Hall Ticket Number:

I/IV B.Tech (Supplementary) DEGREE EXAMINATION

November, 2019 — **Common to all branches**

First Semester — **Engineering Mathematics - I**

Time: Three Hours — **Maximum:** 60 Marks

Answer Question No.1 compulsorily. (1X12 = 12 Marks)

Answer ONE question from each unit. (4X12=48 Marks)

1. Answer all questions (1X12=12 Marks)
   a) Define Rank of a matrix.
   b) Find the eigen values of $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$
   c) Define Orthogonal matrix.
   d) State Rolle's theorem.
   e) Define Skew-Hermitian matrix .
   f) What is the condition for $u = f(x, y)$ to have maximum and minimum.
   g) What is the period of sin2x.
   h) Write the Fourier sine series of the function f(x) in (0,L)
   i) What is the sum of the Fourier series at a point of discontinuity.
   j) Define double integral.
   k) Write the spherical polar co-ordinate system.
   l) Evaluate $\int_0^2\int_0^1 y\, dydx$

UNIT I

2. a) Find the Rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ — 6M
   b) Find the eigen values and eigen vectors of = $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$ — 6M

(OR)

3. a) Test for consistency and solve — 6M
   $3x + 3y + 2z = 1\,, x + 2y = 4\,, 10y + 3z = -2\,, 2x - 3y - z = 5.$
   b) Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ — 6M

UNIT II

4. a) Reduce the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to the diagonal form. — 6M
   b) Show that $\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \frac{b-a}{\sqrt{1-b^2}}\,, 0 < a < b < 1.$ — 6M

(OR)

5. a) Use McLaurin's expansion (up to third degree) to show that $\log(1 + x) < x - \frac{x^2}{2} + \frac{x^3}{3}\,, x > 0.$ — 6M
   b) Divide 24 into three parts such that the product of the first , square of second and cube of the third may be maximum. — 6M

UNIT III

6. a) Find the Fourier series of the function $f(x) = x, -\pi \le x \le \pi$. — 6M
   b) Expand f(x) = cosx as a half range sine series in (0,π) — 6M

(OR)

7. a) Find the Fourier series of the function $f(x) = \pi \sin \pi x$ *for* $0 < x < 1$. — 6M
   b) Find the complex form of the Fourier series of f(x) = sinx, 0<x<2π . — 6M

UNIT IV

8. a) Find the area of the region bounded by the parabolas $y^2 = 4ax$ *and* $x^2 = 4ay$. — 6M
   b) Change the order of integration and hence evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2\, dy\, dx.$ — 6M

(OR)

9. a) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integration. — 6M
   b) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dy\, dx.$ — 6M

1.
2. a) Define Rank of a matrix.

b) Find the eigen values of $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

c) Define Orthogonal matrix.

d) State Rolle's theorem.

e) Define Skew-Hermitian matrix .

f) what is the condition for $u = f(x, y)$ to have maximum and minimum.

g) Find the integrating factor of $\log x\, dy + (y - \log x)dx = 0$.

h) Define Orthogonal trajectories of curves.

I) State Newton's law of cooling.

j) Write the general form of Euler- Cauchy equation.

k) Find the general solution of $y'' + 6y' + 9y = 0$.

l) Find the wronskian of the functions $u = \cos 2x\, , v = \sin 2x$.

2 a) Find the Rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ 6M

b) Find the eigen values and eigen vectors of $= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$. 6M

(or)

3 a)Test for consistency and solve

$3x + 3y + 2z = 1\, , x + 2y = 4\, , 10y + 3z = -2\, ,2x - 3y - z = 5.$ 6M

b)Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$. 6M

4 a) Reduce the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to the diagonal form. 6M

b) Show that $\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \frac{b-a}{\sqrt{1-b^2}}\, ,0 < a < b < 1.$ 6M

(or)

5 a) Use Maclaurin's expansion (up to third degree) to show that $\log(1 + x) < x - \frac{x^2}{2} + \frac{x^3}{3}\, ,x > 0.$

b) Divide 24 into three parts such that the product of the first , square of second and cube of the third may be maximum. 6M

6 a) Solve the differential equation $y \sin 2x\, dx - (1 + y^2 + cos^2x)dy = 0.$ 6M

b) Find the orthogonal trajectories of a confocal and coaxial parabolas $y^2 = 4a(x + a)$. 6M

(or)

7 a) Solve the differential equation $\frac{dy}{dx} + y = x^3 y^6$. 6M

b)) If 30% of radioactive substance disappeared in 10 days . How long will it take for 90% of it to disappear? 6M

8) a) Solve the differential equation $y'' + 5y' + 6y = 0,$ given $y(0) = 0, y'(0) = 15$. 6M

b) Solve the differential equation $y'' - 6y' + 9y = \frac{e^{3x}}{x^2},$

by the method of variation of parameters. 6M

(or)

9 a) Solve the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 2\,x^2 + 3e^{-x}$ 6M

b) The damped LCR circuit is governed by the equation $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = 0$ where L , R , C are positive constants . Find conditions under which the circuit is over damped , under damped and critically damped . Find also the critical resistance. 6M

Scheme of valuation :

1 a) Definition 1 Mark

b) Eigen values 3 ,2 ,5. 1 Mark

c)) Definition 1 Mark

d) Statement 1 Mark

e) Definition 1 Mark

f) $\frac{\partial u}{\partial x} = 0 , \frac{\partial u}{\partial y} = 0$ 1 Mark

g) Integrating factor = x. 1 Mark

h) Definition 1 Mark

i) Statement 1 Mark

j) Definition 1 Mark

k) $y = (c_1 + c_2 x)e^{-3x}$ 1 Mark

l) Wronskian = 2 1 Mark

2 a) Reduce into echelon form 5 marks

Rank 1 Mark

b) $|A\text{-}\lambda\, I| = 0$ 1 Mark

eigen values (1 ,1 ,-2)	2 Marks
eigen vectors	3 marks
3 a) $A\,X = B$	1Mark
Echelon form	3 Marks
Solution $x = 2, y = 1\,, z = \ -4$	2 Marks
b) Finding inverse	6 Marks
4 a) Eigen values	3 marks
Diagonalization	3 marks
b)) $f(x) = \sin^{-1} x$	1 Mark
Apply LMVT	2 Marks
Inequality	3 Marks
5 a) $f(x) = \ \log(1 + x)$	1 Mark
Maclaurin's expansion	3 Marks
Inequality	2 Marks
b) $x + y + z = 24$	1 Mark
$f(x, y, z) = xy^2z^3$	1 Mark
$F = \ xy^2z^3 + \lambda\,(x+y+z-24)$	1 Mark
Solution	3 Marks
6 a) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$	2 Marks
Solution	4 Marks
b) Form the differential equation	2 Marks
Changing y' by $\frac{-1}{y'}$	1 Mark
Orthogonal trajectory	3 Marks
7 a) Bernoulli's form	1 Mark
Substitution	2 Marks
Solution	3 Marks
b) Law	1 Mark
Finding C value	1 Mark
Finding K value	1 Mark
Solution	2 Marks
8 a) Roots	2 Marks
Solution	2 Marks
$C_{1\,,}\ C_2$ Values	2 Marks
b) Finding complementary function	2 Marks
Finding particular integral	4 Marks
9 a) Finding complementary function	2 Marks
Finding particular integral	4 Marks

b) Finding over damping , under damping and critically damping

the critical resistance. 6 Marks.