18MA003

Hall Ticket Number:

II/IV B.Tech. (Regular) DEGREE EXAMINATION

November, 2019 **Common to CE/CSE/ECE/EEE/EIE**

Third Semester **Probability and Statistics**

Time: Three Hours **Maximum:** 50 Marks

Answer Question No.1 compulsorily. (10X1 = 10 Marks)

Answer ONE question from each unit. (4X10=40 Marks)

(10X1=10 Marks)

1. Answer all questions
   a) Define probability density function. 1M
   b) If mean of a random variable X is 5, calculate E(2X+3). 1M
   c) Find $Z_{0.075}$ 1M
   d) What is the test statistics for two variances. 1M
   e) Write the formula for maximum error of estimate in one mean. 1M
   f) What is the probability density of Gamma distribution. 1M
   g) Define finite population correction factor 1M
   h) Write the $(1-\alpha)100\%$ large confidence interval for $\mu_1 - \mu_2$. 1M
   i) Write the critical region for testing two proportions. 1M
   j) Write the test statistics for testing for intercept β_0. 1M

UNIT I

2. a) If the probability density of a random variable is given by

$$f(x)=\begin{cases} x & \text{for} \quad 0<x<1 \\ 2-x & \text{for} \quad 1<x<2 \\ 0 & \text{eslewhere} \end{cases}$$

Find the probabilities that a random variable having this probability density will take on a value (i) between 0.2 and 0.8 (ii) between 0.6 and 1.2. Also find its mean. 5M

b) A manufacture knows that, on average, 2% of the electric toasters that he makes will require repairs within 90 days after they are sold. Use the normal approximation to binomial distribution to determine the probability that among 1,200 of these toasters at least 30 will require repairs within the first 90 days after they are sold. 5M

(OR)

3. a) Write the probability density function of the Uniform distribution and also derive its mean and variance . 5M

3. b) If two random variables have the joint density $f(x)=\begin{cases} x_1x_2 & \text{for } 0<x_1<2,\ 0<x_2<1 \\ \text{elsewhere} \end{cases}$.

Find the probabilities that

(i) Both random variables will take on values less than 1.

(ii) First random variable will take on a value less than 1 and second random variable will take on a values greater than 1. 5M

UNIT II

4. a) The specifications for a certain kind of ribbon call for a mean breaking strength of 180 pounds. If five pieces of the ribbon have a mean breaking strength of 169.5 pounds with a standard deviation of 5.7 pounds, test the null hypothesis $\mu = 180$ pounds against the alternative hypothesis $\mu < 180$ pounds at the 0.01 level of significance. Assume that the population distribution is normal. 5M

b) A company claims that its light bulbs are superior to those of its main competitor. If a study showed that a sample of $n_1 = 40$ of its bulbs has a mean lifetime of 1647 hours of continuous use with a standard deviation of 27 hours, while a sample of $n_2 = 40$ bulbs made by its main competitor had a mean lifetime of 1638 hours of continuous use with a standard deviation of 31 hours, does this substantiate the claim at the 0.05 level of significance. 5M

(OR)

5. The following random samples are measurements of the heat-producing capacity of specimens of coal from two mines:

Mine 1: 8,260 8,130 8,350 8,070 8,340

Mine 2: 7,950 7,890 7,900 8,140 7,920 7,840

Use the 0.01 level of significance to test whether the difference between the means of these two samples is significant. Also construct a 95% confidence interval for the difference of means 10M

UNIT III

6. a) If 12 determinations of the specific heat of iron have a standard deviation of 0.0086, test the null hypothesis that $\sigma = 0.010$ for such determinations. Use the alternative hypothesis $\sigma \neq 0.010$ and the level of significance $\alpha = 0.01$. 5M

b) The owner of a machine shop must decide which of two snack-vending machines to install in his shop. If each machine is tested 250 times and the first machine fails to work 13 times and the second machine fails to work 7 times, test at the 0.05 level of significance whether the difference between the corresponding sample proportions is significant. 5M

(OR)

7. The following are the numbers of mistakes made in 5 successive days for 4 technicians working for a photographic laboratory:

Technician I	Technician II	Technician III	Technician IV
6	14	10	9
14	9	12	12
10	12	7	8
8	10	15	10
11	14	11	11

Test at the level of significance $\alpha = 0.01$ whether the differences among 4 sample means can be attributed to chance.. 10M

UNIT IV

8. a) The following table shows how many weeks a sample of 6 persons have worked at an automobile inspection station and the number of cars each one inspected between noon and 2 P.M. on a given day:

Number of Weeks employed, x	2	7	9	1	5	12
Number of Cars inspected, y	13	21	23	14	15	21

Find the equation of the least squares line $y = \beta_0 + \beta_1 x$. 5M

b) The following are the number of minutes it took 10 mechanics to assemble a piece of machinery in the morning, x and in the late afternoon, y:

x	11.1	10.3	12.0	15.1	13.7	18.5	17.3	14.2	14.8	15.3
y	10.9	14.2	13.8	21.5	13.2	21.1	16.4	19.3	17.4	19.0

Calculate sample correlation coefficient. 5M

(OR)

9. The following are the measurements of the air velocity and evaporation coefficient of burning fuel droplets in an impulse engine:

Air Velocity, x	20	60	100	140	180	220	260	300	340	380
Evaporation Coefficient, y	0.18	0.37	0.35	0.78	0.56	0.75	1.18	1.36	1.17	1.65

Fit a straight line to those data by the method of least squares, and use it to estimate the evaporation coefficient of a droplet when the air velocity is 190.

Test the null hypothesis $\beta_1 = 0$ against the alternative hypothesis $\beta_1 \neq 0$ at the 0.05 level of significance. 10M

18MA003

II/IV B.Tech (Regular) Scheme of valuation (Probability & statistics) ①
common to CE/CSE/ECE/EEE/EIE

NOV, 2019 Third Semester 18MA 003

(N. Karunakar).

(a) $f(x)$ is a probability densition function if

(i) $f(x) \geq 0$ (ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(b) Given $E(x) = 5$

Consider $E(2x+3) = 2E(x)+3$

$= 2\times 5+3 = 13$

(c) $z_{0.075} = 1.44$

(d) $F = \frac{S_1^2}{S_2^2}$ or $\frac{S_2^2}{S_1^2}$

(e) maximum error of estimate $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$.

(f) probability density function of Gamma distribution is

$$f(x) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} & \text{for } x>0, \alpha>0, \beta>0 \\ 0 & \text{elsewhere.} \end{cases}$$

(g) population correction factor is $\frac{N-n}{N-1}$.

(h) $(\bar{x}_1-\bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} < \mu_1-\mu_2 < (\bar{x}_1-\bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$

(i) Critical region for testing two proportions is

$$\cancel{\frac{x_1}{n_1} - \frac{x_2}{n_2} \pm z_{\alpha/2}\sqrt{\frac{\frac{x_1}{n_1}\left(1-\frac{x_1}{n_1}\right)}{n_1} + \frac{\frac{x_2}{n_2}\left(1-\frac{x_2}{n_2}\right)}{n_2}}}$$

Alternative hypothesis	Reject Null hypothesis if
$P_1 > P_2$	$Z > Z_\alpha$
$P_1 < P_2$	$Z < -Z_\alpha$
$P_1 \neq P_2$	$Z > Z_{\alpha/2}$ (or) $Z < -Z_{\alpha/2}$

(j) Test statistic for β_0 is

$$Z_0 = \frac{\hat{\beta}_0 - \beta_0}{\sqrt{\sigma^2\left(\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\right)}}$$

.) The Probability density function is given by $f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2-x & \text{for } 1 \le x < 2 \\ 0 & \text{elsewhere} \end{cases}$

i) $P(0.2 < X < 0.8) = \int_{0.2}^{0.8} f(x)dx$

$= \int_{0.2}^{0.8} x\,dx = \int_{0.2}^{0.8} x\,dx = \left[\frac{x^2}{2}\right]_{0.2}^{0.8} = 0.3$

ii) $P(0.6 < x < 1.2) = \int_{0.6}^{1.2} f(x)dx$

$= \int_{0.6}^{1} f(x)dx + \int_{1}^{1.2} f(x)dx$

$= \int_{0.6}^{1} x\,dx + \int_{1}^{1.2} (2-x)dx$

$= \left[\frac{x^2}{2}\right]_{0.6}^{1} + 2\left[x\right]_{1}^{1.2} - \left[\frac{x^2}{2}\right]_{1}^{1.2}$

$= 0.5$

$E(x) = \int_{-\infty}^{\infty} x f(x)dx$

$= \int_{-\infty}^{0} x f(x)dx + \int_{0}^{1} x f(x)dx + \int_{1}^{2} x f(x)dx + \int_{2}^{\infty} x f(x)dx$

$= \int_{0}^{1} x^2 dx + \int_{1}^{2} x(2-x)dx$

$= \int_{0}^{1} x^2 dx + \int_{1}^{2} 2x\,dx - \int_{1}^{2} x^2 dx = 1$

1) Given that $n = 1200$, $p = 2\% = 0.02$,

$$q = 1 - p = 0.98,$$

$$\mu = np = (1200)(0.02) = 24$$

$$\sigma = \sqrt{npq} = \sqrt{(1200)(0.02)(0.98)} = 4.8497$$

The Probability that among 1200 of these toasters atleast 30 will require repairs with in the first 90 days after they are sold is

$$P(x \geq 30) = P(x \geq 30 - 0.5)$$

$$= P(x \geq 29.5)$$

$$= P\left(\frac{x-\mu}{\sigma} \geq \frac{29.5-\mu}{\sigma}\right)$$

$$= P\left(z \geq \frac{29.5-24}{4.8497}\right)$$

$$= P(z \geq 1.1271)$$

$$= 1 - F(1.1271)$$

$$= 1 - F(1.13)$$

$$= 1 - 0.8790$$

$$P(x \geq 30) = 0.1292$$

3.a) For the uniform distribution, the Probability density function

3\. is $f(x) = \begin{cases} \frac{1}{\beta-\alpha} & \text{for } \alpha < x < \beta \\ 0 & \text{elsewhere} \end{cases}$

Mean, $\mu = \int_{-\infty}^{\infty} x\, f(x)dx$

$$= \int_{-\infty}^{\alpha} x\, f(x)dx + \int_{\alpha}^{\beta} x\, f(x)dx + \int_{\beta}^{\infty} x\, f(x)dx$$

$$= \int_{-\infty}^{\alpha} x\,(0)\,dx + \int_{\alpha}^{\beta} x\left(\frac{1}{\beta-\alpha}\right)dx + \int_{\beta}^{\infty} x\,(0)dx$$

$$= \frac{1}{\beta-\alpha}\left[\frac{x^2}{2}\right]_{\alpha}^{\beta} = \frac{1}{\beta-\alpha}\left[\frac{\beta^2-\alpha^2}{2}\right] = \frac{\alpha+\beta}{2}$$

Mean of uniform distribution, $\boxed{\mu = \frac{\alpha+\beta}{2}}$

variance, $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

$$= \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$$

$$= \mu_2' - \mu^2$$

Consider $\mu_2' = \int_{-\infty}^{\infty} x^2 f(x)dx$

$$= \int_{-\infty}^{\alpha} x^2 f(x)dx + \int_{\alpha}^{\beta} x^2 f(x)dx + \int_{\beta}^{\infty} x^2 f(x)dx$$

$$= \int_{\alpha}^{\beta} x^2 \left(\frac{1}{\beta-\alpha}\right) dx$$

$$= \frac{1}{\beta-\alpha} \left(\frac{x^3}{3}\right)_{\alpha}^{\beta} = \frac{1}{\beta-\alpha}\left(\frac{\beta^3-\alpha^3}{3}\right)$$

$$= \frac{1}{3}\left(\beta^2+\alpha^2+\alpha\beta\right)$$

NOW $\sigma^2 = \mu_2' - \mu^2$

$$= \frac{1}{3}\left(\beta^2+\alpha^2+\alpha\beta\right) - \left(\frac{\alpha+\beta}{2}\right)^2$$

$$= \frac{1}{12}(\beta-\alpha)^2$$

variance of uniform distribution, $\boxed{\sigma^2 = \frac{(\beta-\alpha)^2}{12}}$

3.b) Given that $f(x_1, x_2) = \begin{cases} x_1 x_2 & \text{for } 0<x_1<2,\ 0<x_2<1 \\ 0 & \text{elsewhere} \end{cases}$

i) $P(X_1<1, X_2<1) = \int_{x_1=-\infty}^{1} \int_{x_2=-\infty}^{1} f(x_1, x_2)\, dx_2\, dx_1$

$$= \int_{x_1=0}^{1} \int_{x_2=0}^{1} x_1 x_2\, dx_2\, dx_1$$

$$= \frac{1}{4}.$$

(ii) $P(X_1<1, X_2>1) = 0.$

1a) step (i) Null hypothesis, $H_0 : \mu = 180$ Pounds

Alternative hypothesis, $H_1 : \mu < 180$ Pounds

step (ii) level of significance $\alpha = 0.01$

step (iii) criterion: since $n = 5$, $\mu < \mu_0$

Reject the null hypothesis if $t < -t_\alpha$

Here $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$; $t_\alpha = t_{0.01}$ at $\nu = n-1$

step (iv) calculations:

Given $n = 5$, $\mu_0 = 180$, $\bar{x} = 169.5$, $s = 5.7$

$$t = \frac{169.5 - 180}{5.7/\sqrt{5}} = -4.12$$

at $\nu = 4$, $t_{0.01} = 3.747$

step (v) Decision: $t = -4.12 < -t_\alpha = -3.747$

Hence $t < -t_\alpha$, we reject the Null hypothesis.

4.b) **Step (i)** Null hypothesis $H_0 : \mu_1 - \mu_2 = 0$

Alternative hypothesis $H_1 : \mu_1 - \mu_2 > 0$

Step (ii) level of significance, $\alpha = 0.05$

Step (iii) Criterion:

we reject the Null hypothesis if $z > z_\alpha$

here $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - \delta}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$ and $z_\alpha = z_{0.05}$

Step (iv) calculations:-

Given $n_1 = 40$, $\bar{x}_1 = 1647$, $\sigma_1 = 27$,

$n_2 = 40$, $\bar{x}_2 = 1638$, $\sigma_2 = 31$, $\delta = 0$

$$z = \frac{(1647 - 1638) - 0}{\sqrt{\dfrac{(27)^2}{40} + \dfrac{(31)^2}{40}}} = 1.38$$

$F(z_\alpha) = F(z_{0.05})$

$= 1 - 0.05$

$= 0.9500$

$= F(1.645)$

$\Rightarrow z_\alpha = 1.645$

Step v : Decision:-

$z = 1.385 < z_\alpha = 1.645$

$\therefore$ we accept the Null hypotheses.

5. step(i): Null hypothesis, $H_0: \mu_1 - \mu_2 = 0$

Alternative hypothesis, $H_1: \mu_1 - \mu_2 \neq 0$

step(ii): level of significance, $\alpha = 0.01$

step(iii): criteria:

we reject the null hypothesis if $t < -t_{\frac{\alpha}{2}}$ or $t > t_{\alpha/2}$

Here $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - \delta}{\sqrt{(n_1-1)s_1^2 + (n_2-1)s_2^2}} \sqrt{\dfrac{n_1 n_2 (n_1+n_2-2)}{n_1+n_2}}$

step(iv): calculations:

Given that $n_1 = 5$, $n_2 = 6$

$$\bar{x}_1 = \frac{1}{5}[8260 + 8130 + 8350 + 8070 + 8340] = 8230$$

$$\bar{x}_2 = \frac{1}{6}[7950 + 7890 + 7900 + 8140 + 7920 + 7840] = 7940$$

$$s_1^2 = \frac{1}{4}\left[(8260-8230)^2 + (8130-8230)^2 + (8350-8230)^2 + (8070-8230)^2 + (8340-8230)^2\right]$$

$$= 15{,}750$$

$$s_2^2 = \frac{1}{5}\left[(7950-7940)^2 + (7890-7940)^2 + (7900-7940)^2 + (8140-7940)^2 + (7920-7940)^2 + (7840-7940)^2\right]$$

$$= 10{,}920$$

$$t = \frac{(8230-7940) - 0}{\sqrt{4(15750) + 5(10920)}} \sqrt{\frac{(5)(6)(5+6-2)}{5+6}} = 4.1897$$

(5-1)

$t_{\frac{\alpha}{2}} = t_{0.005}$ at $\nu = n_1+n_2-2$ is 3.25

Step(v): $t = 4.1897 > t_{\frac{\alpha}{2}} = 3.25$

$\therefore$ we reject Null hypothesis.

Small sample confidence interval for the difference of mean is:

$$(\bar{x}_1 - \bar{x}_2) - t_{\frac{\alpha}{2}} \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} \sqrt{\frac{1}{n_1}+\frac{1}{n_2}} < \mu_1 - \mu_2 <$$

$$(\bar{x}_1 - \bar{x}_2) + t_{\frac{\alpha}{2}} \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$t_{\frac{\alpha}{2}} = t_{0.025} = 2.262$

$$(8230-7940) - (2.262)\sqrt{\frac{(4)(15750)+5(10920)}{5+6-2}} \sqrt{\frac{1}{5}+\frac{1}{6}} < \mu_1 - \mu_2 <$$

$$(8230-7940) + (2.262)\sqrt{\frac{4(15750)+5(10920)}{5+6-2}} \sqrt{\frac{1}{5}+\frac{1}{6}}$$

$$(290) - 156.5630 < \mu_1 - \mu_2 < 290 + 156.5630$$

$$\boxed{133.4370 < \mu_1 - \mu_2 < 446.5630}$$

6.a) Step(i): Null hypothesis, $H_0 : \sigma = 0.010$

Alternative hypothesis $H_1 : \sigma \neq 0.010$

Step(ii): level of significance, $\alpha = 0.01$

Step(iii): criterion:

we reject the Null hypothesis if $\chi^2 < \chi^2_{1-\frac{\alpha}{2}}$ or $\chi^2 > \chi^2_{\frac{\alpha}{2}}$

here $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$

step(iv): calculations:

Given that $n = 12$, $s = 0.0086$, $\sigma_0 = 0.010$

$$\chi^2 = \frac{(12-1)(0.0086)^2}{(0.010)^2} = 8.1356$$

$\chi^2_{\frac{\alpha}{2}} = \chi^2_{0.005}$ at $\nu = n-1$ is 26.757

$\chi^2_{1-\frac{\alpha}{2}} = \chi^2_{0.9950}$ at $\nu = n-1$ is 2.603

step(v): Decision:

$\chi^2 = 8.1356 > \chi^2_{1-\frac{\alpha}{2}} = 2.603$

$\therefore$ we accept the Null hypothesis

$\chi^2 = 8.1356 < \chi^2_{\frac{\alpha}{2}} = 26.757$

$\therefore$ we accept the Null hypothesis.

6.b) step(i) Null hypothesis, $H_0: P_1 = P_2$

Alternative hypothesis $H_1: P_1 \neq P_2$

step(ii): level of significance $\alpha = 0.05$

step(iii): criteria:

we reject the null hypothesis if $z > z_{\frac{\alpha}{2}}$ or $z < -z_{\frac{\alpha}{2}}$

here $z = \dfrac{\frac{x_1}{n_1} - \frac{x_2}{n_2}}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$ where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$

Step(iv) calculations:-

Here $x_1 = 13$, $x_2 = 7$, $n_1 = 250$, $n_2 = 250$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{13+7}{500} = 0.04$$

$$Z = \frac{\frac{13}{250} - \frac{7}{250}}{\sqrt{(0.04)(1-0.04)\left(\frac{1}{250}+\frac{1}{250}\right)}} = 1.3693$$

$$F(z_{\frac{\alpha}{2}}) = F\left(z_{\frac{0.05}{2}}\right) = F(z_{0.025})$$

$$= 1 - 0.025 = 0.9750 = F(1.96)$$

$$\boxed{z_{\frac{\alpha}{2}} = 1.96}$$

Step(v): Decision:-

$Z = 1.3693 < z_{\frac{\alpha}{2}} = 1.96$

$\therefore$ we accept the Null hypothesis

$Z = 1.3693 > -z_{\frac{\alpha}{2}} = -1.96$

we accept the Null hypothesis.

7. Step(i) Null hypothesis $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

Alternative hypothesis $H_1: \mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$

Step(ii) level of significance $\alpha = 0.01$

Step(iii) To reject the Null hypothesis if $F > F_\alpha(k-1, N-k)$

Step(iv): calculations:

Given that $k = 4$, sample size $n_1 = 5$, $n_2 = 5$, $n_3 = 5$, $n_4 = 5$

$T_1 = 49$, $T_2 = 59$, $T_3 = 55$, $T_4 = 50$,

grand Total $T_\cdot = T_1 + T_2 + T_3 + T_4$

$= 213$

$N = n_1 + n_2 + n_3 + n_4 = 20$

ANOVA Table :-

Source of variation	Degrees of freedom	Sum of Squares	Mean Squares	F
Technicians	$K-1 = 3$	$SST_r = 12.95$	$MST_r = 4.3167$	$\frac{MST_r}{MSE} = 0.6798$
error	$N-K = 16$	$SSE = 101.60$	$MSE = 6.35$	
Total	$N-1 = 19$	$SST = 114.55$		

$$SST_r = \frac{\sum_{i=1}^{k} T_i^2}{n_i} - C$$

$$= \frac{\sum_{i=1}^{4} T_i^2}{n_i} - C$$

$$= \frac{(49)^2}{5} + \frac{(59)^2}{5} + \frac{(55)^2}{5} + \frac{(50)^2}{5} - 2268.48 = 12.95$$

$$C = \frac{T_\cdot^2}{N} = \frac{(213)^2}{20} = 2268.48$$

$$SSE = SST - SSTr$$

$$= 114.55 - 12.95 = 101.60$$

$$SST = \sum_{i=1}^{k} \sum_{j=1}^{n_i} y_{ij}^2 - C$$

$$= (6)^2 + (14)^2 + (10)^2 + (8)^2 + (11)^2 + (14)^2 + (9)^2 + (12)^2 + (10)^2 + (14)^2$$

$$+ (10)^2 + (12)^2 + (7)^2 + (15)^2 + (11)^2 + 9^2 + 12^2 + (8)^2 + (10)^2 + (11)^2$$

$$- 2268.45$$

$$= 114.55$$

$$MSTr = \frac{SSTr}{k-1} = \frac{12.95}{4-1} = 4.3167$$

$$MSE = \frac{SSE}{N-k} = \frac{101.60}{16} = 6.35$$

$$F_\alpha (k-1, N-k) = F_{0.01} (4-1, 20-4)$$

$$= F_{0.01} (3, 16)$$

$$= 5.29$$

step(v): Decision:

$$F = 0.6798 \ngtr F_\alpha (k-1, N-k) = 5.29$$

$\therefore$ we ~~reject~~ accept the null hypothesis.

8.a) Given that $n = 6$

x	y	x^2	xy
2	13	4	26
7	21	49	147
9	23	81	207
1	14	1	14
5	15	25	75
12	21	144	252

$\Sigma x = 36$ $\quad \Sigma y = 107$ $\quad \Sigma x^2 = 304$ $\quad \Sigma xy = 721$

$$\beta_0 = \bar{y} - \beta_1 \bar{x} = 12.4471$$

here $\bar{y} = \dfrac{13+21+23+14+15+21}{6} = 17.8333$

$$\bar{x} = \frac{2+7+9+1+5+12}{6} = 6$$

$$\beta_1 = \frac{S_{xy}}{S_{xx}} = 0.8977$$

$$S_{xy} = \Sigma xy - \frac{\Sigma x \Sigma y}{n} = 79$$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 88$$

$\therefore$ the least squares line is $\hat{y} = \beta_0 + \beta_1 x$

$$\hat{y} = 12.4471 + 0.8977x$$

8.b) Given that $n = 10$

x	y	x^2	y^2	xy
11.1	10.9	123.21	118.81	120.99
10.3	14.2	106.09	201.64	146.26
12.0	13.8	144.00	190.44	165.60
15.1	21.5	228.01	462.25	324.65
13.7	13.2	187.69	174.24	180.84
18.5	21.1	342.25	445.21	390.35
17.3	16.4	299.29	268.96	283.72
14.2	19.3	201.64	372.49	274.06
14.8	17.4	219.04	302.76	257.52
15.3	19.0	234.09	361.01	290.70
$\Sigma x = 142.3$	$\Sigma y = 166.8$	$\Sigma x^2 = 2085.31$	$\Sigma y^2 = 2897.80$	$\Sigma xy = 2434.69$

$$S_{xy} = \Sigma xy - \frac{\Sigma x \Sigma y}{n} = 61.126$$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 60.381$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n} = 115.576$$

sample correlation coefficient $r = \frac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}} = 0.732$

9. Given that $n = 10$

x	y	x^2	y^2	xy
20	0.18	400	0.0324	3.6
60	0.37	3600	0.1369	22.2
100	0.35	10,000	0.1225	35.0
140	0.78	19,600	0.6084	109.20
180	0.56	32,400	0.3136	100.80
220	0.75	40,000	0.5625	150.00
260	1.18	67,600	1.3924	306.80
300	1.36	90,000	1.8496	408.00
340	1.17	1,15,600	1.3689	397.80
380	1.65	1,44,400	2.7225	627.00
$\Sigma x =$ 2,000	$\Sigma y =$ 8.35	$\Sigma x^2 =$ 5,32,000	$\Sigma y^2 =$ 9.1097	$\Sigma xy =$ 2175.40

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 1,32,000$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n} = 2.13745$$

$$S_{xy} = \Sigma xy - \frac{\Sigma x \Sigma y}{n} = 505.40$$

$$\beta_1 = \frac{S_{xy}}{S_{xx}} = 0.00383$$

$$\beta_0 = \bar{y} - \beta_1 x = 0.069$$

$\therefore$ the least squares line is $\hat{y} = \beta_0 + \beta_1 x$

$= 0.069 + 0.00383x$

For $x = 190$, $\hat{y} = 0.80 \ mm^2/sec$

<u>Step(i)</u> Null hypothesis $H_0 : \beta_1 = 0$

Alternative hypothesis $H_1 : \beta_1 \neq 0$

<u>Step (ii)</u> level of significance $\alpha = 0.05$

<u>Step(iii)</u> <u>criteria:</u>

we reject the Null hypothesis if $|t_0| > t_{\frac{\alpha}{2}}$ at $\nu = n-2$

here $t_0 = \dfrac{\hat{\beta}_1}{\sqrt{\dfrac{MS_{Res}}{S_{xx}}}}$

<u>Step(iv)</u> <u>calculations:</u>

$MS_{Res} = \dfrac{SS_{Res}}{n-2} = 0.025297$, $t_0 = \dfrac{0.00383}{\sqrt{\dfrac{0.025297}{1{,}32{,}000}}} = 8.7488$

$SS_{Res} = S_{yy} - \dfrac{S_{xy}^2}{S_{xx}} = 0.20238$, $\boxed{t_0 = 8.7488}$

$t_{\frac{\alpha}{2}} = t_{0.025}$ at $\nu = 8$ is 2.306

<u>Step (v)</u> <u>Decision :</u>

$t_0 = 8.7488 > t_{\frac{\alpha}{2}} = 2.306$

$\therefore$ we reject the Null hypothesis.