Solution cum Scheme of Evaluation

III/IV B.Tech (Regular /Supplementary) Degree Examination

14ME505

OPERATIONS RESEARCH

Mechanical Engineering

November, 2019

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14ME505

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III/IV B. Tech (Regular/Supplementary) DEGREE EXAMINATION

November, 2019 **Mechanical Engineering**

Fifth Semester **Operations Research**

Time: Three Hours **Maximum:**60 Marks

Answer Question No.1 compulsorily. (1X12 = 12 Marks)

Answer ONE question from each unit. (4X12=48 Marks)

1 Answer all questions (1X12=12 Marks)

a) What is the role of O.R in Engineering?
Marketing management, Production Management, Finance management etc

b) What is unbound solution, and how does it occur in graphical method.
The feasible region will not be a closed polygon in such case. It will have open boundaries in one or more sides.

c) Under what condition is it possible for an LPP to have more than one optimal solution?
If the slope of objective function equation is parallel to slope of the binding constraint equation.
(Or)
If the coefficient of any non basic variable is zero at the optimal level, there exists an alternative optimal solution

d) When does degeneracy happen in transportation problem?

When the allocations are less than m+n-1, these exists degeneracy.

e) What is an unbalanced assignment problem?
If the cost matrix is not a square matrix, i.e., the no of rows are not equal to no of columns it is called unbalanced assignment problem.

f) List out the methods used to find Basic feasible solution in transportation problem.
North west corner method, Least cost method, Vogel's Approximation Method (VAM)

g) What are different types of Queuing system?
Single server vs Multi server model
Infinite population model vs Finite population model
Based on arrival pattern and service pattern
Based on Service
Based on queue discipline

h) Define Jockeying and Reneging.
In a parallel queue, a customer jumps from one queue to another is referred to as Jockeying
After joining the queue if the customer loses patience and left the system he is said to have reneged.

i) What is Kendall notation?
A convenient notation for summarizing the characteristics of the queuing situation is given by the following format known by name Kendall's notation.
(a /b /c): (d/ e/f)

a = Arrivals distribution

b = Departures (service time) distribution

c = Number of parallel servers (=1, 2,...., ∞)

d = Queue discipline

e = Maximum number (finite or infinite) allowed in the system

(in queue plus in-service)

f = Size of the calling source (finite or infinite)

j) What is meant by critical path?
The path which passes through activity nodes where E and L values are same.

k) Define Total float and Free float.

Total Float: It is the length of time by which an activity can be delayed if all its preceding activities are completed at their earliest possible time and all successor activities can be delayed until their latest permissible time.
The time within which an activity must be scheduled is computed from LS and ES values for each activity's start event and end event. That is for each activity (i,j) the total float is equal to the latest allowable time for the event at the end of the activity minus the earliest time for an event at the beginning of the activity minus the activity duration. That is,

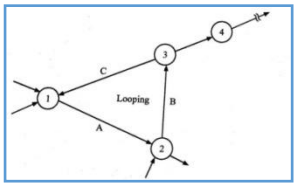
$$Total\ Float\ (TF_{ij}) = (L_j - E_i) - t_{ij} = LS_{ij} - ES_{ij} = LF_{ij} - EF_{ij}$$

Free Float: This is the length of time by which the completion time of any non-critical activity can be delayed without causing any delay in its immediate successor activities. The amount of free float time for a non-critical activity (i,j) is computed as follows:

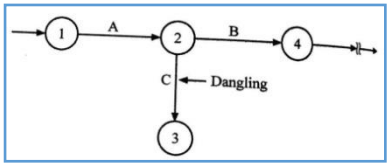
$$Free\ float\ (FF_{ij}) = (E_j - E_i) - t_{ij}$$
$$= Min\ \{ES_{ij}, for\ all\ immediate\ successors\ of\ activity\ (i,j)\} - EF_{ij}$$

l) Explain the following errors of project network diagram: i) Dangling ii) cycling.
Looping (cycling) and dangling are considered as faults in a network. Therefore these must be avoided.

(i) A case of endless loop in a network diagram which is also known as looping is shown in figure. where activities A, B and C form a cycle. Due to precedence relationships, it appears from figure that every activity in looping (or cycle) is a predecessor of itself.



(ii) A case of disconnect activity before the completion of all activities which is also known as dangling is as shown in below figure. In this case, activity C does not give any result as per the rules of the network.



UNIT I

2 Suppose an industry is manufacturing two types of products P1 and P2. The profits per Kg of the two products are Rs.30 and Rs.40 respectively. These two products require processing in three types of machines. The following table shows the available machine hours per day and the time required on each machine to produce one Kg of P1 and P2. Formulate the problem in the form of linear programming model and find out the number of units required for products P1 and P2 to maximize the profit. 12M

Profit/Kg	P1	P2	Total available Machine hours/day
Machine 1	3	2	600
Machine 2	3	5	800
Machine 3	5	6	1100
Profit	Rs.30	Rs.40	

Solution:

Let x1 be the no of kgs of product P1 produced and x2 be the no of kgs of product P2 produced;

To produce these units of P1 and P2, it requires

$3x_1 + 2x_2$ Processing minutes on Machine 1

$3x_1 + 5x_2$ processing minutes on Machine 2

$5x_1 + 6x_2$ processing minutes on Machine 3

Since machine 1 is available for not more than 600 minutes and machine Machine 2 is available for not more than 800 minutes and machine 3 is available for not more than 1100 minutes per day, the constraints are

$3x_1 + 2x_2 \leq 600$

$3x_1 + 5x_2 \leq 800$

$5x_1 + 6x_2 \leq 1100$

and $x_1, x_2 \geq 0$.

Since the profit from P1 is Rs 30 and from P2 is Rs 40, the total profit is $30x_1 + 40x_2$. As the objective is to maximize the profit, the objective function is:

$$Maximize\ Z = 30x_1 + 40x_2$$

The complete formulation of the LPP is

$$Maximize\ Z = 30x_1 + 40x_2$$

$subject\ to\ the\ constraints$

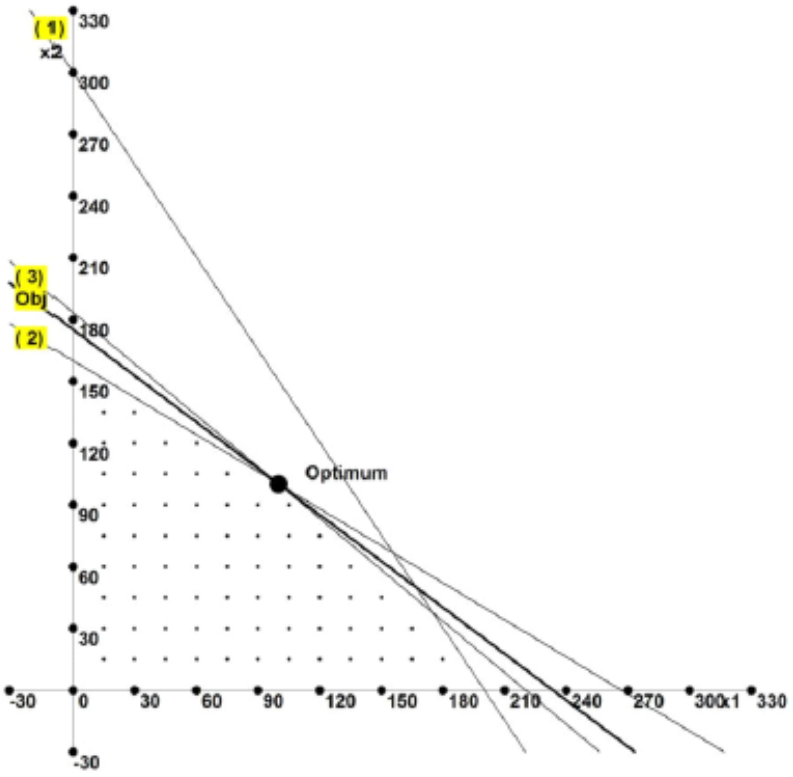
$3x_1 + 2x_2 \leq 600$

$3x_1 + 5x_2 \leq 800$

$5x_1 + 6x_2 \leq 1100$

and $x_1, x_2 \geq 0$

Summary of Optimal Solution:
Objective Value = 7000.00
x1 = 100.00
x2 = 100.00



The answer is x1=100 and x2=100 with Max z= 7000.

(OR)

3 By using penalty method solve 12M

$$Maximize\ z = 3x_1 - x_2$$

subject to constraints,

$$2x_1 + x_2 \geq 2; x_1 + 3x_2 \leq 3;$$

$x_2 \leq 4; x_1, x_2 \geq 0.$

	x1	x2		
Maximize	3.00	-1.00		
Subject to				
(1)	2.00	1.00	>=	2.00
(2)	1.00	3.00	<=	3.00
(3)	0.00	1.00	<=	4.00
Lower Bound	0.00	0.00		
Upper Bound	infinity	infinity		
Unrestr'd (y/n)?	n	n		

Iteration 1

Basic	x1	x2	Sx3	Rx4	sx5	sx6
z (max)	-203.00	-99.00	100.00	0.00	0.00	0.00
Rx4	2.00	1.00	-1.00	1.00	0.00	0.00
sx5	1.00	3.00	0.00	0.00	1.00	0.00
sx6	0.00	1.00	0.00	0.00	0.00	1.00
Lower Bound	0.00	0.00				
Upper Bound	infinity	infinity				
Unrestr'd (y/n)?	n	n				

Basic	Solution
z (max)	-200.00
Rx4	2.00
sx5	3.00
sx6	4.00

Iteration 2

Basic	x1	x2	Sx3	Rx4	sx5	sx6
z (max)	0.00	2.50	-1.50	101.50	0.00	0.00
x1	1.00	0.50	-0.50	0.50	0.00	0.00
sx5	0.00	2.50	0.50	-0.50	1.00	0.00
sx6	0.00	1.00	0.00	0.00	0.00	1.00
Lower Bound	0.00	0.00				
Upper Bound	infinity	infinity				
Unrestr'd (y/n)?	n	n				

Basic	Solution
z (max)	3.00
x1	1.00
sx5	2.00
sx6	4.00

Iteration 3

Basic	x1	x2	Sx3	Rx4	sx5	sx6
z (max)	0.00	10.00	0.00	100.00	3.00	0.00
x1	1.00	3.00	0.00	0.00	1.00	0.00
Sx3	0.00	5.00	1.00	-1.00	2.00	0.00
sx6	0.00	1.00	0.00	0.00	0.00	1.00
Lower Bound	0.00	0.00				
Upper Bound	infinity	infinity				
Unrestr'd (y/n)?	n	n				

Basic	Solution
z (max)	9.00
x1	3.00
Sx3	4.00
sx6	4.00

The answer is x1= 3; x2=0 and max Z=9.

UNIT II

4 A steel company has three LD furnaces and five medium merchant rolling mills. Transportation cost (Rs per ton) for shipping steel from furnaces to rolling mills is shown in the following table. What is the optimal shipping schedule? 12M

	Mill 1	Mill 2	Mill 3	Mill 4	Mill 5	Capacities (tons)
Furnace 1	4	2	3	2	6	8
Furnace 2	5	4	5	2	1	12
Furnace 3	6	5	4	7	3	14
Requirement (tons)	4	4	6	8	8	

Name	D1	D2	D3	D4	D5
S1	4.00	2.00	3.00	2.00	6.00
S2	5.00	4.00	5.00	2.00	1.00
S3	6.00	5.00	4.00	7.00	3.00
Demand	**4.00**	**4.00**	**6.00**	**8.00**	**8.00**

	D6 DummyD	**Supply**
S1	0.00	**8.00**
S2	0.00	**12.00**
S3	0.00	**14.00**
Demand	**4.00**	

Iteration 2: **ObjVal** **80.00**

	Name		D1	D2	D3	D4
			v1=4.00	v2=2.00	v3=2.00	v4=2.00
S1		u1=0.00	4.00 0.00	2.00 **4** 0.00	3.00 -1.00	2.00 **4** 0.00
S2		u2=0.00	5.00 -1.00	4.00 -2.00	5.00 -3.00	2.00 **4** 0.00
S3		u3=2.00	6.00 **4** 0.00	5.00 -1.00	4.00 **6** 0.00	7.00 -3.00
	Demand		**4**	**4**	**6**	**8**

	D5	D6 DummyD	**Supply**
	v5=1.00	v6=-2.00	
S1	6.00 -5.00	0.00 -2.00	**8**
S2	1.00 **8** 0.00	0.00 -2.00	**12**
S3	3.00 **0** 0.00	0.00 **4** 0.00	**14**
	8	**4**	

Iteration 1: **ObjVal** **80.00**

	Name		D1 v1=1.00	D2 v2=2.00	D3 v3=-1.00	D4 v4=2.00
S1		u1=0.00	4.00 -3.00	2.00 **4** 0.00	3.00 -4.00	2.00 **4** 0.00
S2		u2=0.00	5.00 -4.00	4.00 -2.00	5.00 -6.00	2.00 **4** 0.00
S3		u3=5.00	6.00 **4** 0.00	5.00 2.00	4.00 **6** 0.00	7.00 **0** 0.00
	Demand		**4**	**4**	**6**	**8**

	D5 v5=1.00	D6 DummyD v6=-5.00	**Supply**
S1	6.00 -5.00	0.00 -5.00	**8**
S2	1.00 **8** 0.00	0.00 -5.00	**12**
S3	3.00 3.00	0.00 **4** 0.00	**14**
	8	**4**	

The optimum allocation is given by x_{12} =4; x_{14}=4

X_{24}=4; x_{25}=8

X_{31}=4' x_{33}=6; x_{34}=0; x_{36}=4

With total cost is given by Rs 80.

(OR)

5 Solve the following assignment problem to minimize the total time for performing all the jobs. 12M

Jobs/ Workers	**1**	**2**	**3**	**4**	**5**
A	5	2	4	2	5
B	2	4	7	6	6
C	6	7	5	8	7
D	5	2	3	3	4
E	8	3	7	8	6
F	3	6	3	5	7

The revised matrix after row and column reduction is given by

Jobs/ Workers	**1**	**2**	**3**	**4**	**5**	**6**
A	3	0	1	0	5	0
B	0	2	4	4	6	0
C	4	5	2	6	7	0
D	3	0	0	1	4	0
E	6	1	4	6	6	0
F	1	4	0	3	7	0

The final table after revising the matrix is given by

Jobs/ Workers	1	2	3	4	5	6
A	3	0	1	**0**	1	1
B	**0**	2	4	4	2	1
C	3	4	1	5	2	**0**
D	3	0	0	1	**0**	1
E	5	**0**	3	5	1	0
F	1	4	**0**	3	3	1

The optimum assignment schedule is given by;

$$A \to 4, \quad B \to 1, \quad C \to 6, \quad D \to 5, \quad E \to 2 \quad F \to 3$$

The optimum (minimum) assignment cost = 2+2+0+4+3+3 = 14 units of cost

UNIT III

6 In a store with one server, 9 customers arrive on an average of 5 minutes, service is done for 10 customers in 5 minutes. Find 12M

i) The average no. of. Customers in the system.
ii) The average queue length
iii) Average time customers spends in the store.
iv) The average time a customer's waits before being served

Solution:

Arrival rate λ = 9/5 =1.8 customers/minute
Service rate μ = 10/5 = 2 customers/minute

1. Average number of customers in the system

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{1.8}{2 - 1.8} = 9$$

2. Average number of customers in the queue or average queue length

$$L_q = \frac{\lambda}{\mu}.L_s = \frac{\lambda}{\mu}.\frac{\lambda}{\mu - \lambda} = \frac{1.8}{2} \times \frac{1.8}{2 - 1.8} = 8.1$$

3. Average time a customer spends in the system

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{2 - 1.8} = 5\ minutes$$

4. Average time a customer waits before being served

$$W_q = \frac{\lambda}{\mu}.W_s = \frac{\lambda}{\mu}.\frac{1}{\mu - \lambda} = \frac{1.8}{2} \times \frac{1}{2 - 1.8} = 4.5\ minutes$$

(OR)

7 Vehicles are passing through a toll gate at the rate of 70 per hour. The average time to pass through the gate is 45 seconds. The arrival rate and service rate follow Poisson distribution. There is a complaint that the vehicles wait for a long duration. The authorities are willing to install one more gate to reduce the average time to pass through the toll gate to 35 seconds if the idle time of the toll gate is less than 9% and the average queue length at the gate is more than 8 vehicle, check whether the installation of the second gate is justified? 12M

Here $\lambda = \frac{70}{hr} = 70\ vehicles/hr$

$\mu = \frac{1}{45} \times 60 \times 60 = 80\ vehiclews/hr$

$$\rho = \frac{\lambda}{\mu} = \frac{70}{80} = 0.875$$

Average number of vehicles in the queue

$$L_q = \frac{\lambda}{\mu} . \frac{\lambda}{\mu - \lambda} = \frac{70}{80} . \frac{70}{(80 - 70)} = 6.125\ vehicles$$

Expected time to pass through the gate = 35 seconds

$$\mu' = \frac{1}{35} \times 60 \times 60 = 102.85\ vehicles/hr$$

Now revised utilization ratio

$$\rho' = \frac{\lambda}{\mu'} = \frac{70}{102.85} = 0.68$$

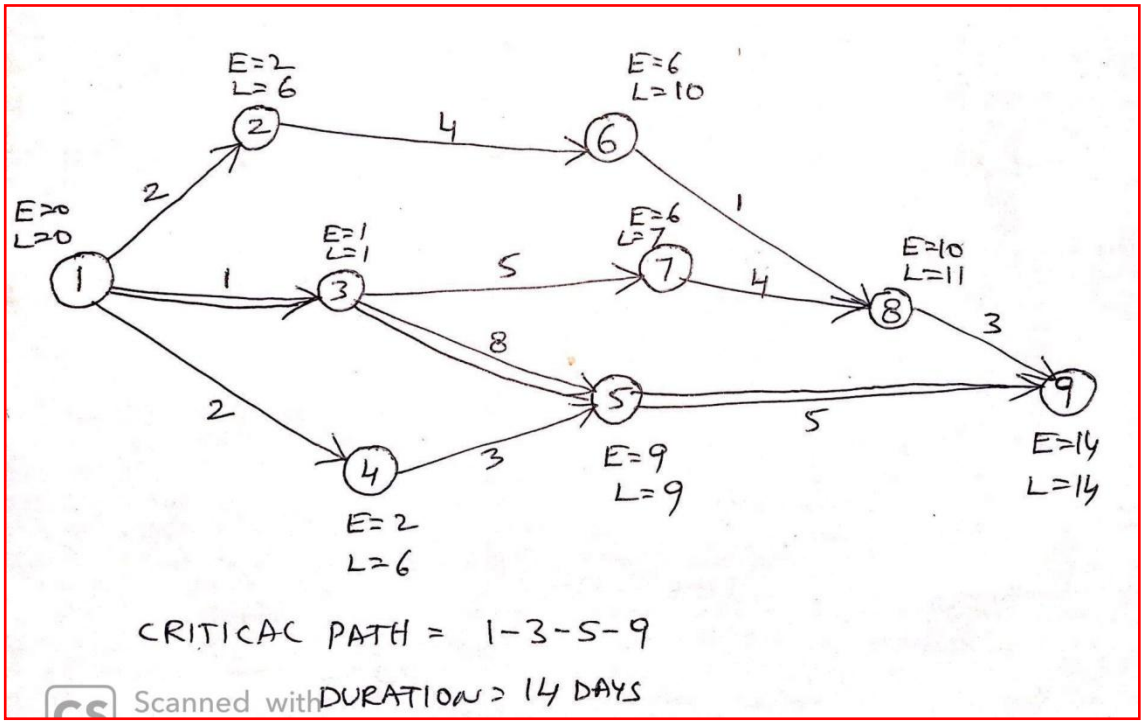
Percentage of idle time = $1 - \rho' = 1 - 0.68 = 0.32$

Average queue length is less than 8 vehicles and idle time is more than 9% . Therefore, the installation of the new gate is not justified.

UNIT IV

8 Draw the project network diagram and find the critical path from the following information of a boiler erection project. 12M

Activity	1-2	1-3	1-4	2-6	3-7	3-5	4-5	5-9	6-8	7-8	8-9
Activity time (Days)	2	1	2	4	5	8	3	5	1	4	3

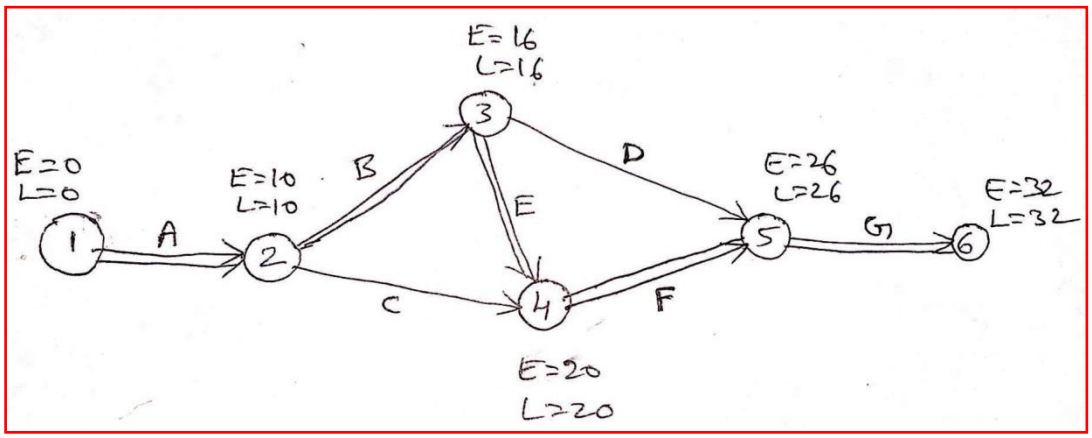


(OR)

9 Draw the PERT network for a project consisting of 7 tasks (A to G) in which the following precedence relationship must hold (X < Y means X must be completed before Y can start). A < B ; A < C ; B < D ; B < E ; C < F ; D < G ; E < F ; F < G 12M

Task	**A**	**B**	**C**	**D**	**E**	**F**	**G**
Expected Time (hrs.)	10	6	7	4	4	6	6
Standard deviation (hrs.)	2	2/3	1/3	2/3	2/3	2/3	2/3

Determine the critical path, expected minimum duration and variance of the project. Also find the probability that the project is completed 2 hours earlier than expected time.



Task	Expected Time (hrs)	Standard deviation	variance
A	10	2	4
B	6	2/3	4/9
C	7	1/3	1/9
D	4	2/3	4/9
E	4	2/3	4/9
F	6	2/3	4/9
G	6	2/3	4/9

The critical path is 1-2-3-4-5-6

Expected minimum duration is 32 hours

Standard deviation of the project duration is 2+2/3+2/3+2/3+2/3=14/3 = 4.66 hours

Variance of the project is 4+4/9+4/9+4/9+4/9 =5.77 hours

The probability that the project is completed 2 hours earlier than expected is given by

$$Prob\left\{Z \leq \frac{T_s - T_e}{\sigma} = \frac{32-30}{4.66}\right\} = Prob\{Z \leq 0.428\} = 0.3343$$

The probability that the project is completed 2 hours earlier than expected is 33.43%.

Scheme of Evaluation

1 Answer all questions (1X12=12 Marks)

a) Explanation of role of OR- 1 Mark
b) Unbound solution Definition – ½ mark, occurrence ½ Mark
c) Explanation for an LPP to have more than one optimal solution- 1Mark
d) Degeneracy in transportation problem-1Mark
e) Unbalanced assignment problem -1 Mark.
f) The three methods used to find Basic feasible solution in transportation problem. – 1Mark.
g) Different types of Queuing system- 1Mark
h) Jockeying- ½ mark and Reneging– ½ mark
i) Definition of Kendall notation– 1Mark
j) Definition of critical path– 1Mark
k) Total float - ½ mark and Free float -½ mark.
l) i) Dangling - ½ mark ii) cycling- ½ mark

UNIT I

2 Formulation of Linear Programming Problem – 4 Marks
Solution – 8 Marks

(OR)

3 Conversion to Standard form-3 Marks
Solution -9 Marks

UNIT II

4 Basic Feasible solution -5 Marks
Optimal Solution-7 Marks

(OR)

5 Balancing of Assignment Matrix- 2 Marks
Solution – 10 Marks

UNIT III

6 Calculation of size of queue- 4 Marks
Time required to complete hair cut- 4 Marks
average number of customers in saloon A & B- 4 Marks

(OR)

7 Calculation of Arrival Pattern-2 Marks
Calculation of service Pattern-2 Marks
Solution - 8 Marks

UNIT IV

8 Network diagram- 4 Marks
Calculation of time estimates- 4 Marks
Critical path Calculation-4 marks

(OR)

9 Network diagram- 4 Marks
Calculation of time estimates- 3 Marks
Critical path Calculation-3 marks
probability that the project is completed 2 hours earlier than expected time-2 Marks