Hall Ticket Number:

III/IV B.Tech (Regular\Supplementary) DEGREE EXAMINATION

November, 2019 — **Electrical and Electronics Engineering**

Fifth Semester — **Signals & Systems**

Time: Three Hours — **Maximum:** 60 Marks

Answer Question No.1 compulsorily. (1X12 = 12 Marks)

Answer ONE question from each unit. (4X12=48 Marks)

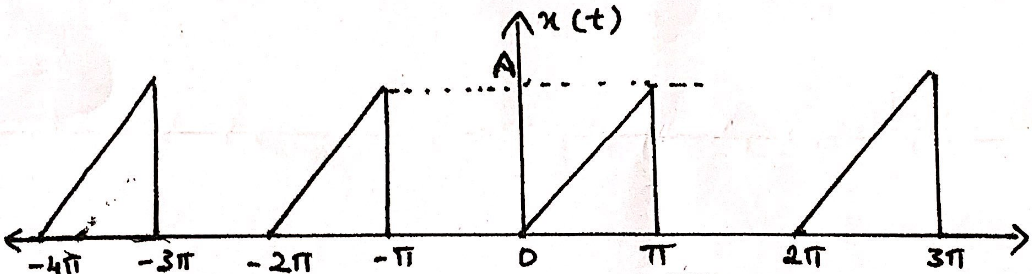
1. Answer all questions (1X12=12 Marks)
   a) Differentiate energy and Power signals.
   b) Determine the fundamental period of a signal $x(t) = \cos^2(3\pi t)$. $= 1/3$
   c) Define Deterministic and random signals.
   d) What are the conditions for causality and stability of LTI system?
   e) Draw the frequency response of an ideal Band pass filter.
   f) State Parseval' energy theorem.
   g) Give the relationship between convolution and correlation.
   h) Define Energy Spectral Density.
   i) What are different sources of Noise?
   j) Define Joint Probability and conditional Probability.
   k) State Baye's theorem.
   l) What is a Gaussian Random variable?

UNIT I

2. a) Determine whether the following system is Linear, Time-invariant, causal and stable with appropriate test conditions $y(t) = t\,x(t-2)$ — 6M

   linear
   time variant
   causal

   b) Find the trigonometric Fourier series for the wave form shown below. — 6M



$a_0 = A/4$

$a_n = -\frac{2A}{\pi^2 n^2}$

$b_n = A/n\pi$

(OR)

3. a) Find the Fourier transform of a signal $x(t) = t+1;\ -1 < t < 0$
   $= 1-t;\ 0 < t < 1$
   $= 0;$ otherwise — 6M

   $\frac{2(1-\cos\omega)}{\omega^2}$

   b) State and prove Sampling theorem. — 6M

UNIT II

4. a) Compute the convolution of the following signals $x(t) = e^{-t}u(t-2)$ and $h(t) = e^{-|t|}$ — 6M
   b) Derive the relationship between autocorrelation and power spectral density — 6M

(OR)

5. a) Explain about different properties of impulse function. — 6M
   b) Derive the relationship between bandwidth and rise time — 6M

UNIT III

6. a) Calculate the equivalent noise bandwidth of a Low pass RC filter. — 6M
   b) Evaluate the thermal noise voltage developed across a resistor of 1KΩ. The bandwidth of the measuring instrument is 8 MHz and ambient temperature is 28^0C. — 6M
   c) Write short notes on Noise Power spectral Density.

(OR)

7. a) Explain classification of noise. — 6M
   b) Derive the expression for noise figure of a multistage amplifier. — 6M

UNIT IV

8. A random variable X is given by $f_X(x) = \frac{\pi}{16}\cos\left(\frac{\pi x}{8}\right);\ -4 \le x \le 4$
   $= 0;$ otherwise — 12M

   Determine mean, variance and standard deviation

(OR)

9. a) Explain about cumulative distribution function. — 6M
   b) Explain about probability density function. — 6M

III/IV B.Tech Regular Degree Examination

EEE Department

Nov-2019

14EE506A Signals & Systems

Fifth Semester

a)

Total energy $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

Energy signals $P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$

1) $\boxed{E = \text{finite} \Rightarrow P = 0} \longrightarrow$ Energy Signal.

2) Energy Signals are absolutely integrable signals

3) Total energy 'E' = area under $|x(t)|^2$ graph

Power signals

1) $\boxed{P = \text{finite} \Rightarrow E = \infty} \longrightarrow$ Power signal

2) Periodic signals are Power signals but vice versa is not true.

3) $P = (RMS)^2 \Rightarrow RMS = \sqrt{P}$

(b) $x(t) = \cos^2(3\pi t)$

$= \frac{1}{2} \cos(6\pi t)$

$\omega = 2\pi f \Rightarrow \omega = \frac{2\pi}{T}$ $T = \frac{2\pi}{\omega} = \frac{2\pi}{6\pi} = \frac{1}{3}$

(c). Deterministic signals

signals which can be defined exactly

Random signals

Non deterministic signals are random signals

(d) Stable: if every bounded i/p produces a bounded o/p (or) Impulse response function is absolutely integrable

LTI: o/p depends on the current & past not on future

(e) $H(j\omega) = \begin{cases} 0 & |\omega| < \omega_1 \\ 1 & \omega_1 \le |\omega| < \omega_2 \\ 0 & |\omega| > \omega_2 \end{cases}$

If $x(t) \longleftrightarrow X(j\omega)$

(f) $E_x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |x(j\omega)|^2 \, d\omega$

(g) Correlation is measurement of the similarity between two signals

Convolution is measurement of effect of one signal on the other signal

(h) Energy spectral density
Energy concerted around a finite time interval

(i) industrial noise

Thermal noise noise.

(k) Bayes theorem
Bayes theorem describes the probability of an event based on prior knowledge of condition that might be related to the event.

(l) Gaussian random variable

In probability theory the normal distribution is a very common continuous probability distribution.

UNIT-I

Determine whether the following system is Linear, Time-invariant, causal and stable with appropriate test conditions $y(t) = tx^2(t-2)$

Form = 3½
Solu = 3

$y(t) = tx^2(t-2)$

* let input $x_1(t)$ produces an output $y_1(t)$

$$y_1(t) = t\,x_1^2(t-2)$$

let input $x_2(t)$ produces an output $y_2(t)$

$$y_2(t) = tx_2^2(t-2)$$

The linear combination of above two equations can be written as

$$\underbrace{ay_1(t) + by_2(t)}_{\text{weighted sum of output}} = \underbrace{atx_1^2(t-2) + bt\,x_2^2(t-2)}_{\text{weighted sum of input}}$$

$\boxed{\text{Therefore Given system is linear}}$

* $y(t,T) = tx^2(t-2-T)$

$$= tx^2(t-T-2)$$

$$y(t-T) = (t-T)\,x^2(t-T-2)$$

$$y(t,T) \neq y(t-T)$$

$\boxed{\because \text{The given system is time variant}}$

* $y(t) = tx^2(t-2)$

for $t=1$, $y(1) = x^2(1-2) = x^2(-1)$

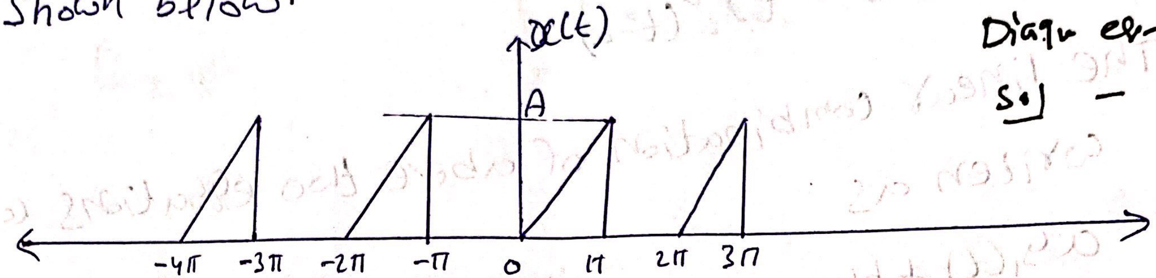
$t=2$, $y(2) = 2x^2(0)$

$t=1$, $y(-1) = -x^2(-3)$

for all values of 't' The output depends only on present and past values of input so system is causal

2(b) Find the trigonometric ~~Fouries~~ Fourier series for the wave form shown below.

Diagram — 2M.
Sol — 4M



The given waveform is periodic with a period $T = 2\pi$

$t_0 = 0, \quad t_0 + T = 2\pi$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$$

The wave form is described by

$$x(t) = \begin{cases} (A/\pi)t, & 0 \le t \le \pi \\ 0, & \pi \le t \le 2\pi \end{cases}$$

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{2\pi}\int_0^{2\pi} x(t)\,dt$$

$$= \frac{1}{2\pi}\int_0^{\pi} \frac{A}{\pi} t\,dt = \frac{A}{2\pi^2}\left[\frac{t^2}{2}\right]_0^{\pi}$$

$$\boxed{a_0 = \frac{A}{4}}$$

$$a_n = \frac{2}{T}\int_0^T x(t)\cos n\omega_0 t\, dt$$

$$= \frac{2}{2\pi}\int_0^{\pi} \frac{A}{\pi} t\cos nt\, dt = \frac{A}{\pi^2}\int_0^{\pi} t\cos nt\, dt$$

$$= \frac{A}{\pi^2}\left[\left(\frac{t\sin nt}{n}\right)_0^{\pi} - \int_0^{\pi}\frac{\sin nt}{n}dt\right]$$

$$= \frac{A}{\pi^2}\left[\frac{0-0}{n} + \left(\frac{\cos nt}{n^2}\right)_0^{\pi}\right]$$

$$= \frac{A}{n^2\pi^2}(\cos n\pi - \cos 0)$$

$$\boxed{a_n = \begin{cases} -\dfrac{2A}{\pi^2 n^2} & \text{for odd } n \\ 0 & \text{for even } n \end{cases}}$$

$$b_n = \frac{2}{T}\int_0^T x(t)\sin n\omega_0 t\, dt$$

$$= \frac{2}{2\pi}\int_0^{2\pi} x(t)\sin nt\, dt = \frac{1}{\pi}\int_0^{\pi}\frac{A}{\pi}t\sin nt\, dt$$

$$= \frac{A}{\pi^2}\int_0^{\pi} t\sin nt\, dt = \frac{A}{\pi^2}\left[\left(\frac{-t\cos nt}{n}\right)_0^{\pi} - \int_0^{\pi}\frac{-\cos nt}{n}dt\right]$$

$$= \frac{A}{\pi^2}\left[\frac{-\pi\cos n\pi}{n} + \left(\frac{\sin nt}{n^2}\right)_0^{\pi}\right]$$

$$= -\frac{A}{n\pi}\cos n\pi$$

$$b_n = \begin{cases} A/n\pi & \text{for odd } n \\ -A/n\pi & \text{for even } n \end{cases}$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n\cos n\omega_0 t + b_n\sin n\omega_0 t$$

$$x(t) = \frac{A}{4} - \frac{2A}{\pi^2}\sum_{n=odd}^{\infty}\frac{\cos nt}{n^2} + \frac{A}{\pi}\sum_{n=1}^{\infty} -(-1)^n\frac{\sin nt}{n}$$

9(

$$x(t) = \frac{A}{4} - \frac{2A}{\pi^2}\left[\cos t + \frac{\cos 3t}{9} + \frac{\cos 5t}{25} + \cdots\right]$$

$$+ \frac{A}{\pi}\left[\sin t - \frac{\sin 2t}{2} + \frac{\sin 3t}{3} - \frac{\sin 4t}{4} + \cdots\right].$$

3(a) Find the Fourier transform of a signal

$$x(t) = \begin{cases} t+1; & -1<t<0 \\ 1-t; & 0<t<1 \\ 0; & \text{otherwise} \end{cases}$$

$$\cancel{X(\omega) =} \int_{-1}^{0}$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{-1} x(t)e^{-j\omega t} dt + \int_{-1}^{0} x(t)e^{-j\omega t} dt + \int x(t)e^{-j\omega t} dt + \int_{1} x(t)e^{-j\omega t} dt$$

$$= \int_{-1}^{0} (t+1)e^{-j\omega t} dt + \int_{1}^{0} (1-t)e^{-j\omega t} dt$$

$$= \int_{-1}^{0} te^{-j\omega t} dt + \int_{-1}^{0} e^{-j\omega t} dt + \int_{0}^{\phi} e^{-j\omega t} dt - \int_{\phi}^{\phi} te^{-j\omega t} dt$$

$$= \left(\frac{te^{-j\omega t}}{-j\omega}\right)_{-1}^{0} - \left(\frac{e^{-j\omega t}}{(-j\omega)^2}\right)_{-1}^{0} + \left(\frac{e^{-j\omega t}}{-j\omega}\right)_{-1}^{0} + \left(\frac{e^{-j\omega t}}{-j\omega}\right)_{0}^{\phi}$$

$$\left(-\frac{te^{-j\omega t}}{-j\omega}\right)_{0}^{\phi} + \left(\frac{e^{-j\omega t}}{(-j\omega)^2}\right)_{0}^{\phi}$$

~~$x(\omega) = 0 +$~~

$$X(\omega) = 0 - \cancel{\frac{e^{j\omega}}{j\omega}} + \frac{1}{\omega^2} - \frac{e^{j\omega}}{\omega^2} - \cancel{\frac{1}{j\omega}} + \cancel{\frac{e^{j\omega}}{j\omega}} - \cancel{\frac{e^{-j\omega}}{j\omega}} + \cancel{\frac{1}{j\omega}}$$

$$+ \cancel{\frac{e^{-j\omega}}{j\omega}} - 0 - \frac{e^{-j\omega}}{\omega^2} + \frac{1}{\omega^2}$$

$$= \frac{2}{\omega^2} - \frac{e^{j\omega}}{\omega^2} - \frac{e^{-j\omega}}{\omega^2}$$

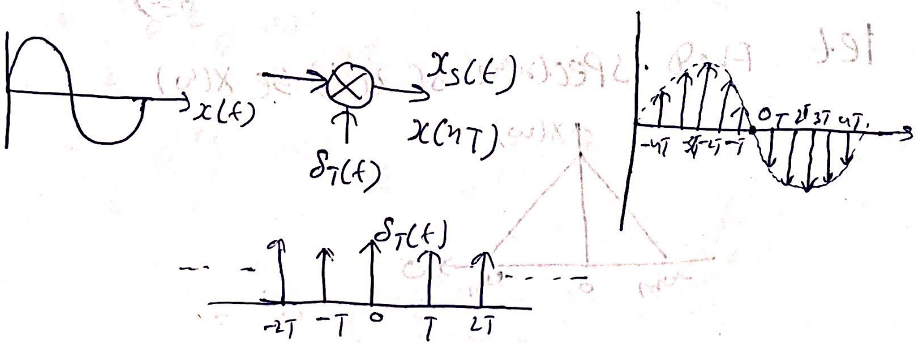
$$= \frac{2 - (e^{j\omega} + e^{-j\omega})}{\omega^2} = \frac{2 - 2\cos\omega}{\omega^2}$$

$$\boxed{X(\omega) = \frac{2(1-\cos\omega)}{\omega^2}}$$

3(b) State and prove sampling theorem

Sampling theorem :- The sampling theorem states that a band limited signal $x(t)$ with $X(\omega) = 0$ for $|\omega| \geq \omega_m$ can be represented into and uniquely determined from its samples $x(nT)$ if the sampling frequency $f_s \geq 2f_m$, where "f_m" is the highest frequency component present in it.

Proof:-



It $x(t)$ be a band limited signal with ω_m is the maximum frequency in the signal $x(t)$.

$x_s(t) \rightarrow$ sampled signal

$\delta_T(t) \rightarrow$ Train of impulse used to sample $x(t)$

$\Rightarrow x_s(t) = x(t)\,\delta_T(t)$ where $\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$

$\delta_T(t)$ can be expressed in exp. fourier series as follows

$\delta_T(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_s t}$, where $C_n = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\, e^{-jn\omega_s t}\, dt$

$= \frac{1}{T}$ { ~~from prop of~~ from property of $\delta(T)$ }

$\Rightarrow \delta_T(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T} e^{jn\omega_s t}$

$\Rightarrow x_s(t) = x(t)\,\delta_T(t) = \frac{1}{T}\sum_{n=-\infty}^{\infty} x(t)\, e^{jn\omega_s t}$

Taking F.T on Both sides

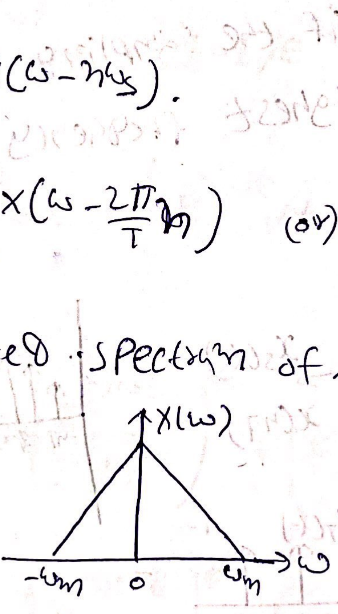
$X_s(\omega) = F\left[\frac{1}{T}\sum_{n=-\infty}^{\infty} x(t)\, e^{jn\omega_s t}\right]$

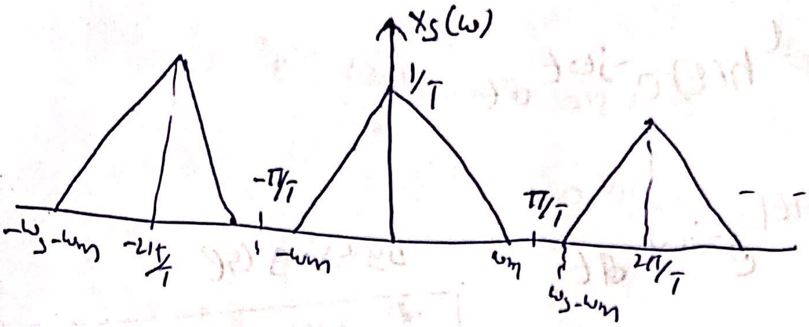
$= \frac{1}{T}\sum_{n=-\infty}^{\infty} F[x(t)e^{jn\omega_s t}]$

$= \frac{1}{T}\sum_{n=-\infty}^{\infty} X(\omega - n\omega_s).$

$X_s(\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty} X(\omega - \frac{2\pi}{T}n)$ (or) $X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$

Let Freq spectrum of $x(t)$ be $X(\omega)$





⇒ for the sampled signal spectrum to be not overlapping

⇒ $\omega_s - \omega_m \geq \omega_m$

$\omega_s \geq 2\omega_m$ (or) $f_s \geq 2f_m$

which is the sampling theorem

if $f_s = f_m$ → Nyquist sampling

$f_s < 2f_m$ → Aliasing (or) overlaping of spectra of $X(\omega)$

UNIT - II

4ⓐ compute the convolution of the following signals

$x(t) = e^{-t}u(t-2)$ and $h(t) = e^{-|t|}$

given $x(t) = e^{-t}u(t-2)$, $h(t) = e^{-|t|}$

by using Time convolution theorem

$$x(t) * h(t) \longleftrightarrow X(\omega)H(\omega)$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt \qquad x(t) = \begin{cases} e^{-t}, & 2 \leq t \leq \infty \\ 0 & \text{otherwise} \end{cases}$$

$$= \int_{2}^{\infty} e^{-t} e^{-j\omega t}\,dt$$

$$= \int_{2}^{\infty} e^{-t(1+j\omega)}\,dt = \left(\frac{e^{-t(1+j\omega)}}{-(1+j\omega)}\right)_{2}^{\infty}$$

$$= 0 - \left(-\frac{e^{-2(1+j\omega)}}{1+j\omega}\right)$$

$$\boxed{X(\omega) = \frac{e^{-2(1+j\omega)}}{1+j\omega}}$$

$$H(\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-|t|}\, e^{-j\omega t}\, dt$$

by using the

$$\boxed{\int_{-\infty}^{\infty} e^{-a|t|}\, e^{-j\omega t}\, dt = \frac{2a}{a^2+\omega^2}}$$

that means

$$e^{-a|t|} \xleftrightarrow{F.T} \frac{2a}{a^2+\omega^2}$$

$$\boxed{H(\omega) = \frac{2}{1+\omega^2}}$$

$$y(t) = x(t) * h(t)$$

$$= X(\omega)\, H(\omega)$$

$$y(t) = \frac{e^{-2(1+j\omega)}}{1+\omega^2} \times \frac{2}{1+\omega^2}$$

$$\boxed{y(t) = \frac{2e^{-2(1+j\omega)}}{(1+\omega^2)^2}}$$

4(b) Derive the relationship between autocorrelation and Power spectral density.

The autocorrelation function of a power signal $x(t)$ in terms of Fourier series coefficients is given as

$$R(\tau) = \sum_{n=-\infty}^{\infty} C_n C_{-n} e^{jn\omega_0\tau}$$

$$R(\tau) = \sum_{n=-\infty}^{\infty} |C_n|^2 e^{jn\omega_0\tau}$$

Taking fourier transform on both sides we have

$$F[R(\tau)] = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left(|C_n|^2 e^{jn\omega_0 \tau} \right) e^{-j\omega\tau} d\tau$$

Interchanging the order of integration and summation we get

$$F[R(\tau)] = \sum_{n=-\infty}^{\infty} |C_n|^2 \int_{-\infty}^{\infty} e^{-j\tau(\omega - n\omega_0)} d\tau$$

$$= 2\pi \sum_{n=-\infty}^{\infty} |C_n|^2 \delta(\omega - n\omega_0)$$

$$F[R(\tau)] = S(\omega)$$

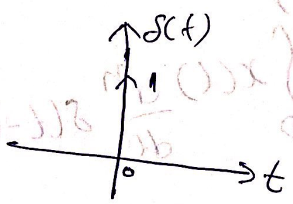
$$\boxed{R(\tau) \xleftrightarrow{F.T} S(\omega)}$$

Hence proved

(2) Explain about different properties of impulse function.

impulse function is

$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$$



~~graph~~

properties of impulse function

① area of unit impulse function is unity

$$\boxed{\int_{-\infty}^{\infty} \delta(t)\, dt = 1}$$

② weighted impulse function area is that weighted.

$$\boxed{\int_{-\infty}^{\infty} A_0 \delta(t)\, dt = A_0}$$

③ always impulse signals are even functions

$$\boxed{\delta(t) = \delta(-t)}$$

④ $\int \delta(t)\,dt = u(t)$

⑤ The time scaled impulse function is $\delta(at)$ is

$$\boxed{\delta(at) = \frac{1}{|a|}\delta(t)}$$

⑥ $$\boxed{x(t)\,\delta(t-t_1) = x(t_1)\,\delta(t-t_1)}$$

⑦ $$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_1)\,dt = \int_{-\infty}^{\infty} x(t_1)\,\delta(t-t_1)\,dt$$

$$= x(t_1)\int_{-\infty}^{\infty} \delta(t-t_1)\,dt$$

$$\boxed{\int_{-\infty}^{\infty} x(t)\,\delta(t-t_1)\,dt = x(t_1)}$$

⑧ $$\int_{-\infty}^{\infty} x(t)\,\frac{d^n}{dt^n}\delta(t-t_1)\,dt = (-1)^n \frac{d^n}{dt^n} x(t)\Big|_{t=t_1}$$

when $x(t)\Big|_{t\to\infty} = 0$ (or) Infinite

⑨ The Fourier transform of $\delta(t)$ is $\underline{1}$

$$\boxed{\delta(t) \xleftrightarrow{F.T} 1}$$

Derive the relationship between bandwidth and rise time.

we know that the transfer function of an ideal LPF is given by

$$H(\omega) = |H(\omega)| e^{-j\omega t_d}$$

$$H(\omega) = \begin{cases} 1, & |\omega| < \omega_c \\ 0, & |\omega| > \omega_c \end{cases}$$

ω_c is called the cutoff frequency

$$H(\omega) = \begin{cases} e^{-j\omega t_d} & -\omega_c \le \omega \le \omega_c \\ 0 & \omega > |\omega_c| \end{cases}$$

The impulse response h(t) of the LPF is obtained by taking the inverse Fourier transform of the transfer function (H(ω))

$$h(t) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\omega t_d} e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega(t-t_d)} d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega(t-t_d)}}{j(t-t_d)}\right]_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega_c(t-t_d)} - e^{-j\omega_c(t-t_d)}}{j(t-t_d)}\right]$$

$$= \frac{1}{\pi(t-t_d)} \sin \omega_c(t-t_d)$$

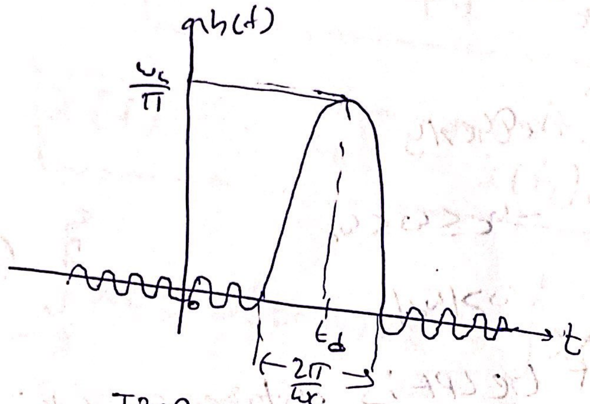
$$h(t) = \frac{\omega_c}{\pi}\left[\frac{\sin \omega_c(t-t_d)}{\omega_c(t-t_d)}\right]$$

$$= \frac{\omega_c}{\pi} \operatorname{sinc} \omega_c(t-t_d)$$

The impulse response of ideal LPF is shown in below fig.

In this impulse response $h(t)$ is non-zero for $t<0$, even though the input $\delta(t)$ is applied at $t=0$. That is the impulse response begins before the input is applied. In real life no system exhibits such type of characteristics.

So the impulse response is known, the step response can be obtained by convolution.



Impulse response of ideal LPF

The step response $y(t) = h(t) * u(t) = \int_{-\infty}^{t} h(\tau)\,d\tau$

$$y(t) = \int_{-\infty}^{t} \frac{\omega_c}{\pi} \frac{\sin \omega_c(\tau - t_d)}{\omega_c(\tau - t_d)}\,d\tau$$

Put $x = \omega_c(\tau - t_d)$

$dx = \omega_c\,d\tau \Rightarrow d\tau = \frac{dx}{\omega_c}$

$$y(t) = \int_{-\infty}^{\omega_c(t-t_d)} \frac{\omega_c}{\pi} \frac{\sin x}{x} \frac{dx}{\omega_c} = \frac{1}{\pi}\int_{-\infty}^{\omega_c(t-t_d)} \frac{\sin x}{x}\,dx$$

$$y(t) = \frac{1}{\pi} Si[x]_{-\infty}^{\omega_c(t-t_d)}$$

$$y(t) = \frac{1}{\pi}\left\{ Si[\omega_c(t-t_d)] - Si(-\infty)\right\}$$

$$= \frac{1}{\pi}\left\{ Si[\omega_c(t-t_d)] + \pi/2\right\}$$

$$\boxed{\begin{aligned} Si(\infty) &= \pi/2 \\ Si(-\infty) &= -\pi/2 \end{aligned}}$$

$$\boxed{y(t) = \frac{1}{2} + \frac{1}{\pi} Si[\omega_c(t-t_d)]}$$

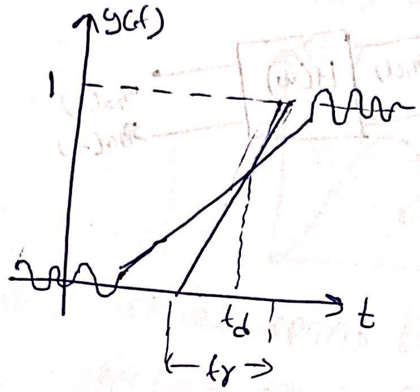
If $\omega_c \to \infty$, then the response is

$$y(t) = \frac{1}{2} + \frac{1}{\pi} Si(\infty) = \frac{1}{2} + \frac{1}{\pi}(\pi/2) = 1$$

If $\omega_c \to -\infty$, then the response is

$$y(t) = \frac{1}{2} + \frac{1}{\pi} Si(-\infty) = \frac{1}{2} + \frac{1}{\pi}(-\pi/2) = 0$$

The step response of LPF is shown in ~~fig~~ below fig.



The rise time t_r is defined as the time required for the response to reach from 0% to 100% of the final value. To find it draw a tangent at $t=t_d$ with the line $y(t)=0$ and $y(t)=1$

$$\left.\frac{dy(t)}{dt}\right|_{t=t_d} = \left.\frac{\omega_c \sin\omega_c(t-t_d)}{\pi\ \omega_c(t-t_d)}\right|_{t=t_d} = \frac{\omega_c}{\pi}$$

$\cancel{\frac{1}{t_r} = \frac{\pi}{\omega_c}}$

$$\frac{1}{t_r} = \frac{\omega_c}{\pi}$$

$$\boxed{t_r = \frac{\pi}{\omega_c}}$$

for a low pass filter

Cutt off frequency = Band width.

So the rise time is inversely proportional to the bandwidth.

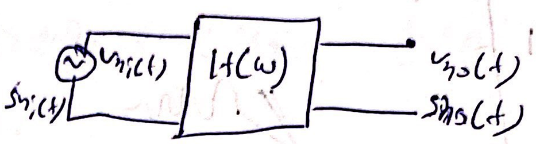
$\boxed{\text{Band width} \times \text{Rise time} = \text{constant}}$

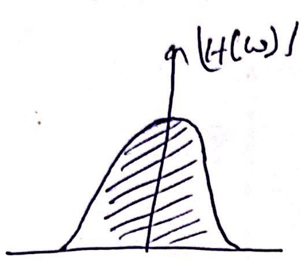
RC filter.

Equivalent noise Bandwidth of a RC LPF:-

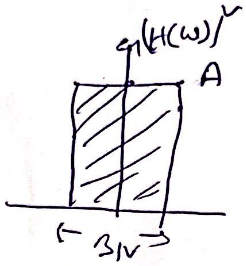
with the help of equivalent noise bandwidth, noise power can be specified at the output of a linear LPF.

we have $S_{no}(\omega) = S_{ni}(\omega)|H(\omega)|^2$





pratical system



ideal system.

we have $v_{no}^2 = P_o = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{ni}(\omega)|H(\omega)|^2 d\omega.$

$$P_o = \frac{1}{\pi}\int_0^{\infty} S_{ni}(\omega)|H(\omega)|^2 d\omega.$$

let input noise PSD is taken as constant ie $S_{ni}(\omega) = C$

$$\Rightarrow P_o = \frac{1}{\pi}\int_0^{\infty} C|H(\omega)|^2 d\omega$$

$$= \frac{C}{\pi}\int_0^{\infty} |H(\omega)|^2 d\omega$$

$P_o = \frac{C}{\pi} \times$ area under the curve $|H(\omega)|^2$

we seen from the frequency spectrum the curve under practical system is equal to are under ideal LPF.

$$\int |H(\omega)|^2 d\omega = A \times B_N$$

A = value of $|H(\omega)|^2$ at $\omega = 0 \Rightarrow |H(0)|^2$

$$\boxed{\therefore B_N = \frac{\int |H(\omega)|^2 d\omega}{|H(0)|^2}}$$

$P_o = v_{no}^2 = \frac{C}{\pi} \times$ area under curve $|H(\omega)|^2$ of actual system.

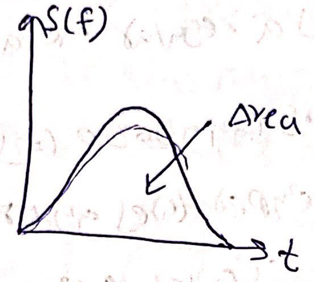
$$P_o = \frac{C}{\pi} B_N \times A$$

$$\boxed{P_o = \frac{C A B_N}{\pi}}$$

$B_N \rightarrow$ noise B.W

write short notes on noise power spectral density.

we know that periodic waveforms are also know as deterministic waveforms since their values are known at all times. periodic wave forms are also called as power wave forms. Another type of power wave form encounted in communications engineering is the random wave form. noise waveforms are examples of random waveforms.



Fourier methods applied to these random power signals result in a power spectral density function. This is a curve which shows the energy distribution as a continuous function of frequency. The units for power spectral density are watts per Hertz, which are equivalent to joules. This means that the total average power is equal to the product of the power spectral density and bandwidth(Hz)

$$S_i(\omega) = \frac{2KTG}{1+(\omega/\alpha)^2}$$

here T is temperature in kelvin
G is the conductance of the resistor.
K is Boltzmann constant
α is average no. of collisions per second.

7(a) Explain classification of noise

noise may be classified in two broad groups
① External noise ② Internal noise

<u>External noise</u> :- External noise may be defined as that type of noise which is generated external to a communication system.

External noise
- Atmospheric noise
- Extraterrestrial noise
  - Solar noise
  - Cosmic noise
- Industrial noise

Atmospheric noise ;- which is also ...
lightning discharges in thunderstorms and other natural electri...
disturbances which occur in the atmosphere.

Extraterrestrial noise
- Solar noise
- Cosmic noise

Solar noise ;- Solar noise is the electrical noise eminating from sun. under steady conditions there is a regular radiation of noise from sun.

Cosmic noise ;- Distant stars can also be considered suns. These distant stars have high temperatures and radiate noise in the same manner as the sun. This is called cosmic nois.

Industrial noise ;- The industrial noise or man made noise is that type of noise which is produced by such sources as automobiles, high-voltage transmission lines and several other heavy electric equipments.

Internal noise
- Short noise
- Partition noise
- Low frequency (or) Flicker noise
- High frequency (or) transit-time noise
- Termal noise.

Short noise ;- The short noise is generated due to the random diffusion of minority carriers (or) simply random generation and recombination of electron-hole pairs.

Partition noise ;- Partition noise is generated in a circuit when a current has to divide between two (or) more paths).

Low frequency noise ;- At low frequencys a particular type of noise appears. The power spectral density of this noise increases as the frequency decreses $S(\omega) \propto \frac{1}{f}$

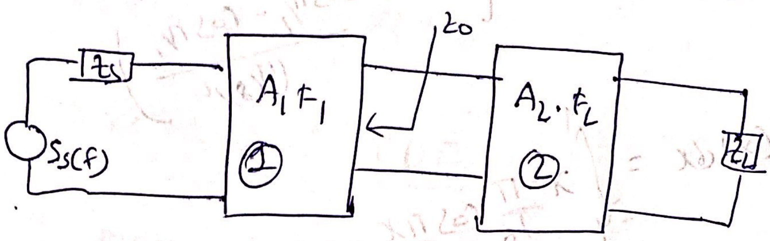
High frequency noise ;- The conductance has a noise current source which is associated with it parallel. Generally this noise is present at high ~~frequency~~. frequency

termal noise ;- The termal noise is the random noise which is generated in a resistor.

Derive the expression for noise figure of a multistage amplifier.

* Let the two stages of the amplifiers have available power gains A_1 and A_2 respectively. Now overall noise figure F of the cascaded amplifier can be determined as under.

* The total available noise power density S_{no} consists of the total noise power density S_{n1} available at the output due to the first stage. It also consists of the total noise power density S_{n2} at the output due to second stage of the amplifier.



* Let F_1 and F_2 be the noise figures of the first stage and second stage of the amplifier respectively.

* Total noise power density S_{n1} available at the output of the first stage is:

$$S_{n1} = \frac{KTF_1}{2} A_1 A_2 \quad -(1)$$

* Let the noise figure of a amplifier stage 2 be F_2.

$$S_{n2} = \frac{KT}{2}(F_2-1)A_2 \quad -(2)$$

$$S_n = S_{n1} + S_{n2} = \frac{KT}{2} F_1 A_1 A_2 + \frac{KT}{2}(F_2-1)A_2$$

$$S_n = \frac{KT}{2}\left(F_1 A_1 A_2 + (F_2-1)A_2\right) \quad -(3)$$

Now the overall noise power density is expressed as

$$S_n = \frac{KT}{2} FA = \frac{KT}{2} F A_1 A_2 \quad -(4)$$

where F = total noise figure.

From eq (3) and (4)

$$\frac{KT}{2} F A_1 A_2 = \frac{KT}{2}\left(F_1 A_1 A_2 + (F_2-1)A_2\right)$$

$$F = F_1 + \frac{(F_2-1)}{A_1}$$

Illy

$$F = F_1 + \frac{F_2-1}{A_1} + \frac{F_3-1}{A_1 A_2} + \frac{F_4-1}{A_1 A_2 A_3} + \cdots$$

UNIT-IV

8 A random variable X is given by $f_X(x) = \frac{\pi}{16}\cos\frac{\pi x}{8}$, $-4 \le x \le 4$

Determine mean, variance and standard deviation.

$$\text{mean} = E(X) = m_x = \int_{-\infty}^{\infty} x f_x(x)\,dx = \int_{-4}^{4} x \frac{\pi}{16}\cos\frac{\pi x}{8}\,dx$$

$$m_x = \frac{\pi}{16}\int_{-4}^{4} x\cos\frac{\pi x}{8}\,dx = \frac{\pi}{16}\left[\left(\frac{x\sin\frac{\pi x}{8}}{\pi/8}\right)_{-4}^{4} - \int_{-4}^{4}\frac{\sin\frac{\pi x}{8}}{\pi/8}\,dx\right]$$

$$= \frac{\pi}{16}\left[\frac{4-4}{\pi/8} + \left.\frac{\cos\frac{\pi x}{8}}{(\pi/8)^2}\right]_{-4}^{4}\right. = \frac{\pi}{16}\left(0 + \frac{\cos\pi/2 - \cos\pi/2}{(\pi/8)^2}\right)$$

$$\boxed{\text{mean} = m_x = 0}$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f_x(x)\,dx = \int_{-4}^{4} x^2 \frac{\pi}{16}\cos\frac{\pi x}{8}\,dx$$

$$E(x^2) = \frac{\pi}{16}\left[\left(\frac{x^2\sin\frac{\pi x}{8}}{\pi/8}\right)_{-4}^{4} - 2\int\frac{x\sin\frac{\pi x}{8}}{\pi/8}\right]$$

$$= \frac{\pi}{16}\left[\frac{16+16}{\pi/8} + 2\left[\left(\frac{x\cos\frac{\pi x}{8}}{(\pi/8)^2}\right)_{-4}^{4} + \left(\frac{\sin\pi x/8}{(\pi/8)^3}\right)_{-4}^{4}\right]\right]$$

$$= \frac{\pi}{16}\left(\frac{32\times 8}{\pi} + 2\left(\frac{1+1}{(\pi/8)^3}\right)\right) = \frac{\pi}{16}\left[\frac{32\times 8}{\pi} + \frac{4\times 8^3}{\pi^3}\right]$$

$$= \pi\left(\frac{16}{\pi} + \frac{128}{\pi^3}\right) = \cancel{16\pi^2 + 128}$$

$$E(x^2) = 16 + \frac{128}{\pi^2} = \frac{16\pi^2 + 128}{\pi^2}$$

$$\text{variance}\ (\sigma_x^2) = E(x^2) - m_x$$

$$\boxed{\sigma_x^2 = \frac{16\pi^2 + 128}{\pi^2}}$$

$$\text{standard deviation}\ (\sigma_x) = \sqrt{\sigma_x^2}$$

$$\boxed{\sigma_x = \frac{\sqrt{16\pi^2 + 128}}{\pi}}$$

Explain about CDF

Cumulative Distribution function (CDF) of a random variable "x" may be defined as the probability that a random variable "x" takes a value less than or equal to x. here x is a the dummy variable.

let us consider the probability of the event $X \le x$. The probability of the this event may be denoted as $P(X \le x)$.

Now according to the definition, the CDF may be written as

$$CDF: F_X(x) = P(X \le x).$$

Properties of CDF

① CDF is the probability distribution function i.e it is defined as the probability of event $(X \le x)$, its value is always between 0 and 1. This means CDF is bounded between 0 and 1

$$0 \le F_X(x) \le 1$$

② $F_X(-\infty) = 0$

$F_X(\infty) = 1$

for $x = -\infty$ means no possible event. Due to this fact $P(X \le -\infty)$ will always be zero. Therefore $F_X(-\infty) = 0$,

for $x = \infty$ means $P(X \le \infty)$. Since $P(X \le \infty)$ includes probability of all possible events.

$$F_X(\infty) = 1.$$

③ $F_X(x_1) \le F_X(x_2)$ if $x_1 \le x_2$

This is property states that CDF, $F_X(x)$ is a monotone non-decreasing function of x.

9(b) Explain about probability density function.

Probability density function (PDF):- The derivative of cumulative distribution function (CDF) with respect to some dummy variable is known as probability density function (PDF). Probability density function (PDF) is generally denoted by $f_X(x)$. Mathematically PDF may be expressed as

$$\boxed{PDF: f_X(x) = \frac{d}{dx} F_X(x)}$$

Properties of probability density function (PDF)

① Probability density function (PDF) is always non zero for all values of x.

Mathematically $f_X(x) \geq 0$ for all values of x.

Proof:- We know that CDF increases monotonically. Therefore the derivative of CDF will always be possitive. Therefore PDF will be always be possitive.

② The area under the PDF curve is always equal to unity.

$$\boxed{\int_{-\infty}^{\infty} f_X(x)\,dx = 1}$$

Proof:- we know $f_X(x) = \frac{d}{dx} F_X(x)$

$$\int_{-\infty}^{\infty} f_X(x)\,dx = \int_{-\infty}^{\infty} \frac{d}{dx} F_X(x)\cdot dx = \left[F_X(x)\right]_{-\infty}^{\infty} = F_X(\infty) - F_X(-\infty)$$

$$= 1 - 0 = 1$$

③ The CDF may be obtained by integrating PDF.

$$F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$$

Proof:- we know $f_X(x) = \frac{d}{dx} F_X(x) \Rightarrow \int_{-\infty}^{x} f_X(x)\,dx = \int_{-\infty}^{x} \frac{d}{dx} F_X(x)$

$$= \left[F_X(x)\right]_{-\infty}^{x} \Rightarrow F_X(x)$$ hence proved

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④ The probability of the event $\{x_1 \leq X \leq x_2\}$ is

$$\boxed{P(x_1 \leq X \leq x_2) = \int_{x_1}^{x_2} f_X(x)\,dx}$$

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