### Hall Ticket Number:

# November, 2019

## **Fourth Semester**

**Common to all branches Engineering Mathematics – IV** 

Time: Three Hours

Answer Question No.1 compulsorily.

Answer ONE question from each unit.

- Answer all questions 1
  - Define Analytic Function. a)
  - If  $x + iy = \sqrt{3}$ , find  $x^2 + y^2$ . b)
  - Write C-R equations in Cartesian coordinates. c)
  - d) State Laurent's series.

e) Find the poles of 
$$f(z) = \frac{z-3}{z^2+2z+5}$$

- Define residue. f)
- Define Uniform distribution. g)
- h) State central limit theorem.
- i) Define sampling distribution.
- Define point estimation. j)
- k) State different types of errors.
- 1) Write the test statistic for single proportion.

### UNIT I

- 2 Find all roots of the equation  $\sqrt[4]{1+i}$ a)
  - 6M Evaluate  $\int_{C} \frac{\log z}{(z-1)^3} dz$  where C is |z-1| = 1/2 using Cauchy's integral formula. b)

### (**OR**)

Construct the analytic function whose real part is  $x^2 - y^2 - y$ 3 a) 6M

Evaluate  $\int_{C} \frac{\sin^2 z}{(z - \pi/6)^3} dz$  where C is |z| = 1 using Cauchy's integral formula. b) 6M

### **UNIT II**

<sup>4</sup> a) Expand 
$$\frac{e^{2z}}{(z-1)^3}$$
 about  $z=1$  6M

Find all poles and residues at each pole for  $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ . 6M b)

(OR) Find the Laurent's series expansion of  $f(z) = \frac{7z - 2}{(z+1)z(z-2)}$  in the region 1 < |z+1| < 3. 5 6M a)

b) Evaluate 
$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx$$
 by method of contour integration. 6M

(1X12 = 12 Marks)

Maximum: 60 Marks

(4X12=48 Marks)

6M

6M

### **UNIT III**

- 6 a) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D. 6M of the distribution.
  - b) A random sample of size 25 from a normal population has the mean 47.5 and standard deviation 8.4. 6M Does this information support or refuse the claims that this mean of the population is  $\mu = 42.1$ .

### (OR)

- 7 a) Find Mean and Variance of Uniform Distribution
  - b) A manufacturer of fuses claims that with a 20% overload, the fuses will blow in 12.40 minutes on 6M the average. To test the claim, a sample of 20 of the fuses was subjected to a 20% overload, and the times it took them to blow had a mean of 10.63 minutes and a standard deviation of 2.48 minutes. If it can be assumed that the data constitute a random sample from a normal population, do they tend to support or refute the manufacturer's claim?

### **UNIT IV**

- a) A trucking firm is suspicious of the claim that the average lifetime of certain tires is at least 28,000 6M miles. To check the claim, the firm puts 40 of these tires on its rucks and gets a mean lifetime of 27,463 miles with a standard deviation of 1,348 miles. What can it conclude if the probability of a Type I error (α) is to be at most 0.01?
  - b) The lapping process which is used to grind certain silicon wafers to the proper thickness is 6M acceptable only if  $\sigma$ , the population standard deviation of the thickness of dice cut from the wafers, is at most 0.50 mil. Use the 0.05 level of significance to test the null hypothesis  $\sigma = 0.50$  against the alternative hypothesis  $\sigma > 0.50$ , if the thickness of 15 dice cut from such wafers have a standard deviation of 0.64 mil.

### (OR)

- 9 a) A random sample of size 100 is taken from a population with  $\sigma = 5.1$ . Given that  $\overline{x} = 21.6$ , construct 6M a 95% confidence interval for the population mean.
  - b) A manufacturer of submersible pumps claims that at most 30% of the pumps require repairs within 6M the first 5 years of operation. If a random sample of 120 of these pumps includes 47 which required repairs within the first 5 years, test the null hypothesis p = 0.30 against the alternative hypothesis p > 0.30 at the 0.05 level of significance.