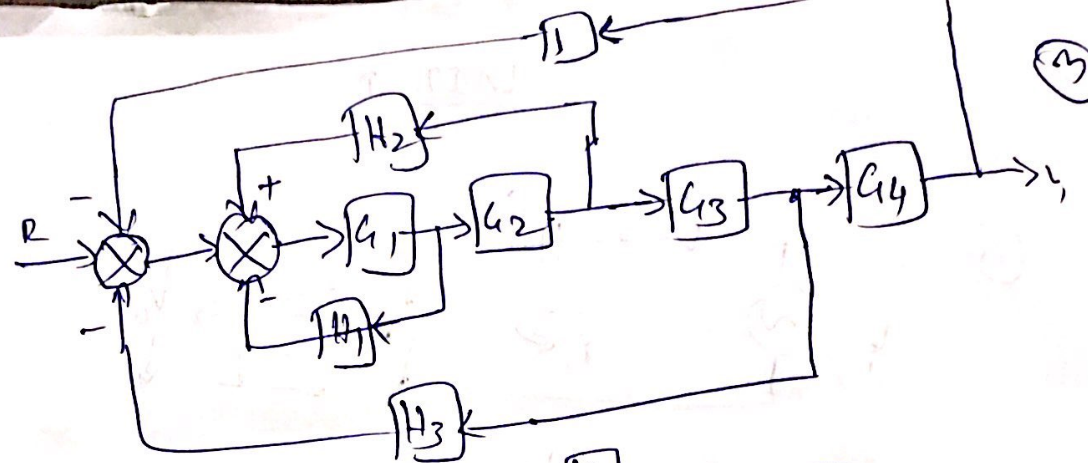
$$\frac{V_0(s)}{V_i(s)}=1\Big/\left[1+\frac{R_1 C_1 s+1}{C_1 R_2 s}+\frac{C_2 \cancel{s} R_1+C_2 \cancel{s}}{C_1 \cancel{s}}\right]$$

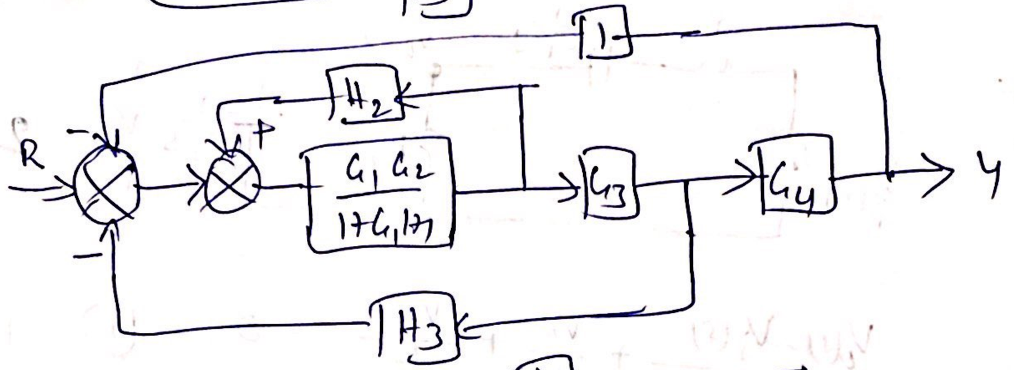
$$=1\Big/\left[\frac{C_1 R_2 s+C_1 R_1 s+1+R_1 R_2 C_2 s+R_2 C_2 s}{C_1 R_2 s}\right]$$

$$\frac{V_0(s)}{V_i(s)}=\frac{C_1 R_2 s}{s\left[C_1 R_2+C_1 R_1+R_2 C_2+R_1 R_2 C_2\right]+1} \quad 2M$$

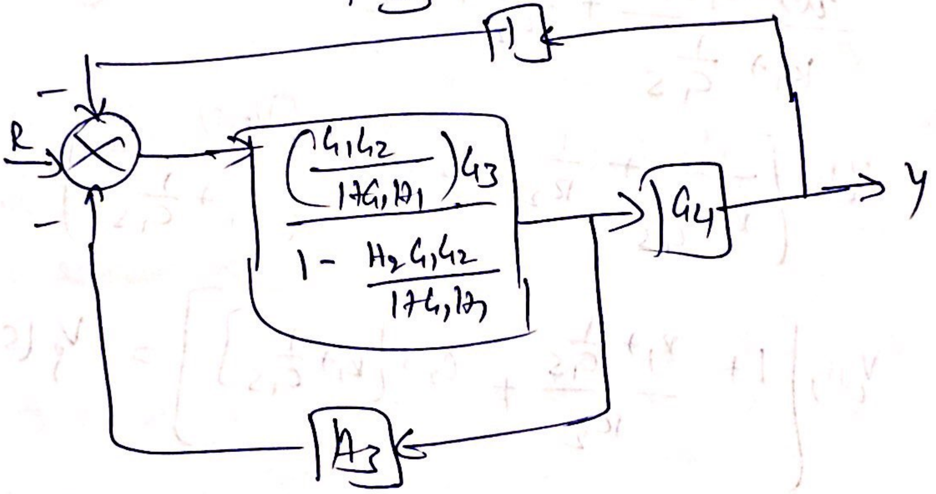
2 b

3 a

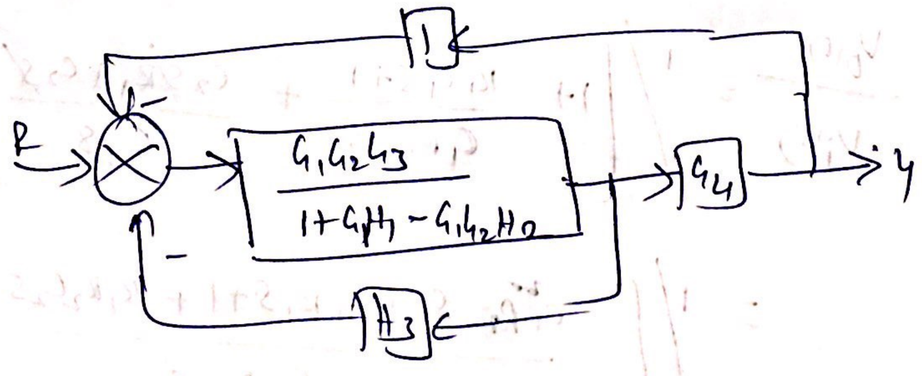




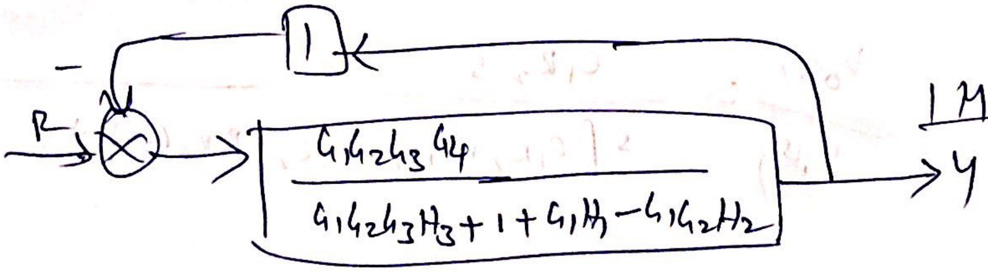
2M



1M



1M



1M

$$\frac{Y(s)}{R(s)} = G_1G_2G_3G_4 \Big/ 1+G_1H_1-G_1G_2H_2+G_1G_2G_3H_3+G_1G_2G_3G_4$$

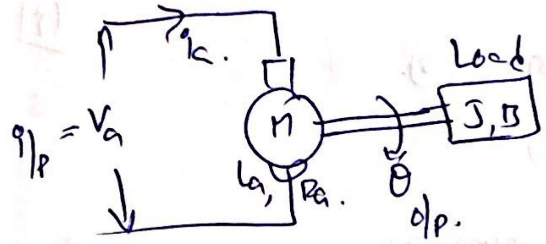
1M

Scanned with CamScanner

(OR) ③

③

a)



1M

where R_a: armature resistance Ω

V_a: armature voltage V

L_a: armature inductance H

i_a: armature Current A

e_b: Back emf V

1M

K_t = Torque Constant N-m/A.

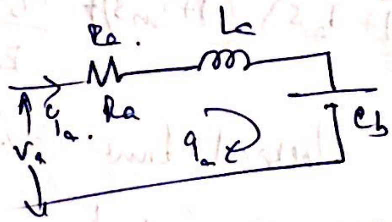
θ : angular displacement of shaft. rad.

J: moment of Inertia kg-m²/rad.

B: frictional Coefficient N-m/(rad/sec).

K_b: Back emf Constant V/rad/sec

the equivalent ckt of armature is shown in fig.

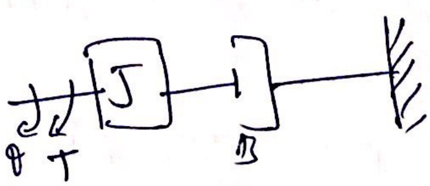


$$i_a R_a + L_a \frac{di_a}{dt} + e_b = V_a \quad \text{— ①}$$

2M

$$T \propto i_a \quad \text{— ②} \quad \therefore T = K_t i_a.$$

$$J \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T \quad \text{— ③}$$



$$\text{But } e_b = K_b \frac{d\theta}{dt}. \quad \text{— ④}$$

apply LTF to above equations

$\therefore\ I_a(s)\left[R_a + sL_a\right] + E_b(s) = V_a(s).$

$T(s) = K_t\, I_a(s)$ (1M)

$J s^2 \theta(s) + B s\, \theta(s) = T(s).$

$E_b(s) = K_b\, s \cdot \theta(s).$

$\therefore$ from the above equation

$$I_a(s) = \frac{Js^2 + Bs}{K_t}\,\theta(s).$$

$$(R_a + sL_a)\, I_a(s) + E_b(s) = V_a(s).$$

$$\left[\frac{(R_a + sL_a)(Js^2 + Bs) + K_b K_t s}{K_t}\right]\theta(s) = V_a(s)$$

$$\therefore \frac{\theta(s)}{V_a(s)} = \frac{K_t}{R_a\left(\frac{sL_a}{R_a}+1\right) Bs\left[1 + \frac{Js^2}{Bs}\right] + K_b K_t s}$$ 1M

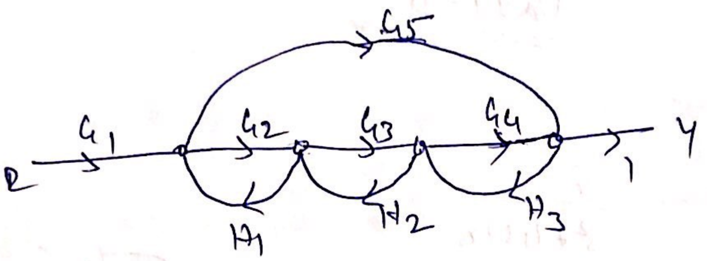
$$= \frac{K_t / R_a B}{s\left[(1 + T_a s)(1 + T_m s) + \frac{K_b K_t}{R_a B}\right]}$$

$\frac{L_a}{R_a} = T_a =$ Electrical time Constant.

$\frac{J}{B} = T_m =$ Mechanical time Constant.

3 b

4



Mason's gain formula.

$$T = \frac{1}{\Delta} \sum_{K} \Delta_K P_K \qquad \text{— 1M}$$

T = Transfer function of the system.

P_K = F/w path gain of k^{th} path — 1M

K = No. of F/w paths in the signal flow graph

Δ = 1 − (Sum of individual loop gains)
+ (Sum of gain products of all possible combinations of Two-non-Touching loops)

− . . .

Δ_K = Δ for that part of graph which is not touching k^{th} F/w path.

$P_1 = G_1G_2G_3G_4$ 1M

$P_2 = G_1G_5$

$$\Delta = 1 - [G_2H_1 + G_3H_2 + G_4H_3 + G_5H_1H_2H_3] \qquad 2M$$
$$+ [G_2H_1\ G_4H_3]$$

$\Delta_1 = 1$

$\Delta_2 = 1 - [G_3H_2]$

$$\therefore T = \frac{G_1G_2G_3G_4 + G_1G_5[1 - G_3H_2]}{1 - G_2H_1 - G_3H_2 - G_4H_3 - G_5H_1H_2H_3 + G_2G_4H_1H_3} \qquad 1M$$

UNIT - II

4 b

(4)

(a) $\frac{Y(s)}{R(s)} = \frac{8}{s^2+4s+8}$

$t_r = \frac{\pi-\theta}{\omega_d}$. where $\omega_d = \omega_n\sqrt{1-\xi^2}$.

$t_p = \frac{\pi}{\omega_d}$.

2M

$\% M_p = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \times 100$.

$t_s = \frac{4}{\xi\omega_n}$.

from the T/F $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

$\omega_n = \sqrt{8} = 2.828$

$2\xi\omega_n = 4$.

$2\times 2.828 \times \xi = 4$

$\xi = 0.707$

$\therefore \theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = \tan^{-1}\frac{\sqrt{1-0.707^2}}{0.707} = 45$

$\theta = 0.785\,rad$

$\therefore t_r = \frac{\pi-0.785}{2.828\sqrt{1-0.707^2}} = \frac{2.356}{1.99} = 1.178$ 1M

$t_p = \frac{\pi}{1.99} = 1.578$ 1M

$\% M_p = e^{\frac{-0.707\cdot\pi}{\sqrt{1-0.707^2}}} \times 100 = 4.325\,\%$. 1M

$t_s = \frac{4}{0.707\times 2.828} = \underline{\underline{2.0006}}$ 1M

(4)

(b) The roots of the characteristic equation which are the poles of the closed loop transferfunction affect the transient response of linear time-invariant systems, particularly the stability. the zeros of the T/F.

1M

Addition of pole to the F/W path T/F. which to the Reduce Stability. it Increase the raise time of the step response. reduce the B/w.

1M

Addition of pole to closed loop T/F- Increase the raise time, reduce the overshoot.

1M

Addition of zero to the closed loop T/F. decrease the raise time. Increase the max. overshoot. of the step response.

1M

Addition of zero to the F/w path T/F. the overshoot is large and the damping is very poor. when zero moves closer to the origin the overshoot Increases but damping Improves.

1M

The Conclusion is that although the characteristic equation roots are generally used to study the relative damping and relative stability of linear control systems, the zeros of the T/F should not be overlooked in their effects on the transient performance of the systems.

Scanned with CamScanner

(OR)

(5) (6)

(5)

(a) $G(s) = \frac{1}{(s+10)(s+20)}$

$i/p = (10 + 10t + 10t^2)\,u(t)$.

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot R(s)}{1+G(s)\cdot H(s)}.$$ 1 M

$$R(s) = 10 + \frac{10}{s^2} + \frac{10\times 2!}{s^3} = 10 + \frac{10}{s^2} + \frac{20}{s^3}$$ 1 M

$H(s) = 1$

$$\therefore e_{ss} = \lim_{s\to 0} \frac{s\times\left[10 + \frac{10}{s^2} + \frac{20}{s^3}\right]}{1 + \frac{1}{(s+10)(s+20)}\times 1}$$ 2 M

$$= \lim_{s\to 0} \frac{s\left[\frac{10s^3 + 10s + 20}{s^3}\right]\times(s+10)\times(s+20)}{(s+10)(s+20)+1}$$

$$= \lim_{s\to 0}\left[\frac{(10s^3+10s+20)(s+10)(s+20)}{s^2\left[1+(s+10)(s+20)\right]}\right]$$ 2M

$= \infty$.

5

b

$$\frac{Y(s)}{U(s)} = \frac{2(s-1)}{(s+1)(s+2)}$$

unit step i/p $u(t) = 1$

$U(s) = 1/s$.

$$\therefore Y(s) = \frac{1}{s} \times \frac{2(s-1)}{(s+1)(s+2)} \qquad \underline{1M}$$

by practial fraction method

$$\frac{2s-2}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} \qquad \underline{2M}$$

$$= \frac{A(s+1)(s+2) + Bs(s+2) + C(s+1)s}{s(s+1)(s+2)}$$

$$2s-2 = A(s^2+3s+2) + B(s^2+2s) + C(s^2+s).$$

$\therefore -2 = 2A \quad \therefore \boxed{A = -1}$

$A+B+C = 0$

$3A + 2B + C = 2$

where $s = -1$

$-B = -4$

$\boxed{B = 4}$

$A+B+C = 0$

$-1+4+C = 0$

$C = -3$

$$\therefore Y(s) = \frac{-1}{s} + \frac{4}{s+1} - \frac{3}{s+2} \qquad \underline{2M}$$

I.L.T to the above equation we get

$$L^{-1}[Y(s)] = y(t) = L^{-1}\left[\frac{-1}{s} + \frac{4}{s+1} - \frac{3}{s+2}\right]$$

$$y(t) = -1 + 4e^{-t} - 3e^{-2t}. \qquad \underline{1M}$$

Scanned with CamScanner

UNIT - III

(6) (a) $s^5 + 4s^4 + 3s^3 + 5s^2 + 2s + 5 = 0$.

s^5	1	3	2
s^4	4	5	5
s^3	$\frac{4\times3 - 5\times1}{4} = 1.75$	0.75	
	1.75	0.75.	
s^2	3.28	5	
s^1	-1.91		
s^0	-5		

2M

2M

System is unstable. 1M

one root at RHS of s-plane. 1M

(6) (b) $G(s) \cdot H(s) = \frac{K}{s(s+1)(s+2)}$.

let $K=1$.

$\therefore G(s) \cdot H(s) = \frac{1}{s(s+1)(s+2)}$.

Type 1

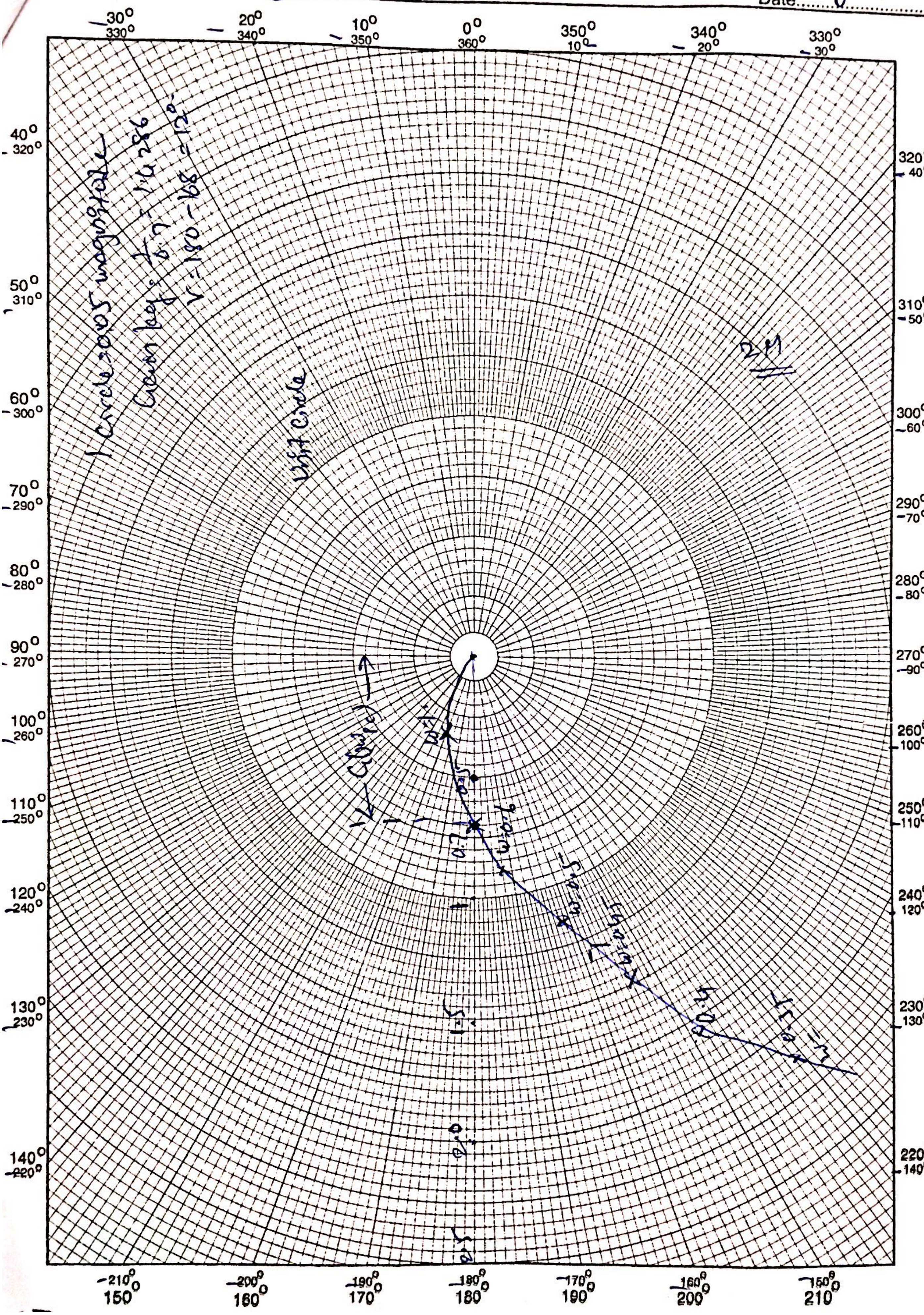
order 3.

$G(j\omega) = \frac{1}{j\omega(1+j\omega)(1+2j\omega)}$ 1M

$|G(j\omega)| \angle G(j\omega) = \frac{1}{\omega\sqrt{1+5\omega^2+4\omega^4}} \quad \angle -90 - \tan^{-1}\omega - \tan^{-1}2\omega$

(6)(b).

Date Pg - 7th.



(7)

ω rad/sec	0.35	0.4	0.45	0.5	0.6	0.7	1
$\lvert G(j\omega)\rvert$	2.2	1.8	1.5	1.2	0.9	0.7	0.3
$\angle G(j\omega)$ deg	-144	-150	-156	-162	-171	-180	-198

2M

Gain Margin $k_g = 1.4286$

$\gamma = +12^\circ$

characteristic equation :-

$$s^3 + 3s^2 + 2s + K = 0$$

$$\begin{array}{c|cc} s^3 & 1 & 2 \\ s^2 & 3 & K \\ s^1 & \frac{6-K}{3} & \\ s^0 & K. & \end{array}$$

1M

$\therefore \frac{6-K}{3} > 0$ $\quad\therefore 6-K>0$ and $K>0.$

$K-6<0$

$K<6$

$0<K<6$ is the range of K.

(OR)

(7)

(a) $G(s) = \dfrac{10}{s(1+0.1s)(1+0.05s)}$ 1M

$\omega_{c_1} = \dfrac{1}{0.1} \doteq 10$ $\quad\therefore \omega_l = 0.1$ rad/sec

$\omega_{c_2} = \dfrac{1}{0.05} = 20.$ $\quad\omega_h = 50$ rad/sec.

Term	Corner frequency rad/sec	Slope rad/sec	Change in slope db/dec
$\frac{10}{j\omega}$	—	-20	
$\frac{1}{1+0.1j\omega}$	$\omega c_1 = \frac{1}{0.1} = 10$	-20	$-20-20=-40$
$\frac{1}{1+0.05j\omega}$	$\omega c_2 = \frac{1}{0.05} = 20$	-20	$-40-20=-60$

1M

at $\omega=\omega_l = 0.1$: $20\log\left|\frac{10}{j\omega}\right|_{\omega=0.1} = 20\log\frac{10}{0.1} = 40$ db

$\omega=\omega c_1 = 10$: $20\log\frac{10}{10} = 0$

1M

$\omega=\omega c_2 = 20$: $A = \left[-40\times\log\frac{20}{10}\right]+0 = -12.04$

$\omega=\omega_h = 50$: $A = \left[-60\times\log\frac{50}{20}\right]-12.04 = -35.916.$

$\phi = -90^\circ - \tan^{-1}0.1\omega - \tan^{-1}0.05\omega.$

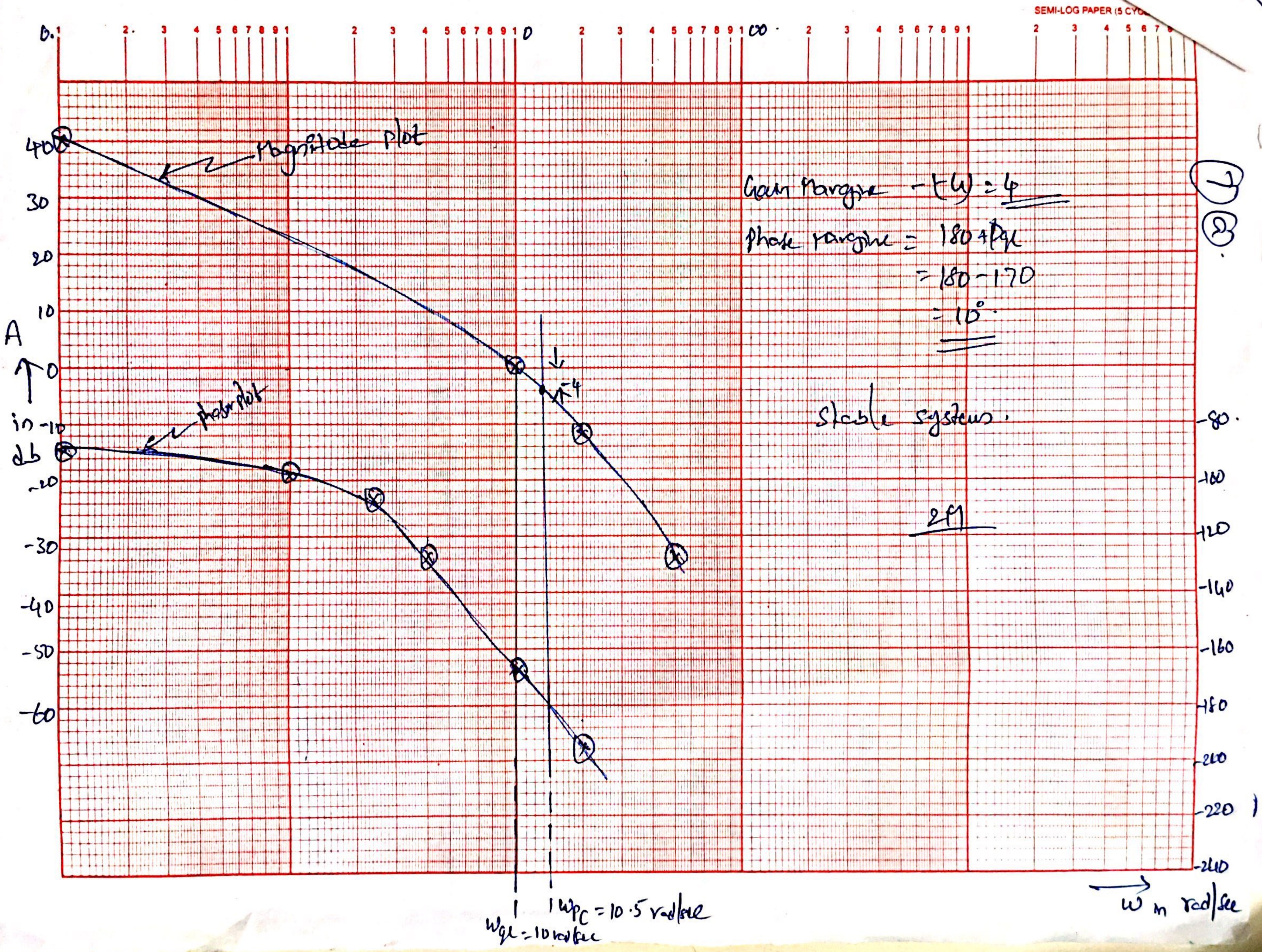
ω rad/sec	ϕ deg
0.1	-90
1	-98
2.5	-111
4	-123
10	-162
20	-198

1M

Gain cross over frequency = 10 rad/sec

phase cross over frequency = 10.5 rad/sec

$\boxed{\text{STABLE}}$ systems.



(7) (8)

Gain margin $- F(\omega) = 4$

Phase margin $= 180 + \phi_{gc}$

$= 180 - 170$

$= 10°$

Stable system.

2M

(7)

(8)

(b) A row of all zeros -:-

Determine the Auxiliary polynomial., A(s) 2M

Differentiate the auxiliary polynomial with respect to s.

so to get d A(s)/ds.

The row of zeros is replaced with coefficients of

d A(s)/ds.

Continue the construction of the array in usual manner.

Determine the Auxiliary polynomial, A(s)

Divide the characteristic equation by Auxiliary polynomial

Construct Routh array using the coefficients of quotient

polynomial.

First element of a row is zero :-

2M

To overcome this problem let 0 → ε and complete

the construction of array in the usual way which are

function of ε.

If no sign change and if no row with all zeros.

then roots are lying LHS. system stable.

2M

If sign changes in first column some of the roots are

RHS of s-plane system is unstable.

ε → 0 if there is a row of all zeros then the roots

on Imaginary axis.

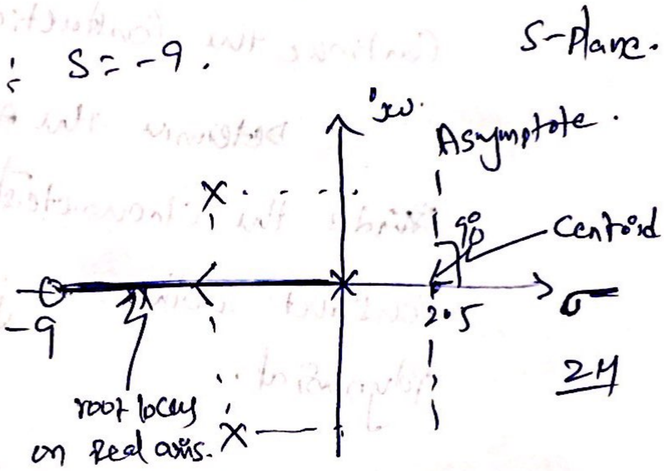
UNIT - IV

(8) $\frac{K(s+9)}{s(s^2+4s+11)}$

Poles of root locus : $s = 0$
(X) $s^2+4s+11 = 0$

$s = 0, \quad s = -2 \pm j2.64$. 1M

Zeros of root locus : $s = -9$.
(O)



2M

Angle of Asymptotes :- $= \frac{\pm 180(2q+1)}{n-m}$, $q = 0, 1, 2 \ldots n-m$

$m = 01$
$n = 3$

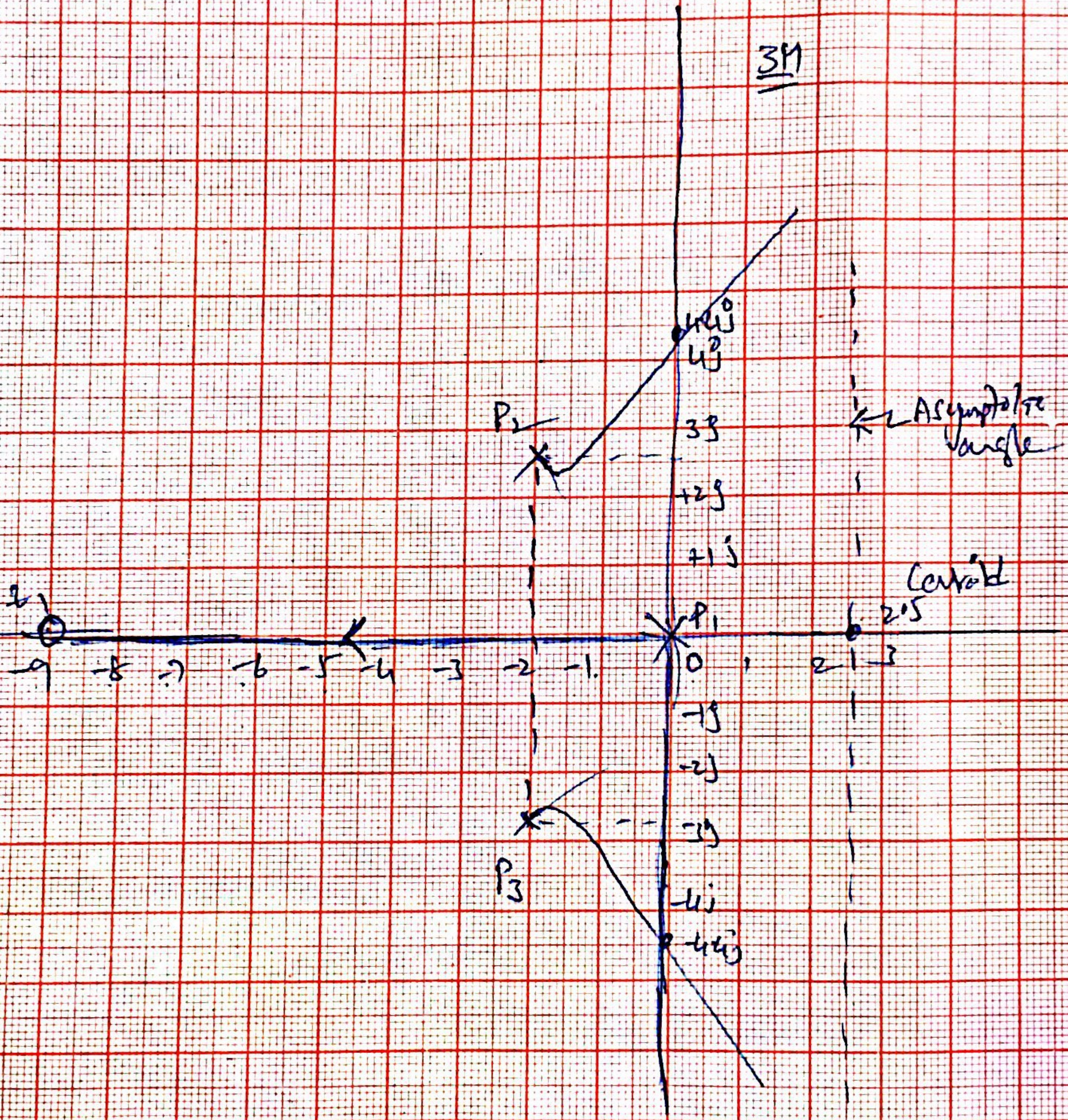
$\therefore$ Angle $= \frac{\pm 180(2q+1)}{3-1} = \pm 90°$. 1M

Centroid $= \frac{\Sigma \text{ poles} - \Sigma \text{ zeros}}{n-m}$

$= \frac{0 - 2 + j2.64 - 2 - j2.64 - (-9)}{3-1}$ 1M

$= 5/2 = 2.5$.

There is no Breakaway or Breakin points.

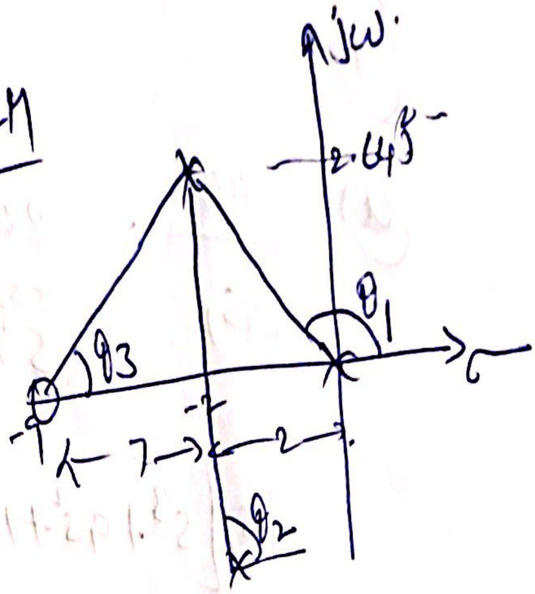

3M
+4j
4j
P2
3j
+2j
+1j
← Asymptote angle
Centroid
2.5
P1
z1
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
-1j
-2j
-3j
P3
-4j
-4j

Angle of departure:-

$\theta_1 = 180 - \tan^{-1}\frac{2.64}{2} = 127.1$ 2M

$\theta_2 = 90$

$\theta_3 = \tan^{-1}\frac{2.64}{7} = 20.7$



angle of departure = $180 - (\theta_1 + \theta_2) + \theta_3$

$= 180 - (127.1 + 90) + 20.7$

$= -16.4°$. 1M

Crossing points

$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{K(s+9)}{s(s^2+4s+11)+(s+9)K}$ 1M

∴ Characteristic equation

$(s^3+4s^2+11s) + Ks + 9K = 0.$

$s = j\omega$.

Imaginary part = 0

∴ $\omega = \pm 4.4$

1M

Real part = 0

$K = 8.8$

$6\omega^2 + 9K = 0$.

1M

(OR)

(9)

(a) $\frac{Y(s)}{U(s)} = \frac{20(s+2)}{s^3+9s^2+11s+3}$.

$Y(s)\left[s^3+9s^2+11s+3\right] = 20(s+2)U(s)$

$\dddot{y} + 9\ddot{y} + 11\dot{y} + 3y = \dot{u}\,20 + 40u$

$x_1 = y$

$x_2 = \dot{y}$ 2M

$x_3 = \ddot{y}$

$\dot{x}_3 + 9x_3 + 11x_2 + 3x_1 =$

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -11 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + u$ (elements etc.).

from Here $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -11 & -9 \end{bmatrix}$ 1M

Canonical form and diagonal forms.

$(\lambda I - A) = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -11 & -9 \end{bmatrix}$

$= \begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 3 & 11 & \lambda+9 \end{bmatrix}$

Scanned with CamScanner

$$|\lambda I - A| = \lambda[\lambda^2+9\lambda+11]+1[3]+0$$

$$\lambda^3+9\lambda^2+11\lambda+3=0$$

$$\therefore \lambda_1 = -0.39,\ \lambda_2 = -7.6\quad \lambda_3 = -1.$$

$$[\lambda_1 I - A] = \begin{bmatrix} -0.39 & 0 & 0 \\ 0 & -0.39 & 0 \\ 0 & 0 & -0.39 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -11 & -9 \end{bmatrix}$$

1M

$$= \begin{bmatrix} -0.39 & -1 & 0 \\ 0 & -0.39 & -1 \\ 3 & 11 & 8.61 \end{bmatrix}$$

$$\therefore m_1 = \begin{bmatrix} 7.64 \\ -3 \\ 1.17 \end{bmatrix}$$

$$[\lambda_2 I - A] = \begin{bmatrix} -7.6 & 0 & 0 \\ 0 & -7.6 & 0 \\ 0 & 0 & -7.6 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -11 & -9 \end{bmatrix}$$

1M

$$= \begin{bmatrix} -7.6 & -1 & 0 \\ 0 & -7.6 & -1 \\ 3 & 11 & 1.4 \end{bmatrix}$$

$$m_2 = \begin{bmatrix} 0.36 \\ -3 \\ 22.8 \end{bmatrix}$$

$$[\lambda_3 I - A] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -11 & -9 \end{bmatrix}$$

1M

$$= \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & -1 \\ 3 & 11 & 8 \end{bmatrix} \quad \therefore m_3 = \begin{bmatrix} 3 \\ -3 \\ 3 \end{bmatrix}$$

$$M = \begin{bmatrix} 7.64 & 0.36 & 3 \\ -3 & -3 & -3 \\ 1.17 & 22.8 & 3 \end{bmatrix}$$

$$\therefore J = M^{-1} A M = \begin{bmatrix} 7.64 & 0.36 & 3 \\ -3 & -3 & -3 \\ 1.17 & 22.8 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -11 & -9 \end{bmatrix} \begin{bmatrix} 7.64 & 0.36 & 3 \\ -3 & -3 & -3 \\ 1.17 & 22.8 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -0.35 & -0.06 & 0 \\ -9.30\times10^{-5} & -7.60 & 0 \\ 4.54\times10^{-3} & 0.06 & -1 \end{bmatrix}$$

(9)

(b) $A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$,

$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Controllability : $Q_c = \begin{bmatrix} B & AB & A^2B & \dots & A^{n-1}B \end{bmatrix} \neq 0.$

2M

$$\therefore AB = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$ 1M

$$A^2 = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ 2 & 3 \end{bmatrix}$$ 1M

$$A^2B = \begin{bmatrix} -1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$\therefore Q_c = \begin{bmatrix} B & AB \end{bmatrix}$

Det of $Q_c = \begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix}$ 2M

$= -1 \neq 0$.

$\therefore$ Given system is Controllable. for zero initial Condition.

HOD, EEE Department

BEC, Bapatla - 522101.

Prepared by me.

1) P. Sampath kumar
Asst professor
EEE Dept
BEC. Bapatla.

12/11/2019

2. Dr M. S. Dinesh

12/11/19

3. M. Suresh Babu