SCHEME OF EVALUATION 18ME301

II/IV B.TECH (REGULAR) DEGREE EXAMINATION

NOVEMBER, 2019 **Mechanical Engineering**

Third Semester **Strength of Materials-I**

Time: Three Hours **Maximum :** 50 Marks

Answer Question No.1 compulsorily. (1X10 = 10 Marks)

Answer ONE question from each unit. (4X10=40 Marks)

1. Answer all questions (1X10=10 Marks)

a What is the difference between normal stress and shear stress?

Normal stress is a result of load applied perpendicular to a member. **Shear stress** however results when a load is applied parallel to an area.

b What is allowable stress?

The **allowable stress** or **allowable** strength is the maximum **stress** (tensile, compressive or bending) that is **allowed** to be applied on a structural material.

Factor of safety $n = \frac{yield\ stress}{Allowable\ stress}$

c What is volumetric strain?

Volumetric strain of a deformed body is defined as the ratio of the change in volume of the body to the deformation to its original volume.

$$\epsilon_v = (\sigma/E) \times (1-2\upsilon)$$

d State Hooke's law.

Hooke's law states that stress is proportional to strain with in elastic limits.

$$\sigma = E\epsilon$$

e What is meant by statically indeterminate structure?

If the equations of static equilibrium are not sufficient for the calculation of axial forces and reactions, these structures are called statically indeterminate. Structures of this type can be analyzed by supplementing the equilibrium equations with additional equations pertaining to the displacements of the structure.

f What are the assumptions made in deriving torsion formula?

1. The material of the bar is homogeneous, perfectly and obeys Hooke's Law.
2. The stress does not exceed the limit of proportionality.
3. Cross-sections rotate as if rigid, i.e every diameter rotates through the same angle. i.e the cross section remains plane and circular.

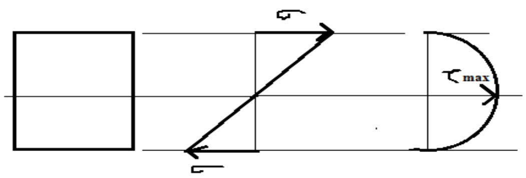
g List the various types of beams

1. Cantilever beam, 2. Simply supported beam, 3. Over hanging beam, 4 fixed beam and 5. Continuous beam

h What is point of contraflexure?

It is the point where the bending moment is zero or changes its sign.

i Sketch the variation of bending stress and shear stress distribution across the depth of a rectangular section beam



j What is plane stress?

If a cubical element considered within a strained material is under the action of stresses acting on only two pairs of parallel planes and the third pair of is free from any stress, the element is said to be under plane stress condition.

UNIT – I

2.a Distinguish between i) Elasticity and Plasticity and ii) Engineering stress and True stress 4 M

Elasticity and Plasticity:

Elasticity is the property of a body to recover its original configuration (shape and size) when you remove the deforming forces. Plastic bodies do not show a tendency to recover to their original configuration when you remove the deforming forces. Plasticity is the property of a body to lose its property of elasticity and acquire a permanent deformation on the removal of deforming force.

Engineering stress and True stress: True stress is defined as the load divided by the cross-sectional area of the specimen at that instant and is a true indication of the internal pressures. Engineering stress is simply a normalizing of the load, and has little physical significance when the actual area is difference than the original.

b A tensile test was conducted on a mild steel bar. The following data was obtained from the test: 6 M

Diameter of the steel bar = 20 mm;
Gauge length of a bar =150 mm;
Load at the elastic limit = 200 kN;
Extension at a load of 100kN=0.2 mm;
Maximum load = 300 KN; Total extension = 50 mm;
Diameter of the rod at the failure = 12.5 mm;
Determine i) Young's modulus, ii) Stress at the elastic limit, iii) Ultimate stress, iv) Percentage elongation and v) Percentage decrease in area

Ans:

Area of the steel bar, A = πr^2 = 314.16 mm^2

(i) Young's modulus, E = $\frac{stress}{strain} = \frac{100x1000}{314.16} X \frac{150}{0.2}$ = = 238.732 KN/ mm^2 2

1

(ii) Stress at elastic = $\frac{stress}{strain} = \frac{200x1000}{314.16}$ = = 636.62 N/ mm^2

(iii) Ultimate stress = $\frac{300x1000}{314.16}$ = = 954.93.62 N/ mm^2 1

1

(iv) Percentage of elongation = $\frac{50}{150} x$ 100 = 33.33%

(v) Percentage of reduction in area = $\frac{A1-A2}{A1} X$ 100 = $\frac{314.16-122.72}{314.16} X$ 100 = 29.1% 1

(OR)

3.a A bar of 30 mm diameter is subjected to a pull of 60 KN. The measured extension on gauge length of 200 mm is 0.1 mm and change in diameter is 0.004 mm. Calculate i) Young's modulus ii) Poisson's ratio and iii) Bulk modulus. 5 M

Ans:

Area of the bar, A = 706.858 mm^2

Stress = $\frac{60x1000}{706.858}$ = = 84.88 N/ mm^2 1

Lateral strain = $\frac{0.004}{30}$ = 0.00013 1

Linear strain = $\frac{0.1}{200}$ = 0.0005

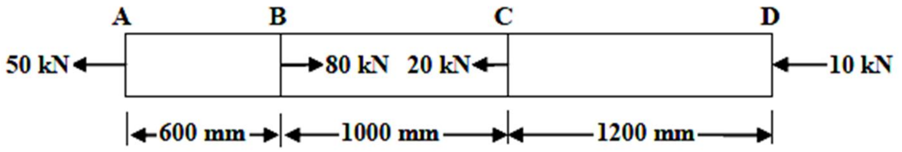
Volumetric strain, $\epsilon_v = \epsilon\,(1- 2\upsilon) = 0.0005\,(1- 2x0.26) = 0.00024$

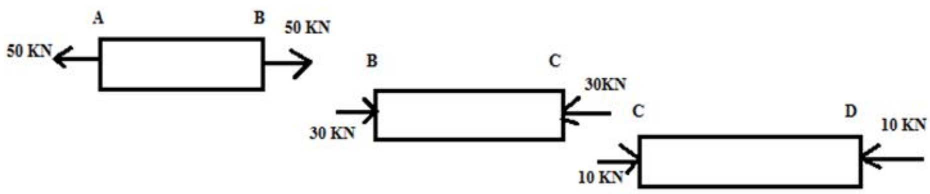
(i) Young's modulus $= \frac{stress}{strain} = \frac{84.88}{0.0005}$ = 169.76 KN/mm^2 1

(ii) Poisson's ratio, $\upsilon = \frac{lateral\ strain}{linear\ strain} = \frac{0.00013}{0.0005}$ = 0.26 1

(iii) Bulk modulus, K = $\frac{169760}{3(0.00024)}$ = 120.3 KN/ mm^2 1

b A brass bar having a cross sectional area of 1000 mm^2 is subjected to axial forces as shown in figure. Determine the total elongation of the bar if E = 105 GPa. 5 M





3

$$\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3$$

$$\Delta l = \frac{1\ X\ 1000}{1000\ X\ 105000}(50\ X\ 600 - 30\ X\ 1000 - 10\ X\ 1200)$$ 2

= - 0.114 mm

UNIT – II

4.a A steel rod of 15 m long is at a temperature of 15°C. Find the free expansion of the length when the temperature is raised to 65 °C. Also find the thermal stress produced when i) the expansion of the rod is prevented ii) the rod is permitted to expand by 6 mm. Take $\alpha = 12 \times 10^{-6}$/°C and E = 200 GPa. 5 M

Solution:

Length of the steel rod, l = 15 m = 15000 mm

Initial temperature t_1 = 15°C

Final temperature t_2 = 65°C

Free expansion of the rod, $\delta = l \times \alpha \times \Delta T = 15000 \times 12 \times 10^{-6} \times (65-15) = 9.0$ mm 1

(i) The expansion of the rod is prevented

$$\delta = \frac{PL}{AE}$$

Thermal stress, $\sigma = \frac{\delta E}{l} = = \frac{7.5 \times 200000}{15000}$ = 120 N/mm2

2

(ii) The rod is permitted to expand by 6 mm

$$(\delta - 6) = \frac{PL}{AE}$$

Thermal stress, σ = $= \frac{(7.5-6)x\ 200000}{15000}$ = 20 N/mm2

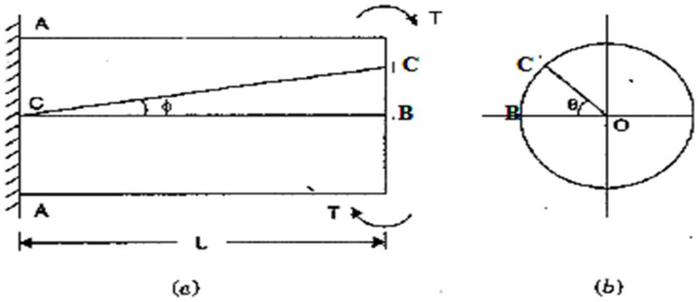
2

b 5 M

Derive the torsion equation for a circular shaft.

Torsion of Circular Bars:

Consider a circular bar of length 'l' and diameter 'd' (or shaft of circular cross-section) subjected to a couple 'T'. A bar loaded in this manner is said to be in pure torsion.



1

During twisting, there will be a rotation about the longitudinal axis of one end of the bar with respect to the other. For instance left hand end of the bar is fixed, then the right hand end will rotate through small angle θ with respect to the left hand end. The angle θ is known as the angle of twist.

A line AB on the surface of the shaft, which is parallel to the axis before strain, takes up the form of a helix AC after straining. Let ϕ be the angle of shear strain on the surface.

Then BC = $\ell\theta$

$$\theta = \frac{BC}{\ell}$$

Where τ = shear stress in the shaft

G = Modulus lf rigidity

But $\phi = \frac{\tau}{G}$

$\therefore \tau = \phi G$

Now from diagram BC = $r\theta = \frac{d}{2}\theta$ Where $\boldsymbol{\theta}$ = Angle of twist

$\therefore \tau = \frac{d}{2}\ x\ \frac{\theta}{\ell}\ x\ G \gg \frac{G\theta}{\ell}$ r = K. r

Where K $=\frac{G\theta}{\ell}$ is a constant for a given shaft

R = d/2 is the shaft radians

Thus the shear stress in the shaft is proportional to he radius to the shaft. At any radius x

$$\frac{\tau_x}{x} = \frac{\tau}{r}$$ 2

$$\frac{\tau_x}{x} = \frac{\tau}{r} = \frac{x}{r} \text{ also } \frac{\tau}{r} = \frac{G\theta}{\ell}$$

Now consider an elementary ring of he shaft at a radius x and of thickness d x.

∴ The shear stress in the ring is τ_x

Total force on the ring = τ_x x area of the ring = $\tau_x\ x\ 2\pi x dx$

Moment of this force about the axis of the shaft

$$= \tau_x\ x\ 2\pi x dx \text{ x } x = 2\pi x^2 dx \text{ x } \tau_x$$

$$= 2\pi x^2 dx \text{ x } \tau \text{ /r x } [\therefore \frac{\tau_x}{x} = \frac{\tau}{r}]$$

Total resisting moment of the shaft c/s is

$$= 2\boldsymbol{\pi}\ \frac{\tau}{\boldsymbol{n}} \int \boldsymbol{x^3}\ \ \boldsymbol{dx} = \boldsymbol{2\pi}\ \tau \text{ /r } [\frac{x^4}{4}]^r{}_0$$

$$= 2\boldsymbol{\pi}\ \frac{\tau}{\boldsymbol{n}} \mathbf{x} \frac{\boldsymbol{r^4}}{\boldsymbol{4}} = \frac{\boldsymbol{\pi r^3}}{\boldsymbol{2}} \mathbf{x}\ \tau$$

But total resisting moment of the section

= Applied torque T

$$\text{T} = \frac{\boldsymbol{\pi r^3}}{\boldsymbol{2}}\tau = \frac{\boldsymbol{\pi d^3}}{\boldsymbol{16}} \mathbf{x}\ \tau = \frac{\tau}{2(\frac{d}{2})}\ \frac{\boldsymbol{\pi d^4}}{\boldsymbol{32}}$$

$$= \frac{\tau}{r} .J \qquad [\therefore J = \frac{\pi d^4}{32}]$$

$$\gg \frac{T}{J} = \frac{\tau}{r}$$

From eq (1) and (2) $[\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{\ell}]$ 2

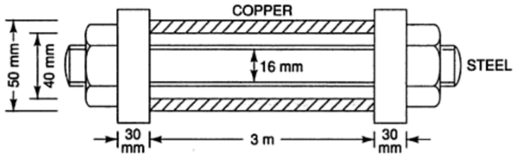
This is well known as Torsion formula for shafts of circular c/s.

(OR)

5. A steel rod of 16 mm diameter and 3m length passes through a copper tube of 50 mm external and 40 mm internal diameter and of the same length. The tube is closed at each end with the help of 30 mm thick steel plates which are tightened by nuts till the length of copper tube is reduced by 0.6 mm. The temperature of the whole assembly id then raised by 56^0C. Determine the stresses in steel and copper before and after the rise of temperature. Assume that the thickness of the steel plates at the ends do not change during tightening of the nuts. Take 10 M

E_s = 210 GPa; E_c = 100 GPa and
α_s = 12 x10^{-6} /^{0}C ; α_c = 17 x 10^{-6} /^{0}C

Solution:



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$$A_s = (\pi/4)16^2 = 64\,\pi\,\text{mm}^2;$$
$$A_c = (\pi/4)\,[50^2 - 40^2] = 225\,\pi\,\text{mm}^2$$

Stresses due to tightening of the nuts

As $\Delta = \dfrac{\sigma_c L}{E_c}$ $\therefore$ $0.6 = \dfrac{\sigma_c \times 3000}{100\,000}$ or $\sigma_c = 20$ MPa (compressive) 1

and as the force in the rod and the tube is the same, $\sigma_s . A_s = \sigma_c . A_c$

or $\sigma_s \times 64\pi = 20 \times 225\,\pi$ or $\sigma_s = 70.3$ MPa (tensile) 1

Stresses due to temperature rise

As the coefficient of expansion of copper is more than that of steel, it expands more. Thus compressive stress is induced in the copper tube and tensile in the steel rod. 2

As $\sigma_s . A_s = \sigma_c . A_c$

$\therefore$ $\sigma_s = (A_c/A_s)\,\sigma_c = (225/64)\,\sigma_c = 3.516\,\sigma_c$

Now, from compatibility equation,

Temperature strain of steel + tensile strain of steel
= Temperature strain of copper – compressive strain of copper 1

i.e. $\alpha_s . L_s t + \dfrac{\sigma_s L_s}{E_s} = \alpha_c L_c t - \dfrac{\sigma_c L_c}{E_c}$.

$$12 \times 10^{-6} \times (3000 + 60) \times 56 + \frac{3.516\,\sigma_c \times 3060}{210\,000}$$
$$= 17 \times 10^{-6} \times 3000 \times 56 - \frac{\sigma_c \times 3000}{100\,000}$$

2

or $2.056 + 0.051\sigma_c = 2.856 - 0.03\,\sigma_c$

or $0.081\,\sigma_c = 0.8$ or $\sigma_c = 9.87$ MPa

and $\sigma_s = 3.516 \times \sigma_c = 3.516 \times 9.87 = 34.7$ MPa

Final stresses

$\sigma_c = 20 + 9.87 = 29.87$ MPa (compressive) 2

and $\sigma_s = 70.3 + 34.6 = 104.9$ MPa (tensile)

UNIT – III

6. A 10 m long simply supported beam carries two point loads of 10 KN and 6 KN at 2 m and 9 m respectively from the left end. It also has a uniformly distributed load of 4 KN/m run for the length between 4 m and 7 m from left end. Draw shear force and bending moment diagrams. 10 M

Support reactions

Taking moments about A, $R_b \times 10 = 10 \times 2 + 4 \times 3 \times 5.5 + 6 \times 9$ or $R_b = 14$ kN

$\therefore R_a = 10 + 12 + 6 - 14 = 14$ kN

Shear force diagram

- Portion AC: $F_x = 14$ kN (constant);
- Portion CD: $F_x = 14 - 10 = 4$ kN (constant)
- Portion DE: $F_x = 4 - 4(x-4)$ (linear) $F_{d(x=4)} = 4$ kN; $F_{e(x=7)} = -8$ kN
  Shear force is zero at, $4 - 4(x-4)$ or $x = 5$ m
- Portion EF: $F_x = 4 - 12 = -8$ kN (constant)
- Portion FB: $F_x = -8 - 6 = -14$ kN (constant)

Shear force diagram has been shown in Fig. 4.10b. 5

Bending moment diagram

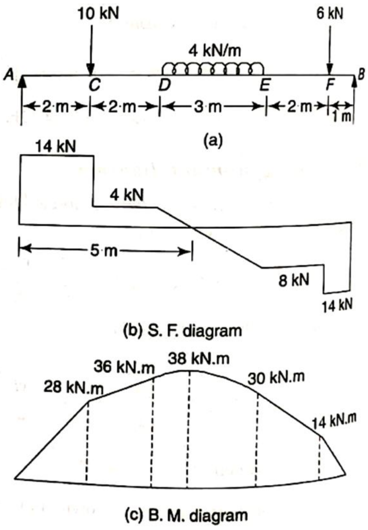
- Portion AC: $M_x = 14x$ (linear); $M_{a(x=0)} = 0$; $M_{c(x=2)} = 28$ kN · m
- Portion CD: $M_x = 14x - 10(x-2)$ (linear); $M_{c(x=2)} = 28$ kN · m; $M_{d(x=4)} = 36$ kN · m
- Portion DE: $M_x = 14x - 10(x-2) - \dfrac{4(x-4)^2}{2}$ ...(parabolic)

Bending moment is maximum at $x = 5$ m (at a point where shear force is zero),

$$\therefore \quad M_{max} = 14 \times 5 - 10(5-2) - \frac{4(5-4)^2}{2} = 38 \text{ kN} \cdot \text{m}$$ 5

$M_{d(x=4)} = 36$ kN · m; $M_{e(x=7)} = 30$ kN · m

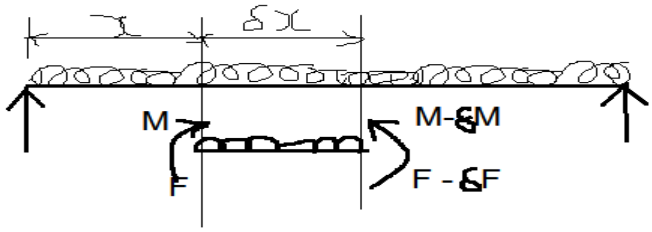
- Portion EF: $M_x = 14x - 10(x-2) - 12(x-5.5)$ ...(linear)
  $M_{e(x=7)} = 30$ kN · m; $M_{f(x=6)} = 14$ kN · m
- Portion FA: $M_x = 14x - 10(x-2) - 12(x-5.5) - 6(x-9)$ ...(linear)
  $M_{e(x=7)} = 14$ kN · m; $M_{f(x=6)} = 0$



(OR)

7.a Derive the relationship between intensity of loading, shear force and bending moment. 5 M

<u>Relationship between intensity of loading, shear force and Bending moment:</u>



1

Consider an elementary length of the beam of length

δx between x and $x + \delta x$ measured from the left hand support.

Let the B.M at x to be M & the B.M at $x + \delta x$ to be M – SM

The S.F at x to be F &

The S.F at $x + \delta x$ to be F – SF 1

The load on the elementary length δx may be taken as uniform whose intensity may be w per unit length.

Consider the equilibrium of the element, all the vertical forces are together zero.

$$-\ (\text{F} - \delta x) + \text{F} - w\,\delta x = 0$$

$$\delta x - w\,\delta x = 0$$

$$\frac{\delta F}{\delta x} = w$$ 1

Taking limit as $\delta x \rightarrow 0$ we get $\frac{dF}{dx} = w$

i.e the first derivative of S.F w.r.t x at a point gives intensity of load at the point

Now taking moments of all the forces about ‘A’

$$(F - \delta x)\ \delta x - (M - \delta M)\ +\ w\delta\ell + \frac{\delta x}{2} + M = 0$$

$$F\ .\ \delta x - \delta x\ .\ \delta x - M + \delta M + w\, x^2/2 + M = 0$$

1

Neglectives small quantities

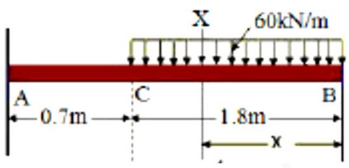
$$F.\ \delta x + \delta M = 0$$

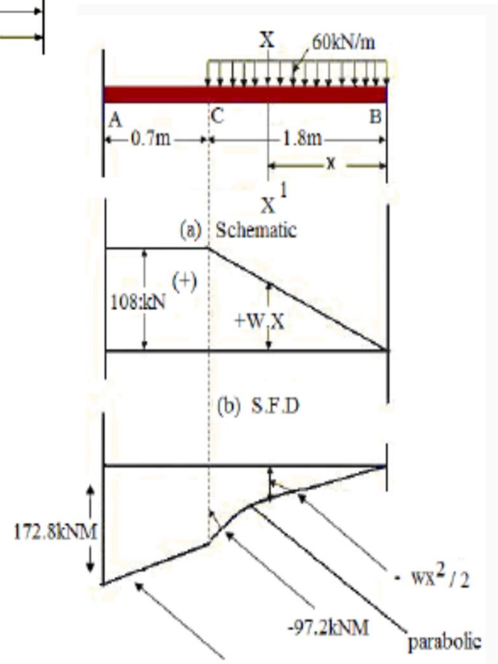
$$\frac{\delta M}{\delta x} = -F \qquad\qquad \frac{dM}{dx} = -F$$

1

$$\therefore \frac{d^2M}{dx^2} = \frac{-df}{dx} = -w$$

b A cantilever beam carries a uniform distributed load of 60 KN/m as shown in figure. Draw the shear force and bending moment diagrams for the beam. 5 M





Solution: Consider a section (X – X’) at a distance x from end B.

S.F. between B and C $F_x = + w.x$ …(1)

A x = 0, $F_B = w.0 = 0$

x= 1.8 m; $F_C = 60 \times 1.8 = 108$ kN

Since, there is no load between points A and C; for this region Fx remains constant. Therefore, from C to A;

$F_x = + 108$ kN

SFD will be triangular from B to C and a rectangle from C to A.

Now,

Bending moment between B and C

$M_x = -(wx).x/2 = -wx^2/2$

At x = 0; $M_B = -w.0/2 = 0$

x = 1.8 m; $M_C = -60/2.(1.8)2 = -97.2$ kN m

For region C to A; $M_x = -w\,(1.8)(x - 1.8/2) = -60 \times 1.8\,(x - 0.9) = -108\,(x - 0.9)$

At x = 1.8 m; $M_C = -108\,(1.8 - 0.9) = -97.2$ kN m

x = 2.5 m; $M_A = -108\,(2.5 - 0.9) = -172.8$ kN m 5

BMD is parabolic in nature from B to C and straight line from C to A.

UNIT – IV

8.a What are the assumptions made in theory of simple bending? 4 M

The following assumptions are made in the theory of simple bending.

1. The material of the beam is homogenous and isotropic.
2. The transverse sections, which were plane before bending, remain plane after bending also. (i.e the load must be perpendicular to the longitudinal axis of the beam)
3. The value of young's modulus of elasticity (E) is the same intension and compression.
4. The material of the beam obeys Hooke's Law and is stress within its elastic limit.
5. Each layer of the beam is free to expand or control independently, of the layer, above and below it.
6. The radius of curvature of the beam is very large in comparison to the cross-sectional dimensions of the beam.

b Abeam of I-section beam 500 mm x 200 mm has a web thickness of 10 mm and a flange thickness of 20 mm. If the shear force acting on the section is 200 KN, find the maximum shear stress developed in the I-section. Also sketch the shear stress distribution across the section. 6 M

Solution. Moment of inertia about neutral axis *N-A* is

$$I=\frac{20\times50^3}{12}-\frac{19\times46^3}{12}$$

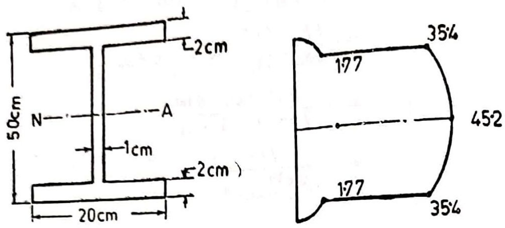
$$=208{,}333{\cdot}3-154{,}115{\cdot}3=54{,}218\ \text{cm}^4$$ 1

Now $\tau=\dfrac{FA\bar{y}}{Ib}$

Shear stress in the flange at the junction with the web

$$=\frac{200\times10^3\times20\times2\times24\times10^{-6}}{53218\times20\times10^{-10}}$$

$$=1{\cdot}17\ \text{MPa}$$



(a) (b) 3

Fig. 7·26

Shear stress in the web at the junction with flange

$$=\frac{200\times10^3\times(20\times2)\times24\times10^{-6}}{54{,}218\times1\times10^{-10}}$$

$$=35{\cdot}4\ \text{MPa}$$

Shear stress at the neutral axis

$$\tau_{max}=\frac{200\times10^3\times(20\times2)\times24\times10^{-6}}{54{,}218\times1\times10^{-10}}+\frac{200\times10^3(1\times23)\times11{\cdot}5\times10^{-6}}{54{,}218\times1\times10^{-10}}$$

$$=45{\cdot}2\ \text{MPa}$$

2

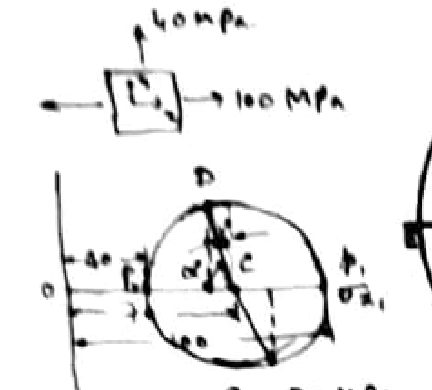
$\sigma_1 = 100\ N/mm^2,\ \sigma_2 = 40\ N/mm^2$

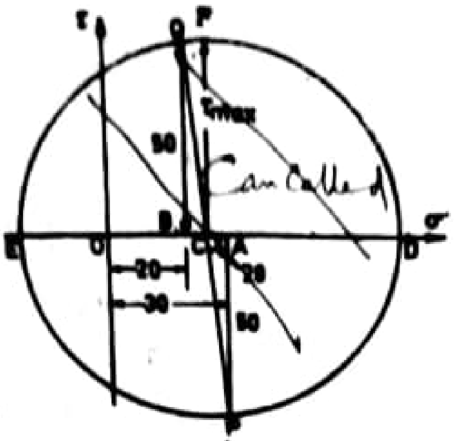
(OR)

9. The state of stress at a point is given by ~~$\sigma_x = 30$ MPa, $\sigma_y = -20$ MPa~~ and $\tau = 50$ MPa. ~~Find the location~~ of principal planes, principal stresses ~~and maximum shear stress~~ using Mohr's Circle for plane stress. 10 M

find Normal, tangential and resultant stress across a plane at $\theta = 48^\circ$

Solution: with the Major principal plane





$R = 30$ MPa

$\alpha = 84^\circ$

$\sigma_{x_1} = 70 - 30 \cos 84^\circ = 66.86$ MPa Scale 1 cm = 10 MPa

By measurement $\tau_{x_1 y_1} = 30 \sin 84^\circ = 29$; $\sigma_{resultant} = \sqrt{(66.86)^2 + 29^2} = 72.88$ MPa 5

$\sigma_{y_1} = 70 + 30 \cos 84 = 70 + 3.136 = 73.136$

Cancelled

i. ~~Maximum Principal stress, σ_1 = OD = 7.5 cm = 75 MPa~~ 1

ii. ~~Minimum Principal stress, σ_2 = OE = 2.5 cm = - 25 MPa~~ 1

iii. ~~Maximum Shear stress, τ_{max} = CF = 5 cm = 50 MPa~~ 1

iv. ~~Position of Principal Planes~~ 1

~~$2\Theta_1 = 84^\circ$~~

Hence

~~$\Theta_1 = 42^\circ$ and $\Theta_2 = 132^\circ$~~ 1