Hall Ticket Number:

II/IV B.Tech (Regular) DEGREE EXAMINATION

NOVEMBER, 2019 **Common to CSE & IT**

Third Semester **Discrete Mathematical Structures**

Time: Three Hours **Maximum :** 50 Marks

Answer Question No.1 compulsorily. (1X10 = 10 Marks)

Answer ONE question from each unit. (4X10=40 Marks)

1. Answer all questions (1X10=10 Marks)

A Simplify A-(A-B)

B Suppose that a ≡b mod n and d is a positive integer such that d divides n. prove that a ≡ b mod d.

C Check the following function is one – to –one or onto.

$$f(x) = x^2 \text{ where } A = \{positive\ intgers\}.$$

D Draw truth table for $(\sim P \cap \sim Q) \rightarrow R$

E Write the negations of the following sentence by changing quantifiers.

"Every complete bipartite graph is not planar"

F What is the coefficient of x^5 in $(1 + x + x^2 + \cdots)^2$?

G Find the general solution to $a_n - 5a_{n-1} + 6a_{n-2} = 0$.

H Define Hamiltonian Graph.

I Differentiate weekly connected and strongly connected components.

J Find out the edge chromatic number for $K_{3,3}$

UNIT – I

2.a In a survey of students at Florida State University the following information was obtained: 260 were taking a statistics course, 208 were taking a mathematics course, 160 were taking a computer programming course, 76 were taking statistics and mathematics, 48 were taking statistics and computer programming, 62 were taking mathematics course and computer programming, 62 were taking mathematics course and computer programming, 30 were taking all 3 kinds of courses, and 150 were none of the 3 courses. 5M

i) How many students were surveyed? 622

ii) How many students were taking a statistics and a mathematics course but not computer programming course? 46

iii) How many were taking a statistics and a computer course but not mathematics course? 18

iv) How many were taking a computer programming and a mathematics course but not statistics course? 32

v) How many students were taking a statistics course but not taking a course in mathematics or in computer programming? 166

2.b Let R be the relation in the natural numbers $N = \{1,2,3, \ldots\}$defined by "$x + 2y = 10$" that is $R = \{(x, y) | x \in N, y \in N, x + 2y = 10\}$. Find 5M

a) The domain and range of R.

b) R^{-1}

c) Draw the digraph for RUR^{-1}

(OR)

3.a Give the transitive closure, the transitive reflexive closure, and the symmetric closure represented as digraph for the following relation. 5M

x R y iff x is an integral multiple of y, on the set {2, 3, 4, 5, 6}.

P.T.O

3.b Let R_1 and R_2 be arbitrary binary relations on set A. Prove or Disprove the following assertions 5M

a) If R_1 and R_2 are reflexive, then $R_1.R_2$ is reflexive.
b) If R_1 and R_2 are Symmetric, then $R_1. R_2$ is symmetric.
c) If R_1 and R_2 are antisymmetric, then $R_1. R_2$ is antisymmetric.
d) If R_1 and R_2 are transitive, then $R_1.R_2$ is transitive.
e) If R_1 and R_2 are irreflexive, then $R_1.R_2$ is irreflexive.

UNIT – II

4.a Prove or disprove the validity of the given argument: 5M

i.
$$\begin{array}{l} \sim t \rightarrow \sim r \\ \sim s \\ t \rightarrow w \\ r \vee s \\ \hline \therefore w \end{array}$$

ii.
$$\begin{array}{l} \sim r \rightarrow (s \rightarrow \sim t) \\ \sim r \vee w \\ \sim p \rightarrow s \\ \sim w \\ \hline \therefore t \rightarrow p \end{array}$$

4.b Verify that $11^{n+2} + 12^{2n+1}$ is divisible by 133 then $11^{n+2} + 12^{2n+2}$ is divisible by 133. 5M

(OR)

5.a Verify that the following argument is valid by using the rules of inference 5M

If Clifton does not live in France, then he does not speak French.
Clifton does not drive a Datsun.
If Clifton lives in France, then he rides a bicycle.
Either Clifton speaks French, or he drives a Datsun.
Hence, Clifton rides a bicycle.

5.b i) Find the number of distinct triples $(x1, x2, x3)$ of nonnegative integers satisfying $x_1 + x_2 + x_3 < 6$ 5M
ii) A teacher wishes to give an examination with 10 questions. In how many ways can the test be given a total of 30 points if each question is to be worth of 2 or more points?

UNIT – III

6.a In $(1 + x^5 + x^9)^{10}$ find i) the coefficient of x^{23} ii) the coefficient of x^{32}. 5M

6.b Solve the following recurrence relation using generating functions 5M

$$a_n - 9a_{n-1} + 20a_{n-2} = 0 \text{ for } n \geq 2 \text{ and } a_0 = -3, a_1 = -10$$

(OR)

7.a Find a particular solution of $a_n - 2a_{n-1} + a_{n-2} = 5 + 3n$. 5M

7.b Solve the divide and conquer relation $a_n - 7a_{n/3} = 2n$ where $n = 3^k$ for $k \geq 1$ and $a_1 = 5/2$. 5M

UNIT – IV

8.a Suppose that G is a connected planar graph. Determine |V| if 5M

(i) G has 35 regions each of degree 6
(ii) G has 14 regions each of which degree 4.

8.b What is Hamiltonian cycle? Give two Hamiltonian cycles in K_5 that have no edges in common? 5M

(OR)

9.a Prove that for any polyhedral graph 5M

i) $|V| \geq 2 + |R|$
ii) $|E| \leq 3|V| - 6$

9.b Prove that Every simple planar graph is 5-colorable. 5M

One mark Questions :-

1.) a) Let us consider

$\Rightarrow x \in (A-(A-B))$

$\Rightarrow x \in A$ and $x \notin (A-B)$

$\Rightarrow \cancel{x \in} A$ and $(\cancel{x \notin} A$

$\Rightarrow x \in A$ and $x \in (A-B)^c$

$\Rightarrow x \in A$ and $x \notin (A-B)$

$\Rightarrow x \in A$ and $(x \notin A$ or $x \in B)$

$\Rightarrow (x \in A$ and $x \notin A)$ or $(x \in A$ and $x \in B)$

$\Rightarrow x \in (A \wedge A^c) \cup x \in (A \wedge B)$

$\Rightarrow x \in (\cancel{A\wedge} A \wedge B)$

1). b) Given that

$a \equiv b \bmod n$

$(a-b) = n.k$ ($\because k$ is an integer)

d divides n

$n = d.l$ ($\because l$ is an integer)

$\Rightarrow a-b \equiv d.l.k$

$a-b \equiv d(p)$ ($\because p = l.k$ is an integer)

i.e. $a \equiv b \bmod d$.

1). c) Given that $f(x) = x^2$ where A: {positive integer}

$f(x)$ is one-to-one but not onto.

1). d) truth table for $(\sim p \wedge \sim Q) \rightarrow R$

P	Q	$\sim P$	$\sim Q$	$\sim P \wedge \sim Q$	$(\sim P \wedge \sim Q) \rightarrow R$
T	T	F	F	F	
T	F	F	T	F	
F	T	T	F	F	
F	F	T	T	T	

1). d) truth table for $(\sim p \wedge \sim Q) \rightarrow R$

P	Q	R	$\sim P$	$\sim Q$	$\sim P \wedge \sim Q$	$(\sim P \wedge \sim Q) \rightarrow R$
T	T	T	F	F	F	T
T	T	F	F	F	F	T
T	F	T	F	T	F	T
F	T	T	T	F	F	T
F	F	F	T	T	T	F
F	F	T	T	T	T	T
F	T	F	T	F	F	T
T	F	F	F	T	F	T

1). e) The Negation of "Every complete bipartite graph is not planar".

$\cancel{\forall x, \{ \lambda(x) \to \sim P(x)}$

$\sim [\forall x, [C(x) \wedge B(x) \to \sim P(x)]]$

1). f) coefficient of x^5 in $(1+x+x^2+\dots)^2$

$\Rightarrow \frac{1}{(1-x)^2}$

coefficient of x^5 is $c(6,5)$.

1). g) Given $a_n - 5a_{n-1} + 6a_{n-2} = 0$

$C(t) = t^2 - 5t + 6$

$= t^2 - 3t - 2t + 6$

$= t(t-3) - 2(t-3)$

$= (t-2)(t-3)$

$a_n = C_1 \cdot 2^n + C_2 \cdot 3^n$.

1.) h) Hamiltonian Graph:-

A Graph G is said to be Hamiltonian if there exists a cycle containing every vertex of G.

1). i) A pair of vertices in a digraph are weakly connected if there is a non directed path between them.

A pair of vertices in a digraph are strongly connected if there is a directed path from x to y and a directed path from y to x.

1.) i.) edge chromatic number for $k_{3,3}$ is 3

unit - I

2) a) i) How meny students were surveyed? 622.

ii) How many students were taking a statistics and a mathematics course but not a computer Progremming course? 46.

iii) How many were taking a statistics and a computer Course but not a mathematics course? 18.

5M

iv) How many were taking a computer progremming and a mathe-matics course but not a statistics course? 32.

v) How many were taking a statistics course but not taking Course in mathemetics or in computer Progremming? 166.

vi) ~~How many were taking a mathematics course but not taking a statistics course~~ or a ~~Computer Programming course? 100.~~

vii) ~~How many were taking a Computer Programming Course but not taking a course in mathematics or in statistics? 80.~~

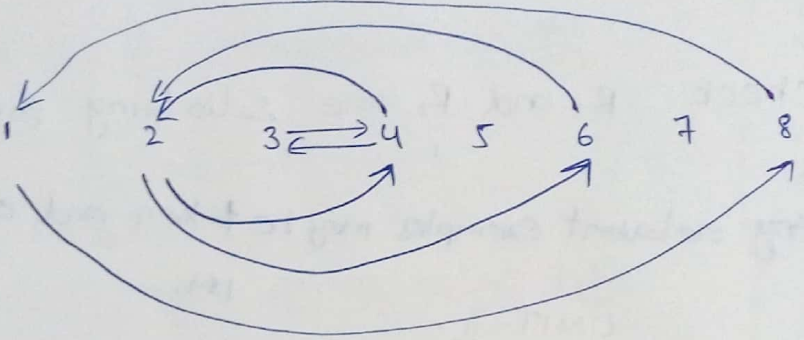
2.b) Let us consider $N = \{1, 2, 3, \dots 9\}$

$R = \{(2,4), (4,3), (6,2), (8,1)\}$

domain of R is $\{2,4,6,8\}$
range of R is $\{4,3,2,1\}$ $\Big\} \to 2M$

$R^{-1} = \{(4,2), (3,4), (2,6), (1,8)\}$ $\} \to 1M$

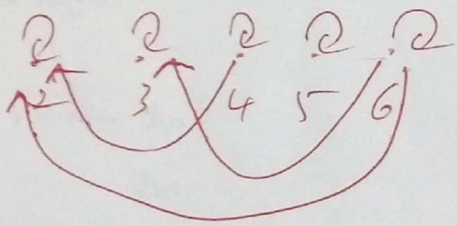
$R \cup R^{-1} = \{(2,4), (4,3), (6,2), (8,1), (4,2), (3,4), (2,6), (1,8)\}$



$\} \to$ 2M

(OR)

3) a) Given set $\{2,3,4,5,6\}$



~~R = {(2,2), (3,3), (4,4), (5,5)~~),

$R = \{(2,2), (3,3), (4,2), (4,4), (5,5), (6,2), (6,3), (6,6)\}$

transitive closure

$R^+ = \{(2,2), (3,3), (4,2), (4,4), (5,5), (6,2), (6,3), (6,6)\}$ $\} \to 3M$

transitive Reflexive closure

$R^* = \{(2,2), (3,3), (4,2), (4,4), (5,5), (6,2), (6,3), (6,6)\}$ $\} \to 1M$

Symmetric closure is $R \cup R^{-1}$

$= \{(2,2), (3,3), (4,2), (4,4), (5,5), (6,2), (6,3), (6,6), (2,4), (2,6), (3,6)\}$ → 1M

3.) b). Let us consider R_1 and R_2 are any two Binary relations.

And check R_1 and R_2 are satisfying given problems.

Note :- Any relevant examples may be taken and each carries 1M.

UNIT - II

4) a) i) Given argument is

$$\begin{array}{l} \sim t \rightarrow \sim r \\ \sim s \\ t \rightarrow w \\ r \vee s \\ \hline \therefore w \end{array}$$

Let us consider

$\sim t \rightarrow \sim r \rightarrow$ (1)

$\sim s \rightarrow$ (2)

$t \rightarrow w \rightarrow$ (3)

$r \vee s \rightarrow$ (4)

Apply law of implication on 4 $[\because \sim p \to q \equiv \sim p \vee q]$

$\sim r \to s \to$ 5

Apply law of hypothetical syllogism 1 & 5

$$\left[\begin{array}{l} \because p \to q \\ \quad q \to r \\ \hline \therefore p \to r \end{array}\right]$$

$\sim t \to s \to$ 6

Apply law of contrapositive on 6

$\sim s \to t \to$ 7

Apply modus ponens on 2 & 7

$$\begin{array}{l} \sim s \\ \sim s \to t \\ \hline \therefore t \to 8 \end{array}$$

→ 3M

Apply Modus ponens 3 and 8

$$\begin{array}{l} t \\ t \to w \\ \hline \therefore w. \end{array}$$

∴ Given argument is valid.

ii) Given arguments are

$\sim r \rightarrow (s \rightarrow \sim t)$

$\sim r \vee w$

$\sim p \rightarrow s$

$\sim w$

$\therefore t \rightarrow p$

Let us consider

$\sim r \rightarrow (s \rightarrow \sim t) \rightarrow (1)$

$\sim r \vee w \rightarrow (2)$

$\sim p \rightarrow s \rightarrow (3)$

$\sim w \rightarrow (4)$

By Applying law of disjunctive syllogism on (2) and (4)

$\sim r \rightarrow (5)$

→ 2M

Apply Modus ponens on (1) and (5)

$$\frac{\begin{array}{c}\sim r \rightarrow (s \rightarrow \sim t) \\ \sim r\end{array}}{s \rightarrow \sim t} \rightarrow (6)$$

Apply law of Hypothetical syllogism on (3) and (6)

$$\frac{\begin{array}{c}\sim p \rightarrow s \\ s \rightarrow \sim t\end{array}}{\therefore \sim p \rightarrow \sim t} \rightarrow (7)$$

Apply law of contrapositive on (7)

$t \rightarrow p$

$\therefore$ Given argument is valid.

4.b) Let $s(n) = 11^{n+2} + 12^{2n+1}$ is divisible by 133.

Basis step:-

if $n=1$, then $s(1) = 11^3 + 12^3 = 1331 + 1728 = 3059$ is divisible by 133. →1M

Inductive hypothesis:-

Assume that, ~~then~~ $s(k) = 11^{k+2} + 12^{2k+1}$ is divisible by 133. →1M

i.e. $11^{k+2} + 12^{2k+1} = 133 \cdot x$

Inductive step:-

prove that $s(k+1)$ is divisible by 133.

i.e. $s(k+1) = 11^{k+3} + 12^{2k+3}$

$= 11^{k+2} \cdot 11 + 12^{2k+1} \cdot 12^2$

$= 11 \cdot 11^{k+2} + 144 \cdot 12^{2k+1}$

$= 11 \cdot 11^{k+2} + (133 + 11) \cdot 12^{2k+1}$

$= 11 \cdot 11^{k+2} + 133 \cdot 12^{2k+1} + 11 \cdot 12^{2k+1}$ →3M

$= 11 \cdot 11^{k+2} + 11 \cdot 12^{2k+1} + 133 \cdot 12^{2k+1}$

$= 11(11^{k+2} + 12^{2k+1}) + 133 \cdot 12^{2k+1}$

$= 11 \cdot 133x + 133 \cdot 12^{2k+1}$

$= 133(11x + 12^{2k+1})$

$= 133 \cdot y$

Thus, $s(k+1)$ is divisible by 133.

$11^{n+2} + 12^{2n+2}$ is not divisible by 133

(OR)

5) a) Let

P: clifton live in france

q: clifton speak french

r: clifton drives a car

s: clifton rides a bicycle.

$\sim p \to \sim q \to (1)$

$\sim r \to (2)$

$p \to s \to (3)$

$q \vee r \to (4)$ $\to$ 2M

$\therefore s$

Applying disjunctive syllogism on (2) and (4)

$$\begin{array}{l} \sim r \\ \underline{q \vee r} \\ \therefore q \to (5) \end{array}$$

Apply law of contrapositive on (1)

$$q \to p \to (6)$$

Apply law of hypothetical syllogism on (3) and (6)

$$\begin{array}{l} q \to p \\ \underline{p \to s} \\ \therefore q \to s \to (7) \end{array}$$

Apply Modus ponens on (5) and (7)

$$\begin{array}{l} q \\ \underline{q \to s} \\ \therefore s \end{array}$$

$\therefore$ Given argument is valid.

3M

5) b) i) the number of distinct triples that satisfying the inequality $x_1 + x_2 + x_3 < 6$ is

count the number of solutions for $x_1 + x_2 + x_3 = n$ where $n = 0, 1, 2, 3, 4, 5$ and sum → 3M

ii) $C(19, 10)$ → 2M

UNIT - III

6) a) i) coefficient of x^{23} is $10! / 1! \, 2! \, 7!$ → 3M

ii) coefficient of x^{32} is $10! / 3! \, 1! \, 6!$ → 2M

b) Given recurrence relation is

$a_n - 9a_{n-1} + 20 a_{n-2} = 0$ for $n \geq 2$ and $a_0 = -3$, $a_1 = -10$.

$a_n = 2.5^n - 5.4^n$.

By using generating functions the general solution is

$a_n = 2.5^n - 5.4^n$.

Note:- for Assign [2M] for writing Generating Sequence of terms

Assign [3M] for finding out general Solution

(OR)

7.a) Given that

$$a_n - 2a_{n-1} + a_{n-2} = 5 + 3n$$

for the above recurrence relation, characteristic polynomial is $c(t) = t^2 - 2t + 1 = (t-1)^2$ of multiplicity 2 → 2M

Now the particular solution is in the form of

$$a_n^P = An^2 + Bn^3$$ → 2M

By simplification $A = 4$; $B = \frac{1}{2}$

Now the particular solution is → 1M

$$a_n^P = 4n^2 + \frac{1}{2} \cdot n^3.$$

7.b) Given that

$$a_n - 7a_{n/3} = 2n \quad \text{where } n = 3^k.$$

Let us consider

$$b_k = a_n = a_{3^k}.$$

then the transformed Relation is

$$b_k - 7b_{k-1} = 2(3^k) \text{ for } k \geq 1 \text{ and } a_1 = b_0 = 5/2.$$ → 1M

→ The linear Relation has the characteristic polynomial $(t-7)$

→ So, that the homogeneous relation has a solution

$b_k^H = B \cdot 7^k$ for some constant B. 2M

→ Now the particular solution of the inhomogenous relation

takes the form $b_k^P = A 3^k$

→ substitution reveals $A = -3/2$

$$b_k = -\frac{3}{2}(3^k) + B \cdot 7^k$$

→ but then the intial condition $b_0 = 5/2$ enables us

to determine $B = 4$ → 2M

then

$$b_k = (-3/2)3^k + (4)\,7^k$$

⇒ Now, the final general solution is

$$a_n = (-3/2) \cdot n + 4 \cdot n^{\log_3 7}.$$

UNIT-IV

8) a)

i) G has 35 regions each of degree 6.

we know that if G is a plane graph, then the sum of degrees of the regions determined by G is $2|E|$.

$$2|E| = 35 \times 6$$

$$|E| = 105$$

we know that

$$|V| - |E| + |R| = 2$$

$$|V| - 105 + 35 = 2$$

$$|V| = 72.$$

$\rightarrow 3M$

ii) G has 14 regions each of which degree 4.

$$2|E| = 14 \times 4$$

$$|E| = 28$$

$$|V| - |E| + |R| = 2$$

$$|V| - 28 + 14 = 2$$

$$|V| = 16.$$

$\rightarrow 2M$

8) b). Hamiltonian cycle:-

A Graph G is said to be Hamiltonian if there exists a cycle containing every vertex of G. Such a cycle is referred to as Hamiltonian cycle. → 1M

Draw any two Hamiltonian cycles in K_5. → 2M with detailed explanation

(OR)

9) a) ~~if~~

$|V| \geq 2 + |R|/2$

we know that $|V| - |E| + |R| = 2$.

$(|E| - |R|) + 2 \geq 2 + |R|/2$

$|E| - |R| \geq |R|/2$

$|E| \geq 3|R|/2$

1) a) We know that

$3|R| \le 2|E|$ for a connected planar graph

$3|R| \le 2(|V|+|R|-2)$

$2|V|+2|R|-4 \ge 3|R|$

$2|V| \ge |R|+4$

$|V| \ge 2+|R|/2$

Given Question is $|V| \ge 2+|R|$ so it is not proved

$\rightarrow$ 3M

b) ii)

we know that

$3|R| \le 2|E|$

$|R| \le 2/3|E|$

$|E|-|V|+2 \le 2/3|E|$

$3|E|-3|V|+6 \le 2|E|$

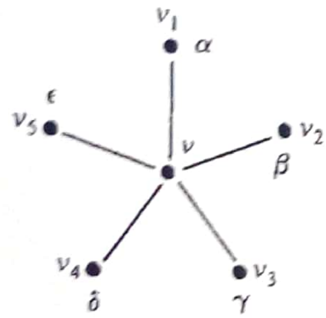
$|E| \le 3|V|-6.$

$\rightarrow$ 2M

Theorem 5.12.1. Every simple planar graph is 5-colorable.

Proof. We use induction on the number of vertices of the graph, and assume the theorem to be true for all planar graphs with at most n vertices.

Let G be a planar graph with $n + 1$ vertices. By the corollary to Euler's



→2M

Figure 5-92

formula, G contains a vertex v whose degree is at most 5. The graph $G - v$ is a planar graph with n vertices, and so can be colored with five colors, by the inductive hypothesis. Our aim is to show how this coloring of the vertices of $G - v$ can be modified to give a coloring of the vertices of G. We may assume that v has exactly five neighbors, and that they are differently colored, since otherwise there would be at most four colors adjacent to v, leaving a spare color which would be used to color v; this would complete the coloring of the vertices of G. So the situation is now as in Figure 5-92, with the vertices $v_1, \ldots, v_5$ colored $\alpha, \beta, \gamma, \delta, \epsilon$, respectively.

If λ and μ are any two colors, we define $H(\lambda,\mu)$ to be the two-colored subgraph of G induced by all those vertices colored λ or μ. We shall first consider $H(\alpha,\gamma)$; there are two possibilities:

(1) If v_1 and v_3 lie in different components of $H(\lambda,\gamma)$ (see Figure 5-93), then we can interchange the colors α and γ of all the vertices in the component of $H(\alpha,\gamma)$ containing v_1. The result of this recoloring is that v_1 and v_3 both have color γ, enabling v to be colored α. This completes the proof in this case.

(2) If v_1 and v_3 lie in the same component of $H(\alpha,\gamma)$ (see Figure 5-94), then there is a circuit C of the form $v \rightarrow v_1 \rightarrow \cdots \rightarrow v_3 \rightarrow v$, the path between v_1 and v_3 lying entirely in $H(\alpha,\gamma)$. Since v_2 lies inside

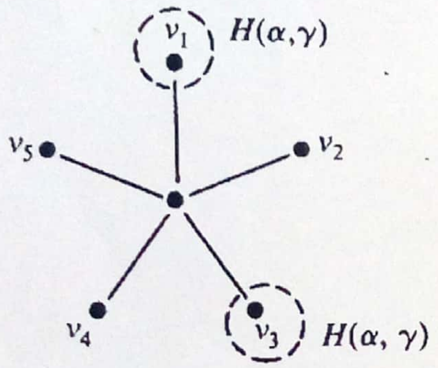


Figure 5-93

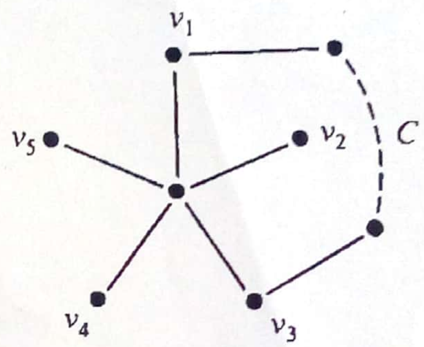


Figure 5-94

→3M

C and v_4 lies outside C, there cannot be a two-colored path from v_2 to v_4 lying entirely in $H(\beta,\delta)$. We can therefore interchange the colors of all the vertices in the component of $H(\beta,\delta)$ containing v_2. The vertices v_2 and v_4 are both now colored δ, enabling v to be colored β. This completes the proof. $\square$