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IV/IV B.Tech (Regular/Supplementary) DEGREE EXAMINATION

November, 2019

Seventh Semester

Time: Three Hours

Mechanical Engineering

Design of Machine Elements-III

Maximum : 60 Marks

(12 x 1 = 12 Marks)

(4 x 12 = 48 Marks)

Answer Question No.1 compulsorily.

Answer ONE question from each unit.

Note: Use of Design Data Book should be Permitted

(12x1=12 Marks)

1. Answer all questions

- State the applications of springs.
- Distinguish between closely and open coiled helical spring.
- What is surge in springs?
- Name the friction materials used in clutches and brakes.
- Mention the advantages of cone clutch?
- What is 'back-stop' band brake?
- What types of stresses induced in a solid flywheel?
- Define coefficient of fluctuation of speed.
- How the limiting speed of flywheel is determined.
- What is piston slap and how can it be controlled?
- Name the materials used for connecting rod.
- Define objective function and constraints.

UNIT I

2. a) Name the commonly used materials for springs. What are the factors which govern the choice of a particular material? 4M

- b) A bumper, consisting of two helical steel springs of circular section, brings to rest a railway wagon of mass 1500 kg and moving at 1.2 m/s. While doing so, the spring compressed by 150 mm. The mean diameter of the coils is 6 times the wire diameter. The permissible shear stress is 400 MPa. Determine i) maximum force in each spring, ii) wire diameter of the spring, iii) mean diameter of the coils and iv) number of active coils. Take $G = 84 \text{ GPa}$. 8M

(OR)

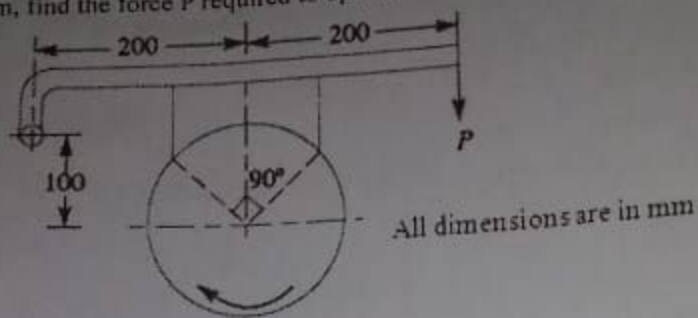
3. a) What is nipping in leaf springs? Discuss its role in spring design. 4M
- b) Design a leaf spring for the rear axle of a tractor trolley. The load on the rear axle of the trolley is 10 kN. The span is 1200 mm and width of clamp is 100 mm. In all, 12 leaves are used out of which two are main leaves and remaining graduated leaves. 8M

UNIT II

4. a) Describe the working principle of centrifugal clutch with the help of a neat sketch. 4M
- b) Design a cone clutch to transmit 10 kW at maximum speed of 100 rpm. The outer cone is of cast iron and forms the part of the I.C. engine flywheel. The overall dimension restricts the mean diameter of the cone to 300 mm. The semi-cone angle is 15° . The inner cone is positioned by means of a centrally placed helical spring. 8M

(OR)

5. a) What are the factors to be considered in brake design? Explain.
b) A single block brake as shown in figure has the drum diameter 250 mm. The angle of contact is 90° and the coefficient of friction between the drum and the lining is 0.35. If the torque transmitted by the brake is 70 N-m, find the force P required to operate the brake.



UNIT III

6. a) What is the function of flywheel? Describe the construction of a flywheel.
b) A single cylinder internal combustion engine working on the four stroke cycle develops 75 kW at 360 rpm. The fluctuation of energy can be assumed to be 0.9 times the energy developed per cycle. If the fluctuation of speed is not to exceed 1 percent and the maximum centrifugal stress in the flywheel is to be 5.5 MPa. Estimate the mean diameter and the cross sectional area of the rim. The material of the rim has density 7100 kg/m^3 .

(OR)

7. A punching machine makes 30 holes/min in a steel plate of 18 mm thickness. The diameter of hole is 25 mm and shear strength of plate is 240 N/mm^2 . If the actual punching operation takes place during one-tenth of a revolution of a crank, design a suitable flywheel which is required to rotate at 9 times the speed of crank shaft. The permissible coefficient of fluctuation of speed is 0.1 and space limitation requires that the diameter of the flywheel should not exceed 1000 mm.

UNIT IV

8. Design a cast iron piston for a 4-stroke I.C. engine, with the following specifications:

Cylinder bore = 100 mm;
Stroke length = 120 mm;
Maximum gas pressure = 4 N/mm^2 ;
Indicated mean effective pressure = 0.75 N/mm^2 ;
Mechanical efficiency = 80 %;
Fuel consumption = 0.15 kg per BHP hour;
Higher calorific value of the fuel = $42 \times 10^3 \text{ kJ/kg}$;
Speed = 200 r.p.m;

Assume any other relevant data required.

(OR)

- a) Find the extreme points of the following function: $f(x_1, x_2) = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2 + 6$.
b) Write a short note on the following optimization techniques:
i) Interval halving method ii) Golden section method

SCHEME FOR EVALUATION

- 1
 - a) Any two Applications of spring — (1m)
 - b) Difference Explanation — (1m)
 - c) Explanation of surge — (1m)
 - d) Any two Material Names — (1m)
 - e) Explanation of cone clutch — (1m)
 - f) back-stop (or) Self locking Explanation — (1m)
 - g) Names of stress in solid flywheel — (1m)
 - h) coefficient of fluctuation of speed & formula — (1m)
 - i) Limiting speed (or) centrifugal stress formula — (1m)
 - j) Explanation of piston slap (or) piston clearance — (1m)
 - k) Any two materials used for connecting rod — (1m)
 - l) Definition of objective function and constraints — (1m)
- 2
 - a) Any two materials of springs — (2m)
Any two Factors — (2m)
 - b) calculation of load — (2m)
wire and coil diameters — (4m)
NO. of coils — (2m)
- 3
 - a) Nipping Explanation with diagram — (4m)
 - b) Thickness of leaves — (4m)
Deflection of spring — (4m)

4 a) Centrifugal clutch working principle with diagram

b) Force required to engage clutch — (5M)
inner and outer dia of clutch — (3M)

5 a) Any four design factors — (4M)

b) Equivalent coefficient of friction — (2M)

Normal reaction — (2M)

Load calculation with diagram — (4M)

6 a) Any two functions of flywheel — (2M)

construction of flywheel — (2M)

b) mean dia of rim — (2M)

Fluctuation of Energy (ΔE) — (2M)

Area of cross section of rim — (4M)

7 a) Fluctuation of Energy (ΔE) — (6M)

Dimension of cross section of the rim — (6M)

8 piston head thickness — (6M)

Ribs design — (1M)

piston rings design — (2M)

skirt design — (1M)

piston pin design — (2M)

9 a) function satisfied points — (2M)

Hessian matrix — (2M)

Nature of x and $f(x)$ — (2M)

b) Internal Helving Method Explanation (a) procedure — (3M)

Golden section method Explanation (a) procedure — (3M)

KEY

- 1 a) Springs are used
- To Absorb shock loads in vehicle suspension, railway buffer springs
 - To store Energy in clocks, Toys
 - To measure force in weighing balance
 - To ~~control force~~ Apply force and control motion in cam and follower
- Any two Applications - (1m)
- b) A helical spring is said to be closely coiled spring when the spring wire is coiled so close that plane containing each coil is almost right angle to the axis of the helix. In other words helix angle is very small. i.e. If helix angle is less than 10° it is close coiled spring. If it is greater than 10° it is open coiled spring. - (1m)
- c) when the Natural frequency of vibrations of the spring coincides with the frequency of external periodic force acts on it resonance occurs. In this state the spring is subjected to a wave of successive compressions of coils that travels from one end to the other and back. This type of vibratory motion is called surge. - (1m)
- d) - asbestos based - bronze based - iron based
- Any two materials - (1m)
- e) The conical surface results in considerable friction force even with a small engaging force due to the wedge action. - (1m)
- f) A back stop brake is a device which is used to prevent the reverse rotation of drum which will cause harmful effects. - (1m)
- g) The solid fly wheel is subjected to two types of stresses
- 1) Radial stress
 - 2) circumferential stress. - (1m)
- h) The ratio of maximum fluctuation of speed to mean speed is called as coefficient of fluctuation of speed. - (1m)

i) The limiting speed of flywheel can be obtained using the formula
 $\sigma = e v^2$ where σ = Limiting Tensile stress
 e = Density of flywheel material. — (1m)

j) The clearance between cylinder liner and piston is excessive piston slap occurs, resulting in piston running with excessive noise. — (1m)

k) The materials used for I/C connecting rod are either medium carbon steels & alloy steels. — (1m)

l) The criterion with respect to which I/C design is optimized when expressed as a function of I/C design variables is known as objective function.

The restrictions that must be satisfied to produce an acceptable design are collectively called design constraints. — (1m)

2 a) There are Four types of materials used for Spring Applications

- 1) patented and cold drawn steel wires (unalloyed)
- 2) oil hardened and tempered spring steel wires.
- 3) oil hardened and tempered steel wires (alloyed)
- 4) stainless steel spring wires

Any two materials — (2m)

b) The selection of spring wire material depends on the following factors.

- 1) Load acting on the spring
- 2) Range of stress through which the spring operated
- 3) The limitations on mass and volume of spring
- 4) Expected fatigue life
- 5) Environmental conditions
- 6) Severity of deformation encountered in making the spring

Any two factors — (2m)

2. b) Given data: $m = 1500 \text{ kg}$, $v = 1.2 \text{ m/s}$, $\delta = 150 \text{ mm}$, $D = 6d$, $\tau = 400 \text{ MPa}$
 $G = 84 \text{ GPa}$

$$KE \text{ of wagon} = \frac{1}{2}mv^2 = 1080 \text{ J}$$

$$\text{Energy absorbed by each spring} = U = \frac{KE}{\text{No. of springs}} = 540 \text{ N}\cdot\text{m}$$

$$\text{using relation } U = \frac{1}{2}Ps \Rightarrow \boxed{P = 7200 \text{ N}} \quad \text{--- (2m)}$$

$$D = 6d \Rightarrow CM = \frac{D}{d} = 6$$

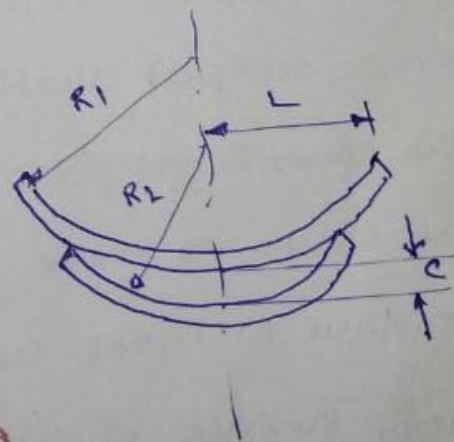
$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = 1.25$$

$$\tau = \frac{8PCK}{\pi d^3} \Rightarrow \boxed{d = 18.54 \approx 20 \text{ mm}} \quad \text{--- (3m)}$$

$$\boxed{D = 6d = 120 \text{ mm}} \quad \text{--- (1m)}$$

$$\delta = \frac{8PC^3n}{Gd} \Rightarrow \boxed{n = \cancel{20.25} \approx 21} \quad \text{--- (2m)}$$

3 a) In Leaf Springs generally all leaves are bent to same radius of ~~curvature~~ curvature. by considering stress in this case like full length leaves are subjected to 50% more than stress in graduated length leaves. To get uniform stress in all leaves the prestressing can be done. The prestressing can be achieved by bending the leaves to different radius of curvature, before they are assembled with the center clip. This is called nipping. In this case the full length leaves has greater radius of curvature than the adjacent leaf. The radius of curvature decreases with shorter leaves. The initial gap between the full length leaves before assembly is called a 'nip'. Such prestressing achieved by a difference in radii of curvature is known as nipping.



Explanation (3m) + Diagram (1m)

3b) Given data: $2P = 10 \text{ kN}$, $2l = 1200 \text{ mm}$, $b = 100 \text{ mm}$, $n_f = 2$, $n_s = 10$

Note: The following assumptions are necessary to solve the problem.

allowable stress = $\sigma = 400 \text{ N/mm}^2$

Young's modulus = $E = 210 \text{ GPa}$

Answers may vary according to assumptions. Hence

marks can be awarded according to procedure.

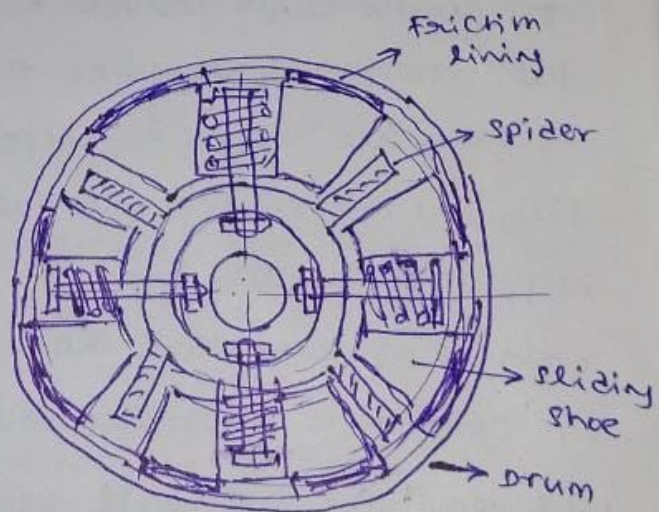
$$\sigma = \frac{18Pl}{(2n_s + 3n_f)bt^2} \Rightarrow t = \frac{7.2}{\sqrt{\sigma}} = 8 \text{ mm} \quad \text{--- (4m)}$$

Deflection of Spring $\delta = \frac{12Pl^3}{(2n_s + 3n_f)Ebt^3} = 46.35 \text{ mm} \quad \text{--- (4m)}$

4a) The centrifugal clutch works on the principle of centrifugal force.

It consists of spider which is mounted on the input shaft and which is provided with four equally spaced radial slides. A sliding shoe is retained in each slide by means of a spring.

The outer surface of the sliding shoe is provided with a lining of friction material (asbestos). The complete assembly of spider, shoes and springs is enclosed in a co-axial drum, which is mounted



on a output shaft. If the speed of the input shaft increases the centrifugal force acting on the sliding shoes increases causing the shoes to move in a radially outward direction. The shoes continue to move with increasing speed until they contact the inner surface of the drum. power is transmitted due to

frictional force between the shoe lining and the inner surface of the drum. when the angular velocity of the shoe decreases the centrifugal force decreases then the clutch is disengaged automatically.

working principle (3m) + Diagram (1m)

4b) Given data: $P = 10 \text{ kW}$, $N = 100 \text{ rpm}$, $\alpha = 15^\circ$, $D_m = 300 \text{ mm}$

Note: the following assumptions are necessary to solve the problem

coefficient of friction $\mu = 0.2$

allowable bearing pressure $p_b = 0.35 \text{ N/mm}^2$

Answers may vary according to assumptions, hence marks can be awarded according to the procedure.

Torque Transmitted

$$T = \frac{60 \times P}{2 \pi N} = 954.93 \times 10^3 \text{ N-mm}$$

— (2m)

Force required to Engage clutch

$$R \cdot T = \frac{\mu P}{4 \sin \alpha} (D+d) = \frac{\mu P}{2 \sin \alpha} D_m \Rightarrow P = 8238.46 \text{ N}$$

— (3m)

Also

$$T = \frac{\pi \mu p_b d (D^2 - d^2)}{8 \sin \alpha}$$

using the relation $D_m = \frac{D+d}{2} = 300 \Rightarrow D = 600 - d$

$$\therefore T = \frac{\pi \mu p_b d (D+d)(D-d)}{8 \sin \alpha} = \frac{\pi \mu p_b d D_m (600-d)}{4 \sin \alpha}$$

solving $d = 27.49 \approx 28 \text{ mm}$

$D = 600 - d = 572 \text{ mm}$

— (3m)

5a) The factors considered in brake design are

- 1) The unit pressure between the braking surfaces
- 2) The coefficient of friction between the braking surfaces
- 3) The radius and peripheral velocity of brake drum.
- 4) The projected area of the friction surfaces
- 5) The ability of the brake to dissipate heat that is equivalent to the energy being absorbed

Any four factors — (4m)

5b) Given data: $D = 250\text{ mm}$, $2\theta = 90^\circ$, $\mu = 0.35$, $T = 70\text{ N-m}$

Equivalent coefficient of friction

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = 0.385$$

— (2m)

~~250 mm~~

$$T = F \times R = F \times \frac{D}{2}$$

$$\therefore F = 560\text{ N} \quad \text{— (1m)}$$

Now

$$F = \mu' N \Rightarrow N = \frac{F}{\mu'} = 1454.54\text{ N}$$

— (1m)

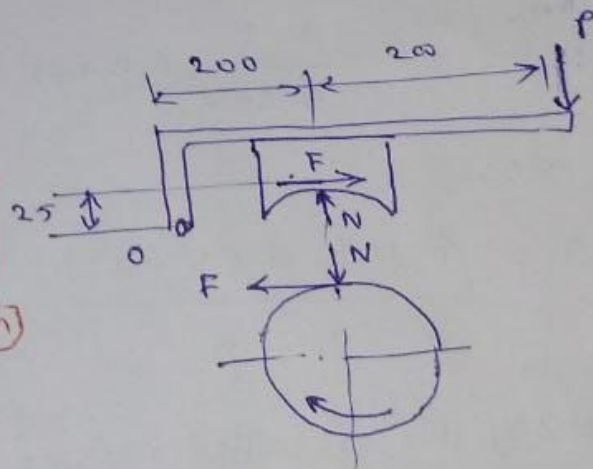
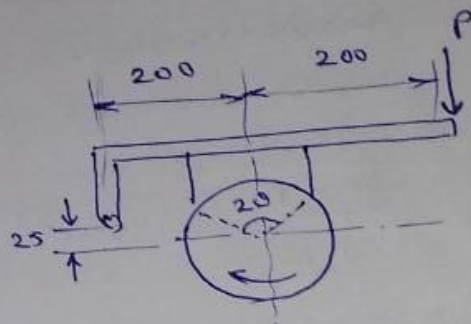
Taking moments about 'O'

$$(-P \times 400) + (N \times 200) - (F \times 25) = 0$$

$$\Rightarrow P = \frac{(N \times 200) - (F \times 25)}{400}$$

$$P = 692.27\text{ N}$$

— (2m)



Free body diagram

— (2m)

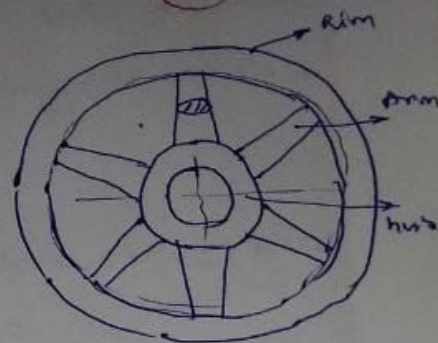
Functions of flywheel

- 1) To store and release energy when needed during work cycle
- 2) To reduce the power capacity of the electric motor or engine
- 3) To reduce the amplitude of speed fluctuations.

Any two functions — (2m)

Construction of flywheel

The flywheel consists of three parts: hub, arm and rim. The arms have elliptical cross section. In small flywheels the arms are replaced by a solid web. In the case of large flywheels the rim and hub cut through the center which is called split flywheel.



Construction — (2m)

6 b) Given data: $P = 75 \text{ kW}$, $N = 360 \text{ rpm}$, $\Delta E = 0.9 E$, $c_s = 0.01$, $\sigma = 5.5 \text{ mpa}$
 $\rho = 7100 \text{ kg/m}^3$

$$T_{\text{mean}} = \frac{60 \times P}{2\pi N} = 1989.436 \text{ N-m}$$

$$E = T_{\text{mean}} \times 4\pi = 25000 \text{ N-m}$$

$$\Delta E = 0.9 E = 22500 \text{ N-m}$$

— (2m)

$$\sigma = \rho v^2 \Rightarrow v = 27.83 \text{ m/sec}$$

$$\text{but } v = \frac{\pi D N}{60} \Rightarrow \boxed{D = 1.476 \text{ m} \text{ or } R = 0.738 \text{ m}} \text{ — (2m)}$$

using the relation

$$\Delta E = I c_s \omega^2$$

$$\text{where } \omega = \frac{2\pi N}{60} = 37.7 \text{ rad/sec}$$

$$I = \Delta E / c_s \omega^2 = 1583.06 \text{ kg-m}^2$$

— (2m)

$$I = m R^2 \Rightarrow m = 2906.61 \text{ kg}$$

$$\text{Now } m = \rho \times A \times \pi D \Rightarrow \boxed{A = 0.088 \text{ m}^2} \text{ — (2m)}$$

7) Given data: No. of holes = 30 holes/min, $t = 18 \text{ mm}$, $d = 25 \text{ mm}$, $r = 24 \text{ mm}$
 punching time = $\frac{1}{10}$ of revolution, flywheel speed = 9 times the crank shaft speed
 $C_s = 0.1$, $D = 1000 \text{ mm}$

$$\text{punching (shear) Area} = \pi dt = 1413.716 \text{ mm}^2$$

$$\text{punching Force} = F = \tau \times A = 339.292 \times 10^3 \text{ N}$$

$$\text{Energy required per hole} = \frac{1}{2} F \times t = \frac{1}{2} F t$$

$$= \frac{1}{2} \times F \times t = 3.053 \times 10^6 \text{ N-mm} = 3.053 \times 10^3 \text{ N-m} \quad \text{--- (2m)}$$

$$\begin{aligned} \text{Energy required per minute} &= 3.053 \times 10^3 \times \text{No. of holes/min} \\ &= 91.608 \times 10^3 \text{ N-m} \end{aligned}$$

punching takes place during $\frac{1}{10}$ th of revolution hence during $\frac{9}{10}$ th revolution Energy stored in flywheel

$$\Delta E = \frac{9}{10} \times \text{Energy per hole} = 2747.7 \text{ N-m} \quad \text{--- (2m)}$$

The flywheel has to rotate 9 times the speed of crank shaft, and there are 30 working strokes per minute.

$$\therefore N = 9 \times 30 = 270 \text{ rpm}$$

$$\omega = \frac{2\pi N}{60} = 28.27 \text{ rad/sec} \quad \text{--- (2m)}$$

$$\Delta E = I C_s \omega^2 \Rightarrow I = 34.38 \text{ kg-m}^2$$

$$I = m R^2 \Rightarrow m = 137.52 \text{ kg}$$

$$m = \rho \cdot b t \cdot \pi D \quad (\text{Assuming } b = 2t), \rho = 7200 \text{ kg/m}^3$$

$$m = \rho \cdot 2t \cdot \pi D \Rightarrow t = 0.055 \text{ m} = 55 \text{ mm}$$

$$b = 2t = 110 \text{ mm} \quad \text{--- (4m)}$$

Note: Answers may vary according to assumptions, hence marks can be awarded according to procedure.

Given data: $D = 100 \text{ mm}$, $L = 120 \text{ mm}$, $P_{\text{max}} = 4 \text{ N/mm}^2$, $P_m = 0.75 \text{ N/mm}^2$
 $\eta = 80\%$, $m = 0.15 \text{ kg/BHP/hr}$, $\text{HCV} = 42 \times 10^3 \text{ kJ/kg}$, $N = 200 \text{ rpm}$

Design of piston head:

Assuming σ_t for CI = 38 N/mm^2

$$t_h = D \sqrt{\frac{3}{16} \cdot \frac{P_{\text{max}}}{\sigma_t}} = 14.048 \text{ mm} \approx \boxed{15 \text{ mm}} \quad \text{--- (2M)}$$

$$n = \frac{N}{2} = 100 \text{ rpm}, \quad A = \frac{\pi}{4} D^2 = 7.85 \times 10^3 \text{ mm}^2$$

$$\text{IP} = \frac{P_m L A N}{60} = 1177.5 \text{ kW}$$

$$\text{BP} = \eta \times \text{IP} = 0.942 \text{ kW}$$

$$m = 0.15 / 60 \times 60 = 4.166 \times 10^{-5} \text{ kg/BP/sec}$$

Assuming $c = 0.05$

$$H = c \times \text{HCV} \times m \times \text{BP} \times 10^3 = 82.4118 \text{ W}$$

Assuming $k = 46.91 \text{ W/m/hr}$, $T_c - T_e = 222^\circ\text{C}$

$$t_h = \frac{H}{12.56 \times k \times (T_c - T_e)} \times 10^3 = 0.63 \text{ mm}$$

Taking largest among the two values $t_h = 15 \text{ mm} \quad \text{--- (4M)}$

Design of ribs

Assuming No. of ribs 4 and $t_r = 0.4 t_h = 6 \text{ mm} \quad \text{--- (1M)}$

Design of piston rings

Assuming $\sigma_s = 96 \text{ N/mm}^2$, $P_s = 0.025 \text{ N/mm}^2$, No. of rings (i) = 5
 comp-3, oil-2

$$b = D \sqrt{\frac{3 P_s}{\sigma_s}} = 2.79 \approx \boxed{3 \text{ mm}}$$

$$h = 0.85 b = 2.5 \text{ mm}, \quad h_{\text{min}} = \frac{D}{10} = 2$$

$$h_1 = 1.1 t_h = 16.5 \text{ mm}, \quad h_2 = 0.875 h = 2.1875 \text{ mm}$$

$$\text{Length of ring section} = (5 \times h) + (4 \times h_2) = \boxed{21.25 \text{ mm}} \quad \text{--- (2M)}$$

Design of piston skirt

Assuming $\mu = 0.1$, $P_b = 0.45 \text{ N/mm}^2$

$$F_s = \mu \left(\frac{\pi}{4} D^2 \right) P_{\text{max}} = 3141.6 \text{ N}$$

$$F_s = P_b D L_s \Rightarrow L_s = 69.81 \approx 70 \text{ mm}$$

$\therefore$ Length of piston = Topland + ring section + skirt length
 $= 16.5 + 21.25 + 70 = 107.75 \text{ mm}$

$D < L > 1.5 D$ Hence Design is safe.

Design of piston pin:

Assuming $P_b = 15 \text{ N/mm}^2$, $L_1 = 0.475 D = 47.5 \text{ mm}$

$$P = \frac{\pi}{4} D^2 P_{\max} = 31.416 \times 10^3 \text{ N}$$

$$P = P_b \times L_1 \times d \Rightarrow d = 44.09 \approx 45 \text{ mm}$$

check for bending stress

$$Z = \frac{\pi}{32} d^3 = 8946.17 \text{ mm}^3$$

$$\sigma_b = \frac{PD}{8Z} = 43.89 \text{ N/mm}^2$$

9a) Given $f(x_1, x_2) = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2 + 6$

$$\frac{\partial f}{\partial x_1} = 3x_1^2 + 4x_1 \Rightarrow x_1(3x_1 + 4) = 0$$

$$\frac{\partial f}{\partial x_2} = 3x_2^2 + 8x_2 \Rightarrow x_2(3x_2 + 8) = 0$$

$\therefore$ function satisfied by the following points $(0, 0) (-\frac{4}{3}, 0) (0, -\frac{8}{3}) (-\frac{4}{3}, -\frac{8}{3})$

Now for sufficient condition

$$\frac{\partial^2 f}{\partial x_1^2} = 6x_1 + 4 \quad \frac{\partial^2 f}{\partial x_2^2} = 6x_2 + 8 \quad \frac{\partial^2 f}{\partial x_1 \partial x_2} = 0$$

The Hessian matrix of f is given by

$$J = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} 6x_1 + 4 & 0 \\ 0 & 6x_2 + 8 \end{bmatrix}$$

$$\therefore J_1 = 6x_1 + 4 \quad J_2 = \begin{vmatrix} 6x_1 + 4 & 0 \\ 0 & 6x_2 + 8 \end{vmatrix} = (6x_1 + 4)(6x_2 + 8)$$

$$F_s = P_b D L_s \Rightarrow L_s = 69.81 \approx 70 \text{ mm}$$

∴ Length of piston = Top land + ring section + skirt length
 $= 16.5 + 21.25 + 70 = 107.75 \text{ mm}$

$D < L > 1.5 D$ Hence Design is safe.

Design of piston pin:

Assuming $P_b = 15 \text{ N/mm}^2$, $L_1 = 0.475 D = 47.5 \text{ mm}$

$$P = \frac{\pi}{4} D^2 P_{\max} = 31.416 \times 10^3 \text{ N}$$

$$P = P_b \times L_1 \times d \Rightarrow d = 44.09 \approx 45 \text{ mm}$$

check for bending stress

$$Z = \frac{\pi}{32} d^3 = 8946.17 \text{ mm}^3$$

$$\sigma_b = \frac{PD}{8Z} = 43.89 \text{ N/mm}^2$$

9a) Given $f(x_1, x_2) = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2 + 6$

$$\frac{\partial f}{\partial x_1} = 3x_1^2 + 4x_1 \Rightarrow x_1(3x_1 + 4) = 0$$

$$\frac{\partial f}{\partial x_2} = 3x_2^2 + 8x_2 \Rightarrow x_2(3x_2 + 8) = 0$$

∴ function satisfied by the following points $(0, 0) (-\frac{4}{3}, 0) (0, -\frac{8}{3}) (-\frac{4}{3}, -\frac{8}{3})$

Now for sufficient condition

$$\frac{\partial^2 f}{\partial x_1^2} = 6x_1 + 4 \quad \frac{\partial^2 f}{\partial x_2^2} = 6x_2 + 8 \quad \frac{\partial^2 f}{\partial x_1 \partial x_2} = 0$$

The Hessian matrix of f is given by

$$J = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} 6x_1 + 4 & 0 \\ 0 & 6x_2 + 8 \end{bmatrix}$$

$$\therefore J_1 = 6x_1 + 4 \quad J_2 = \begin{vmatrix} 6x_1 + 4 & 0 \\ 0 & 6x_2 + 8 \end{vmatrix} = (6x_1 + 4)(6x_2 + 8)$$

Points	J_1	J_2	Nature of J	Nature of x	$f(x)$
$0, 0$	4	32	positive definite	Relative minimum	6
$-\frac{4}{3}, 0$	-4	-32	Indefinite	Saddle point	$194/27$
$0, -\frac{8}{3}$	4	-32	Indefinite	Saddle point	$418/27$
$-\frac{4}{3}, -\frac{8}{3}$	-4	32	Negative definite	Relative maximum	$50/3$

— (2m) —

9 b) i) INTERNAL HALVING METHOD

In the internal Halving method Exactly one-half of the current interval of uncertainty is deleted in Every stage. It requires three experiments in the first stage and two experiments in each subsequent stage. In this method

- 1) The initial interval is divided into four equal parts and label the ends with a, b and x_1, x_2 are the middle points
- 2) Evaluating the $f(x)$ at three interior points and f_1, f_2, f_3 can be obtained
- 3) According to relation between f_1, f_2, f_3 except the small value the remaining boundaries are removed. The iteration continued up to the condition satisfies.

Explanation of procedure — (3m)

ii) GOLDEN SECTION METHOD

The Golden section method is similar to Fibonacci method except that in the Fibonacci method total no. of experiments to be conducted has to be specified before beginning the calculation. whereas this is not required in the golden section method. In the golden section method

- 1) The first two interior locations are obtained by

$$x_1 = 0.382 L \quad \text{and} \quad x_2 = b - x_1$$
 where $L = b - a$, and a, b are the End points.

- 2) Evaluating the function $f(x)$ at the interior points to obtain f_1, f_2 etc.
 - 3) According to the selection between f_1, f_2 ^{except} the smallest value with side boundaries, remaining boundaries are removed.
- Iteration Conti
- 4) Next x_3 can be obtained any the formula as x_2 formula of previous Iteration.
- The Iteration continues up to the Condition satisfies.

Explanation of procedure — (3m)

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