# **Scheme** - **III/IV B.Tech (Regular/Supplementary) DEGREE EXAMINATION**

**February, 2021** **Electronics and Communication Engineering**

**Fifth Semester** **(18EC502)Linear Control Systems**

**Time:** Three Hours **Maximum :** 60 Marks

*Answer Question No.1 compulsorily.* (1X12 = 12 Marks)

*Answer ONE question from each unit.* (4X12=48 Marks)

1. Answer all questions (1X12=12 Marks)

a) Define transfer function CO1
The T.F of a system is defined as the ratio of the Laplace transform of output to Laplacetransform of input with zero initial conditions.

b) What is linear and non-linear control systems CO1
A control system is said to be linear if superposition theorem is applied which satisfies two prpperties 1. Additivity f(x+y)=f(x)+f(y), Homogeneous property f(kx)=kf(x)

c) What is the need for signal flow graph CO1
Block diagrams are very successful for representing control systems, but for complicated systems, the block diagram reduction process is tedious and time consuming. So signal flow graphs are needed which does not require any reduction process because of availability of a flow graph formula, which relates the input and output system variables

d) what are static error constants CO2
Kp, Kv and Ka are called static error constants
Give the equation of a second order system with unit step input.

e) What is the use of RH criterion CO2
It is the easy method to determine the system stability form RH table 1$^{st}$ column elements

f) Give the formula for velocity error constant.

g) What is Bode plot? CO3
The Bode plot is the frequency response plot of the transfer function of a system. A Bode plot consists of two graphs. One is the plot of magnitude of sinusoidal transfer function versus log w. The other is a plot of the phase angle of a sinusoidal function versus log w.

h) What is Polar plot? CO3
The Polar plot of a Sinusoidal transfer function G(jw) is a plot of the magnitude of G(jw)versus the phase angle of G(jw) on polar or rectangular co-ordinates as w is varied from zero to infinity

i) What do you mean by root locus technique CO4
oot locus technique provides a graphical method of plotting the locus of the roots in the S-plane as a given system parameter, is varied over the complete range of values (may be from zero to infinity). The roots corresponding to a particular value of the system parameter can then be located on the locus or the value of the parameter for a desired root location can be determined from the locus

j) What is observability?

k) Define state variable.
The variables involved in determining the state of dynamic system are called state variables. Generally x 1 (t), x 2 (t) ………………… x n (t) are called state variables.

l) Write the state model of a system CO4

$\dot{X}(t) = A\,x(t) + B\,U(t)$ ...State equation

$Y(t) = C\,x(t) + D\,U(t)$ ...Output equation

**This is state model of a system.**

**UNIT I**

2. a) Obtain the transfer function of the block diagram given below? 5M CO1

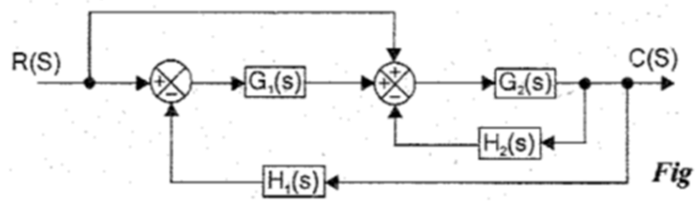
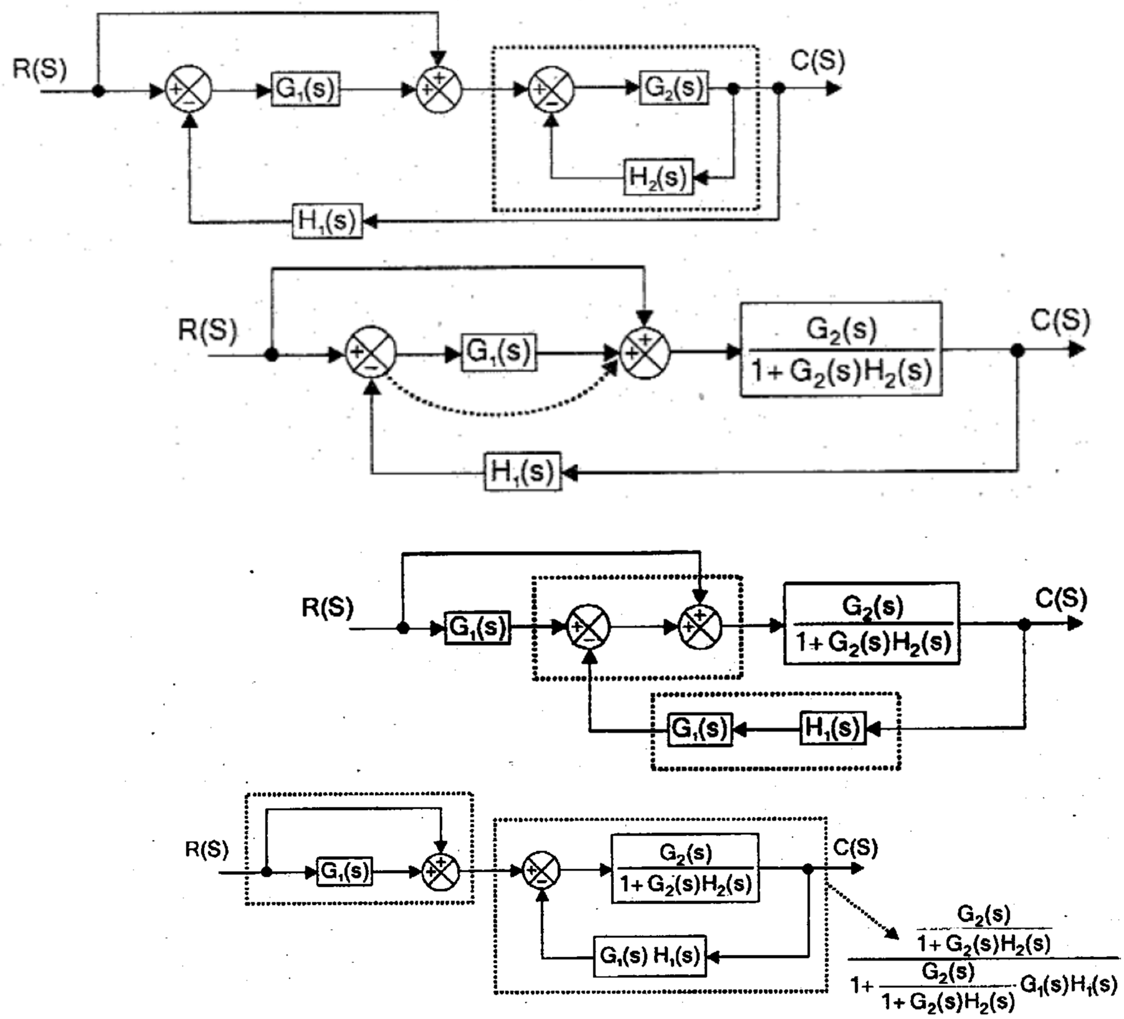
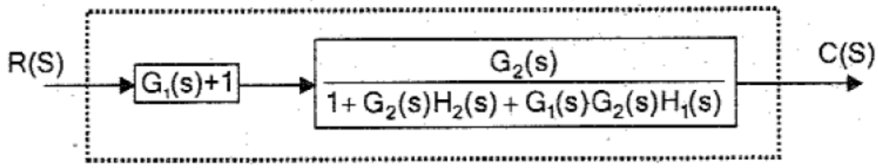


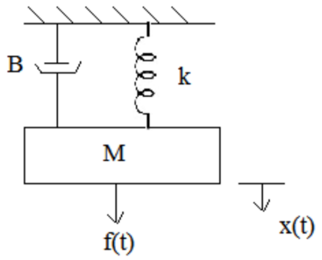

*Fig 1.*






$$\therefore \frac{C(s)}{R(s)} = \frac{G_2(s)\,[G_1(s)+1]}{1+G_2(s)\,H_2(s)+G_1(s)\,G_2(s)\,H_1(s)}$$

b) Find the transfer function for the given mechanical system. 5M CO1




**FORCE BALANCE EQUATIONS OF IDEALIZED ELEMENTS**

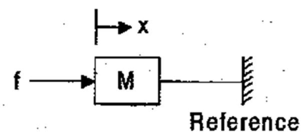
Consider an ideal mass element shown in fig 1.9 which has negligible friction and elasticity. Let a force be applied on it. The mass will offer an opposing force which is proportional to acceleration of the body.

Let, $f$ = Applied force

$f_m$ = Opposing force due to mass

Here, $f_m \propto \frac{d^2x}{dt^2}$ or $f_m = M\frac{d^2x}{dt^2}$

By Newton's second law, $\boxed{f = f_m = M\frac{d^2x}{dt^2}}$ .....(1.2)




*Ideal mass element.*

Consider an ideal frictional element dashpot shown in fig 1.10 which has negligible mass and elasticity. Let a force be applied on it. The dash-pot will offer an opposing force which is proportional to velocity of the body.

Let, $f$ = Applied force

$f_b$ = Opposing force due to friction

Here, $f_b \propto \frac{dx}{dt}$ or $f_b = B\frac{dx}{dt}$

By Newton's second law, $\boxed{f = f_b = B\frac{dx}{dt}}$ .....(1.3)

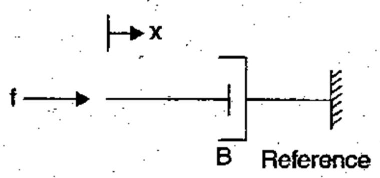



***Fig 1.10 :** Ideal dashpot with one end fixed to reference.*

Consider an ideal elastic element spring shown in fig 1.12, which has negligible mass and friction. Let a force be applied on it. The spring will offer an opposing force which is proportional to displacement of the body.

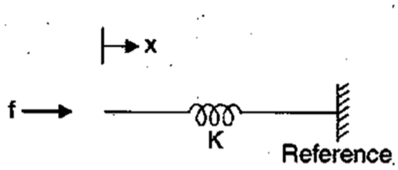



*Fig 1.12 : Ideal spring with one end fixed to reference.*

Let, $f$ = Applied force

$f_k$ = Opposing force due to elasticity

Here $f_k \propto x$ or $f_k = Kx$

By Newton's second law, $\boxed{f = f_k = Kx}$ .....(1.5)

T.F $X(t)/f(t)=1/ MS^2+Bs+K$

**(OR)**

3 (a) Find the overall transfer function of the system whose signal flow graph is shown in fig1.. 5M CO1

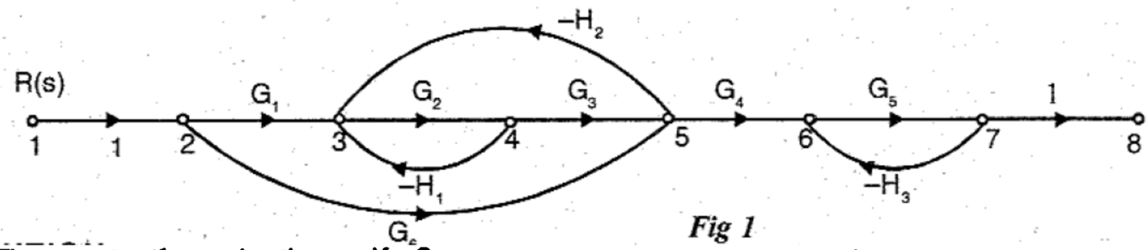



*Fig 1*

There are two forward paths. ∴ K = 2

Let forward path gains be $P_1$ and $P_2$.

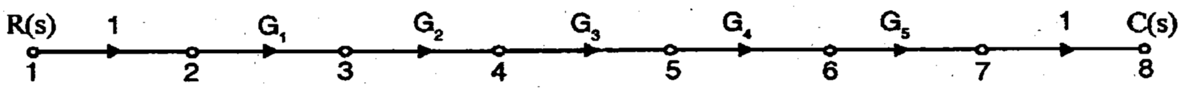



*Fig 2 : Forward path-1.*

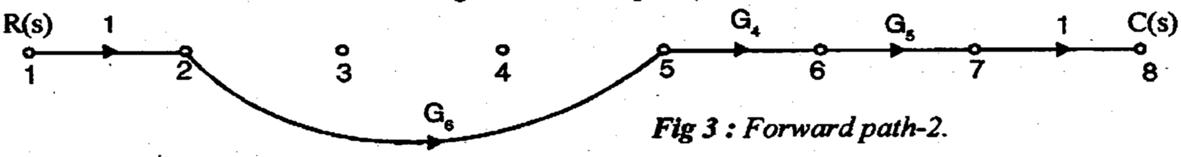



*Fig 3 : Forward path-2.*

Gain of forward path-1, $P_1 = G_1 G_2 G_3 G_4 G_5$

Gain of forward path-2, $P_2 = G_4 G_5 G_6$

**Individual Loop Gain**

There are three individual loops. Let individual loop gains be $P_{11}$, $P_{21}$ and $P_{31}$.

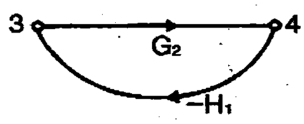



*Fig 4 : Loop-1.*

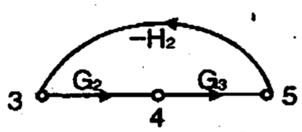



*Fig 5 : Loop-2.*

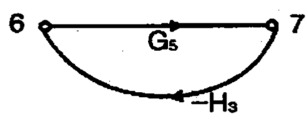



*Fig 6 : Loop-3.*

Loop gain of individual loop-1, $P_{11} = -G_2H_1$

Loop gain of individual loop-2, $P_{21} = -G_2G_3H_2$

Loop gain of individual loop-3, $P_{31} = -G_5H_3$

## Gain Products of Two Non-touching Loops

There are two combinations of two non-touching loops. Let the gain products of two non touching loops be $P_{12}$ and $P_{22}$.

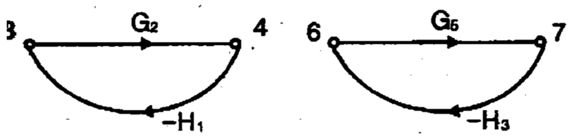



*Fig 7 : First combination of 2 non-touching loops.*

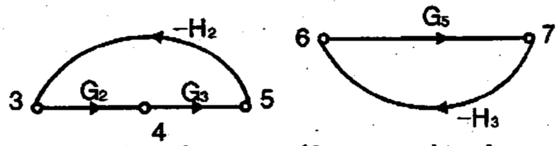



*Fig 8 : Second combination of 2 non-touching loops.*

Gain product of first combination of two non touching loops } $P_{12} = P_{11}P_{31} = (-G_2H_1)(-G_5H_3) = G_2G_5H_1H_3$

Gain product of second combination } $P_{22} = P_{21}P_{31} = (-G_2G_3H_2)(-G_5H_3) = G_2G_3G_5H_2H_3$

## IV. Calculation of Δ and $\Delta_K$

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12} + P_{22})$$

$$= 1 - (-G_2H_1 - G_2G_3H_2 - G_5H_3) + (G_2G_5H_1H_3 + G_2G_3G_5H_2H_3)$$

$$= 1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3$$

$\Delta_1 = 1$, Since there is no part of graph which is not touching with first forward path.

The part of the graph which is non touching with second forward path is shown in fig 9.

$\Delta_2 = 1 - P_{11} = 1 - (-G_2H_1) = 1 + G_2H_1$

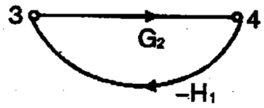



*Fig 9*

## V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2) \quad \text{(Number of forward paths is 2 and so K = 2)}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6(1+G_2H_1)}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6 + G_2G_4G_5G_6H_1}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

$$= \frac{G_2G_4G_5\left[G_1G_3 + G_6/G_2 + G_6H_1\right]}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

**(OR)**

b) Write the differential equations governing the mechanical rotational system shown in fig 1. Obtain the transfer function of the system 5M C01

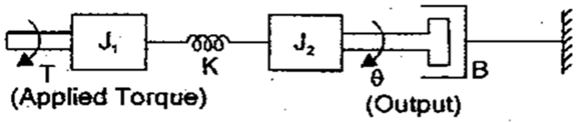



*Fig 1.*

$$J_1\frac{d^2\theta_1}{dt^2}+K(\theta_1-\theta)=T$$

$$J_1\frac{d^2\theta_1}{dt^2}+K\theta_1-K\theta=T \qquad .....(1)$$

**On taking Laplace transform of equation (1) with zero initial conditions we get,**

$$J_1 s^2\,\theta_1(s)+K\theta_1(s)-K\theta(s)=T(s)$$

$$(J_1 s^2+K)\,\theta_1(s)-K\,\theta(s)=T(s) \qquad .....(2)$$

$$\therefore J_2\frac{d^2\theta}{dt^2}+B\frac{d\theta}{dt}+K(\theta-\theta_1)=0$$

$$J_2\frac{d^2\theta}{dt^2}+B\frac{d\theta}{dt}+K\theta-K\theta_1=0$$

**On taking Laplace transform of above equation with zero initial conditions we get,**

$$J_2 s^2\theta(s)+B\,s\,\theta(s)+K\theta(s)-K\theta_1(s)=0$$

$$(J_2 s^2+Bs+K)\,\theta(s)-K\theta_1(s)=0$$

$$\theta_1(s)=\frac{(J_2 s^2+Bs+K)}{K}\,\theta(s) \qquad .....(3)$$

**Substituting for $\theta_1(s)$ from equation (3) in equation (2) we get,**

$$(J_1s^2+K)\,\frac{(J_2s^2+Bs+K)}{K}\,\theta(s)-K\theta(s)=T(s)$$

$$\left[\frac{(J_1s^2+K)\,(J_2s^2+Bs+K)-K^2}{K}\right]\theta(s)=T(s)$$

$$\therefore\frac{\theta(s)}{T(s)}=\frac{K}{(J_1s^2+K)\,(J_2s^2+Bs+K)-K^2}$$

## UNIT II

4. a) Find (i) $w_d$ (ii) $T_r$ (iii) $T_s$ (iv) $M_p$ (v) tp for a system having transfer function $\frac{C(S)}{R(S)} = \frac{10}{s^2+2s+10}$ 5M CO2

Standard form of Second order transfer function $\left.\right\}$ $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

On comparing equation (1) & (2) we get,

$\omega_n^2 = 10$

$\therefore \ \omega_n = \sqrt{10} = 3.162 \text{ rad/sec}$

$2\zeta\omega_n = 2$

$\therefore \ \zeta = \frac{2}{2\omega_n} = \frac{1}{3.162} = 0.316$

$$\theta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta} = \tan^{-1}\frac{\sqrt{1-0.316^2}}{0.316} = 1.249 \text{ rad}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 3.162\sqrt{1-0.316^2} = 3 \text{ rad/sec}$$

$$\text{Rise time, } t_r = \frac{\pi-\theta}{\omega_d} = \frac{\pi-1.249}{3} = 0.63 \text{ sec}$$

$$\text{Percentage overshoot, } \%M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100 = e^{\frac{-0.316\pi}{\sqrt{1-0.316^2}}} \times 100$$

$$= 0.3512 \times 100 = 35.12\%$$

$$\text{Peak overshoot} = \frac{35.12}{100} \times 12 \text{ units} = 4.2144 \text{ units}$$

$$\text{Peak time, } \quad t_p = \frac{\pi}{\omega_d} = \frac{\pi}{3} = 1.047 \text{ sec}$$

$$\text{Time constant, } T = \frac{1}{\zeta\omega_n} = \frac{1}{0.316 \times 3.162} = 1 \text{ sec}$$

$\therefore$ For 5% error, Settling time, $t_s = 3T = 3$ sec

For 2% error, Settling time, $t_s = 4T = 4$ sec

### RESULT

| | | |
|---|---|---|
| Rise time, $t_r$ | = | 0.63 sec |
| Percentage overshoot, $\%M_p$ | = | 35.12% |
| Peak overshoot | = | 4.2144 units, (for a input of 12 units) |
| Peak time, $t_p$ | = | 1.047 sec |
| Settling time, $t_s$ | = | 3 sec for 5% error |
| | = | 4 sec for 2% error |

b) Obtain the response of unity feedback system whose open loop transfer function is G(s) = $\frac{4}{s(s+5)}$ and when the input is unit step. 5M CO2

**SOLUTION**

The closed loop system is shown in fig 1.

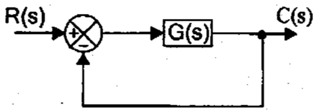



*Fig 1 : Closed loop system.*

The closed loop transfer function, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$

$$\therefore \frac{C(s)}{R(s)} = \frac{\frac{4}{s(s+5)}}{1+\frac{4}{s(s+5)}} = \frac{\frac{4}{s(s+5)}}{\frac{s(s+5)+4}{s(s+5)}} = \frac{4}{s(s+5)+4} = \frac{4}{s^2+5s+4} = \frac{4}{(s+4)(s+1)}$$

The response in s-domain, $C(s) = R(s)\,\frac{4}{(s+1)(s+4)}$

Since the input is unit step, $R(s) = \frac{1}{s}$; $\quad \therefore C(s) = \frac{4}{s(s+1)(s+4)}$

By partial fraction expansion, we can write,

$$C(s) = \frac{4}{s(s+1)(s+4)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}$$

$$A = C(s) \times s\Big|_{s=0} = \frac{4}{(s+1)(s+4)}\Bigg|_{s=0} = \frac{4}{1\times 4} = 1$$

$$B = C(s) \times (s+1)\Big|_{s=-1} = \frac{4}{s(s+4)}\Bigg|_{s=-1} = \frac{4}{-1(-1+4)} = \frac{-4}{3}$$

$$C = C(s) \times (s+4)\Big|_{s=-4} = \frac{4}{s(s+1)}\Bigg|_{s=-4} = \frac{4}{-4(-4+1)} = \frac{1}{3}$$

The time domain response c(t) is obtained by taking inverse Laplace transform of C(s).

Response in time domain, $c(t) = \mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{4}{3}\frac{1}{s+1} + \frac{1}{3}\frac{1}{s+4}\right\}$

$$= 1 - \frac{4}{3}e^{-t} + \frac{1}{3}e^{-4t} = 1 - \frac{1}{3}\left[4e^{-t} - e^{-4t}\right]$$

**(OR)**

5. a) Determine the step, ramp, parabolic error constants of the following feedback control system. $G(S) = \frac{10(s+2)}{s^2(s+1)}$ 5M CO2

**SOLUTION**

**a) To find static error constants**

For a unity feedback system, H(s)=1

Position error constant, $K_p = \underset{s\to 0}{Lt}\, G(s)H(s) = \underset{s\to 0}{Lt}\, G(s) = \underset{s\to 0}{Lt}\, \frac{10(s+2)}{s^2(s+1)} = \infty$

Velocity error constant, $K_v = \underset{s\to 0}{Lt}\, s\,G(s)H(s) = \underset{s\to 0}{Lt}\, s\,G(s) = \underset{s\to 0}{Lt}\, s\frac{10(s+2)}{s^2(s+1)} = \infty$

Acceleration error constant, $K_a = \underset{s\to 0}{Lt}\, s^2G(s)H(s) = \underset{s\to 0}{Lt}\, s^2G(s)$

$$= \underset{s\to 0}{Lt}\, s^2\frac{10(s+2)}{s^2(s+1)} = \frac{10\times 2}{1} = 20$$

b) For the unity feedback system with $G(S) = \frac{K}{s(s+1)\ (s+2)}$, determine the range of K for the system to be stable. 5M CO2

**SOLUTION**

The closed loop transfer function, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{K}{s(s+1)(s+2)}}{1+\frac{K}{s(s+1)(s+2)}} = \frac{K}{s(s+1)(s+2)+K}$

The characteristic equation is, $s(s+1)(s+2)+K=0$

$\therefore s(s^2+3s+2)+K=0 \Rightarrow s^3+3s^2+2s+K=0$

The routh array is constructed as shown below.

The highest power of s in the characteristic polynomial is odd number. Hence form the first row using the coefficients of odd powers of s and form the second row using the coefficients of even powers of s.

| | | | |
|---|---|---|---|
| $s^3$ | : | 1 | 2 |
| $s^2$ | : | 3 | K |
| $s^1$ | : | $\frac{6-K}{3}$ | |
| $s^0$ | : | K | |

Column-1

$s^1 : \frac{3\times2-K\times1}{3}$

$s^1 : \frac{6-K}{3}$

$s^0 : \frac{\frac{6-K}{3}\times K-0\times3}{(6-K)/3}$

$s^0 : K$

For the system to be stable there should not be any sign change in the elements of first column. Hence choose the value of K so that the first column elements are positive.

From $s^0$ row, for the system to be stable, $K>0$

From $s^1$ row, for the system to be stable, $\frac{6-K}{3}>0$

For $\frac{6-K}{3}>0$, the value of K should be less than 6.

$\therefore$ The range of K for the system to be stable is $0<K<6$.

## UNIT III

6. a) Define all the frequency domain specifications 5M CO3

***The frequency domain specifications are,***

1. Resonant peak, $M_r$
2. Resonant Frequency, $\omega_r$
3. Bandwidth, $\omega_b$
4. Cut-off rate
5. Gain margin, $K_g$
6. Phase margin, $\gamma$

**Resonant Peak ($M_r$)**

The maximum value of the magnitude of closed loop transfer function is called the resonant peak,

## Resonant Frequency ($\omega_r$)

The frequency at which the resonant peak occurs is called resonant frequency, $\omega_r$. This is related to the frequency of oscillation in the step response and thus it is indicative of the speed of transient response.

## Bandwidth ($\omega_b$)

The Bandwidth is the range of frequencies for which normalized gain of the system is more than –3 db. The frequency at which the gain is –3 db is called cut-off frequency. Bandwidth is usually defined for closed loop system and it transmits the signals whose frequencies are less than the cut-off frequency. The Bandwidth is a measure of the ability of a feedback system to reproduce the input signal, noise rejection characteristics and rise time. A large bandwidth corresponds to a small rise time or fast response.

## Cut-off Rate

The slope of the log-magnitude curve near the cut off frequency is called cut-off rate. The cut -off rate indicates the ability of the system to distinguish the signal from noise.

### Gain Margin, $K_g$

The gain margin, $K_g$ is defined as the value of gain, to be added to system, in order to bring the system to the verge of instability.

The gain margin, $K_g$ is given by the reciprocal of the magnitude of open loop transfer function at phase cross over frequency. The frequency at which the phase of open loop transfer function is 180° is called the phase cross-over frequency, $\omega_{pc}$.

$$\text{Gain Margin, } K_g = \frac{1}{|G(j\omega_{pc})|} \qquad .....(3.4)$$

The gain margin in db can be expressed as,

$$K_g \text{ in db} = 20 \log K_g = 20 \log \frac{1}{|G(j\omega_{pc})|} \qquad .....(3.5)$$

> *Note :* $|G(j\omega_{pc})|$ *is the magnitude of* $G(j\omega)$ at $\omega = \omega_{pc}$

### Phase Margin ($\gamma$)

The phase margin $\gamma$, is defined as the additional phase lag to be added at the gain cross over frequency in order to bring the system to the verge of instability. The gain cross over frequency $\omega_{gc}$ is the frequency at which the magnitude of the open loop transfer function is unity (or it is the frequency at which the db magnitude is zero).

The phase margin $\gamma$, is obtained by adding 180° to the phase angle $\phi$ of the open loop transfer function at the gain cross over frequency

$$\text{Phase margin, } \gamma = 180° + \phi_{gc}, \qquad .....(3.6)$$

$$\text{where, } \phi_{gc} = \angle G(j\omega_{gc})$$

> *Note :* $\angle G(j\omega_{gc})$ *is the phase angle of* $G(j\omega)$ at $\omega = \omega_{gc}$

b) A unity feedback control system has $G(S) = \frac{10}{s(1+0.4s)(1+0.1s)}$ Draw the Bode plot and determine $w_{gc}$ and $w_{pc}$ from the plot 6M CO3

The sinusoidal transfer function of G(jω) is obtained by replacing s by jω in the given transfer function.

$$\therefore G(j\omega) = \frac{10}{j\omega\,(1+j0.4\omega)(1+j0.1\omega)}$$

The corner frequencies are,

$$\omega_{c1} = \frac{1}{0.4} = 2.5 \text{ rad/sec} \text{ and } \omega_{c2} = \frac{1}{0.1} = 10 \text{ rad/sec}$$

TABLE-1

| Term | Corner frequency rad/sec | Slope db/dec |
|---|---|---|
| $\frac{10}{j\omega}$ | – | – 20 |
| $\frac{1}{1+j0.4\omega}$ | $\omega_{c1} = \frac{1}{0.4} = 2.5$ | – 20 |
| $\frac{1}{1+j0.1\omega}$ | $\omega_{c2} = \frac{1}{0.1} = 10$ | – 20 |

Magnitude in dB=20log10-20logw-20log$\sqrt{1+(0.4w)2}$-20log$\sqrt{1+(0.1w)2}$

W= 0.1, 1.5, 2.5, 3.5, 4.5, 5, 10,15, 20, 25

**PHASE PLOT**

The phase angle of G(jω) as a function of ω is given by,

$$\phi = -90° - \tan^{-1} 0.4\omega - \tan^{-1} 0.1\omega$$

**RESULT**

Gain cross-over frequency = 5 rad/sec.

Phase cross-over frequency = 5 rad/sec.

**(OR)**

7. a) A unity feedback control system has $G(S) = \frac{1}{s(1+s)(1+2s)}$ Draw the polar plot and determine the gain margin(GM) and phase margin(PM). 5M CO3

Given that, G(s) = 1/s(1+s) (1 +2s)

Put s = jω.

$$\therefore G(j\omega) = \frac{1}{j\omega\,(1+j\omega)(1+j2\omega)}$$

The corner frequencies are $\omega_{c1}$ = 1/2 = 0.5 rad/sec and $\omega_{c2}$ = 1 rad/sec.

$$G(j\omega) = \frac{1}{(j\omega)(1+j\omega)(1+j2\omega)} = \frac{1}{\omega\angle 90^\circ \sqrt{1+\omega^2}\,\angle\tan^{-1}\omega \sqrt{1+4\omega^2}\,\angle\tan^{-1}2\omega}$$

$$= \frac{1}{\omega\sqrt{(1+\omega^2)(1+4\omega^2)}} \angle -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

$$\therefore |G(j\omega)| = \frac{1}{\omega\sqrt{(1+\omega^2)(1+4\omega^2)}} = \frac{1}{\omega\sqrt{1+4\omega^2+\omega^2+4\omega^4}} = \frac{1}{\omega\sqrt{1+5\omega^2+4\omega^4}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

**TABLE-1 : Magnitude and phase of G(jω) at various frequencies**

| ω rad/sec | 0.35 | 0.4 | 0.45 | 0.5 | 0.6 | 0.7 | 1.0 |
|---|---|---|---|---|---|---|---|
| $\|G(j\omega)\|$ | 2.2 | 1.8 | 1.5 | 1.2 | 0.9 | 0.7 | 0.3 |
| $\angle G(j\omega)$ deg | -144 | -150 | -156 | -162 | -171 | -179.5 ≈ -180 | -198 |

**TABLE-2 : Real and imaginary part of G(jω) at various frequencies**

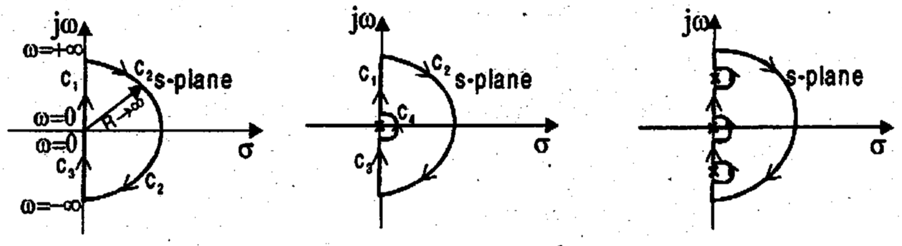
| ω rad/sec | 0.35 | 0.4 | 0.45 | 0.5 | 0.6 | 0.7 | 1.0 |
|---|---|---|---|---|---|---|---|
| $G_R(j\omega)$ | -1.78 | -1.56 | -1.37 | -1.14 | -0.89 | -0.7 | -0.29 |
| $G_I(j\omega)$ | -1.29 | -0.9 | -0.61 | -0.37 | -0.14 | 0 | 0.09 |

## RESULT

Gain margin, $K_g = 1.4286$

Phase margin, $\gamma = +12^\circ$

b) Write the procedure for investigating the stability using the Nyquist Criterion 5M CO3




**a.** *Nyquist Contour when there is no pole on imaginary axis* **b.** *Nyquist Contour when there are poles at origin* **c.** *Nyquist Contour when there are poles on imaginary axis and at origin*

2. The Nyquist contour should be mapped in the G(s)H(s)-plane using the function G(s)H(s) to determine the encirclement $-1+j0$ point in the G(s)H(s)-plane. The Nyquist contour of fig 4.5b can be divided into four sections $C_1$, $C_2$, $C_3$ and $C_4$. The mapping of the four sections in the G(s)H(s)-plane can be carried sectionwise and then combined together to get entire G(s)H(s)-contour.

3. In section $C_1$ the value of $\omega$ varies from 0 to $+\infty$. The mapping of section $C_1$ is obtained by letting $s = j\omega$ in G(s)H(s) and varying $\omega$ from 0 to $+\infty$,

$$\text{i.e. } G(s)H(s)\Big|_{\substack{s=j\omega \\ \omega=0 \text{ to } \infty}} = G(j\omega)\,H(j\omega)\Big|_{\omega=0 \text{ to } \infty}$$

The locus of $G(j\omega)H(j\omega)$ as $\omega$ is varied from 0 to $+\infty$ will be the G(s)H(s)-contour in G(s)H(s)-plane corresponding to section $C_1$ in s-plane. This locus is the polar plot of $G(j\omega)H(j\omega)$. There are three ways of mapping this section of G(s)H(s)-contour, they are,

(i) Calculate the values of $G(j\omega)H(j\omega)$ for various values of $\omega$ and sketch the actual locus of $G(j\omega)H(j\omega)$.

(or)

(ii) Separate the real part and imaginary part of $G(j\omega)H(j\omega)$. Equate the imaginary part to zero, to find the frequency at which the $G(j\omega)H(j\omega)$ locus crosses real axis (to find phase crossover frequency). Substitute this frequency on real part and find the crossing point of the locus on real axis. Sketch the approximate locus of $G(j\omega)H(j\omega)$ from the knowledge of type number and order of the system (or from the value of $G(j\omega)H(j\omega)$ at $\omega = 0$ and $\omega = \infty$).

(iii) Separate the magnitude and phase of $G(j\omega)H(j\omega)$. Equate the phase of $G(j\omega)H(j\omega)$ to $-180^\circ$ and solve for $\omega$. This value of $\omega$ is the phase crossover frequency and the magnitude at this frequency is the crossing point on real axis. Sketch the approximate root locus as mentioned in method (ii).

4. The section $C_2$ of Nyquist contour has a semicircle of infinite radius. Therefore, every point on section $C_2$ has infinite magnitude but the argument varies from $+\pi/2$ to $-\pi/2$. Hence the mapping of section $C_2$ from s-plane to G(s)H(s) plane can be obtained by letting $s = \underset{R\to\infty}{Lt}\, Re^{j\theta}$ in G(s)H(s) and varying $\theta$ from $+\pi/2$ to $-\pi/2$.

Consider the loop transfer function in time constant form and with y number of poles at origin, as shown below.

## UNIT IV

8. a) Explain various steps for constructing the Root Locus in case of a complex pole. 5M CO4

**EXPLANATION FOR THE VARIOUS STEPS IN THE PROCEDURE FOR CONSTRUCTING ROOT LOCUS**

***Step 1 : Location of poles and zeros***

Draw the real and imaginary axis on an ordinary graph sheet and choose same scales both on real and imaginary axis.

The poles are marked by cross "X" and zeros are marked by small circle "o". The number of root locus branches is equal to number of poles of open loop transfer function. The origin of a root locus is at a pole and the end is at a zero.

Let, n = number of poles
m = number of finite zeros

Now, m root locus branches ends at finite zeros. The remaining n−m root locus branches will end at zeros at infinity.

***Step 2 : Root locus on real axis***

In order to determine the part of root locus on real axis, take a test point on real axis. If the total number of poles and zeros on the real axis to the right of this test point is odd number, then the test point lies on the root locus. If it is even then the test point does not lie on the root locus.

***Step 4 : Breakaway and Breakin points***

The breakaway or breakin points either lie on real axis or exist as complex conjugate pairs. If there is a root locus on real axis between 2 poles then there exist a breakaway point. If there is a root locus on real axis between 2 zeros then there exist a breakin point. If there is a root locus on real axis between pole and zero then there may be or may not be breakaway or breakin point.

Let the characteristic equation be in the form,

$$B(s) + K\,A(s) = 0$$

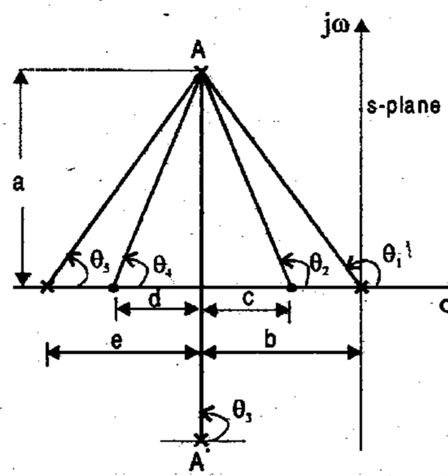
$$\therefore K = \frac{-B(s)}{A(s)}$$

The breakaway and breakin point is given by roots of the equation $dK/ds = 0$. The roots of $dK/ds = 0$ are actual breakaway or breakin point provided for this value of root, the gain K should be positive and real.

***Step 5 : Angle of Departure and angle of arrival***

$$\left.\begin{matrix}\text{Angle of Departure}\\ \text{(from a complex pole A)}\end{matrix}\right\} = 180^\circ - \begin{pmatrix}\text{Sum of angles of vector to the}\\ \text{complex pole A from other poles}\end{pmatrix} + \begin{pmatrix}\text{Sum of angles of vectors to the}\\ \text{complex pole A from zeros}\end{pmatrix}$$

| ***Note : The angles can be calculated as shown in fig 4.9 or they can be measured using protractor.*** |
|---|




$$\theta_1 = 180^\circ - \tan^{-1}\frac{a}{b}$$

$$\theta_2 = 180^\circ - \tan^{-1}\frac{a}{c}$$

$$\theta_3 = 90^\circ$$

$$\theta_4 = \tan^{-1}\frac{a}{d}$$

$$\theta_5 = \tan^{-1}\frac{a}{e}$$

***Fig 4.9 :*** Calculation of angle of departure

**Example:**

Consider the two complex conjugate poles A and A* shown in fig 4.9.(If poles are complex then they exist only as conjugate pairs)

$$\left.\begin{matrix}\text{Angle of departure}\\ \text{at pole A}\end{matrix}\right\} = 180^\circ - (\theta_1 + \theta_3 + \theta_5) + (\theta_2 + \theta_4)$$

$$\left.\begin{matrix}\text{Angle of departure}\\ \text{at pole A *}\end{matrix}\right\} = -\left[\text{Angle of departure at pole A}\right]$$

***Step 6 : Point of intersection of root locus with imaginary axis***

The point where the root loci intersects the imaginary axis can be found by following three methods.

1. By Routh Hurwitz array.
2. By trial and error approach.
3. Letting s = jω in the characteristic equation and separate the real part and imaginary part. Two equations are obtained : *one by equating real part to zero and the other by equating imaginary part to zero.* Solve the two equations for ω and K. The values of ω gives the points where the root locus crosses imaginary axis. The value of K gives the value of gain K at there crossing points. Also this value of K is the limiting value of K for stability of the system.

***Step 7 : Test points and root locus***

Choose a test point. Using a protractor roughly estimate the angles of vectors drawn to this point and adjust the point to satisfy angle criterion. Repeat the procedure for few more test points. Sketch the root locus from the knowledge of typical sketches and the informations obtained in steps 1 through 6.

b) Sketch the root locus of the system whose open loop transfer function is $G(S) = \frac{k}{s(s+2)(s+4)}$. Find the value of K. 5M CO4

**SOLUTION**

***Step 1 :* To locate poles and zeros**

The poles of open loop transfer function are the roots of the equation, s(s+2) (s+4) = 0.

∴ The poles are lying at, s = 0, −2, −4.

Let us denote the poles as $p_1$, $p_2$, and $p_3$.

Here, $p_1 = 0$, $p_2 = -2$, $p_3 = -4$.

The poles are marked by X(cross) as shown in fig 4.23.1.

***Step 2 :* To find the root locus on real axis**

There are three poles on the real axis.

Choose a test point on real axis between s = 0 and s = −2. To the right of this point the total number of real poles and zeros is one, which is an odd number. Hence the real axis between s = 0 and s = −2 will be a part of root locus.

Choose a test point on real axis between s = −2 and s = −4. To the right of this point, the total number of real poles and zeros is two which is an even number. Hence the real axis between s = −2 and s = −4 will not be a part of root locus.

Choose a test point on real axis to the left of s = −4. To the right of this point, the total number of real poles and zeros is three, which is an odd number. Hence the entire negative real axis from s = −4 to −∞ will be a part of root locus.

## *Step 3 :* To find asymptotes and centroid

Since there are three poles the number of root locus branches are three. There is no finite zero. Hence all the three rc locus branches ends at zeros at infinity. The number of asymptotes required are three.

$$\text{Angles of asymptotes} = \frac{\pm 180° (2q+1)}{n-m} \quad ; \quad q = 0, 1, 2, .......n-m.$$

Here, n = 3 and m = 0. $\therefore$ q = 0, 1, 2, 3.

$$\text{When } q = 0, \quad \text{Angles} = \pm\frac{180°}{3} = \pm 60°$$

$$\text{When } q = 1, \quad \text{Angles} = \pm\frac{180° \times 3}{3} = \pm 180°$$

> *Note : It is enough if you calculate the required number of angles. Here it is given by first three values of angles. The remaining values will be repetitions of the previous values.*

$$\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m} = \frac{0-2-4-0}{3} = -2$$

The centroid is marked on real axis and from the centroid the angles of asymptotes are marked using a protractor. The asymptotes are drawn as dotted lines as shown in fig 4.23.1.

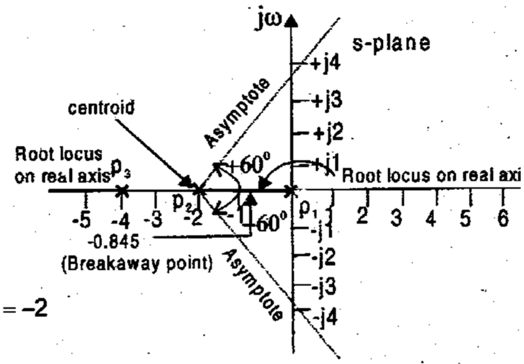



*Fig 4.23.1 : Figure showing the asymptoi root locus on real axis and location of poi centroid ,and breakaway points..*

## *Step 4 :* To find the breakaway and breakin points

$$\left.\begin{array}{l}\text{The closed loop}\\ \text{transfer function}\end{array}\right\} \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{K}{s(s+2)(s+4)}}{1+\frac{K}{s(s+2)(s+4)}} = \frac{K}{s(s+2)(s+4)+K}$$

The characteristic equation is given by,

$$s(s+2)(s+4)+K=0 \quad \Rightarrow \quad s(s^2+6s+8)+K=0 \quad \Rightarrow \quad s^3+6s^2+8s+K=0$$

$$\therefore K = -s^3 - 6s^2 - 8s$$

On differentiating the equation of K with respect to s we get,

$$\frac{dK}{ds} = -(3s^2+12s+8)$$

$$\text{Put} \quad \frac{dK}{ds} = 0$$

$$\therefore -(3s^2+12s+8) = 0 \quad \Rightarrow \quad (3s^2+12s+8) = 0$$

$$s = \frac{-12 \pm \sqrt{12^2 - 4\times 3\times 8}}{2\times 3} = -0.845 \quad \text{or} \quad -3.154$$

Check for K : When s = –0.845, the value of K is given by,

$$K = -[(-0.845)^3 + 6(-0.845)^2 + 8(-0.845)] = 3.08$$

Since K, is positive and real for, s = –0.845, this point is actual breakaway point.

When s = –3.154, the value of K is given by,

$$K = -[(-3.154)^3 + 6(-3.154)^2 + 8(-3.154)] = -3.08$$

Since K, is negative for, s = –3.154, this is not a actual breakaway point.

***Step 5 :*** **To find angle of departure**

Since there are no complex pole or zero, we need not find angle of departure or arrival.

***Step 6 :*** **To find the crossing point of imaginary axis**

The characteristic equation is given by,

$$s^3 + 6s^2 + 8s + K = 0$$

Put $s = j\omega$

$$(j\omega)^3 + 6(j\omega)^2 + 8(j\omega) + K = 0$$

$$-j\omega^3 - 6\omega^2 + j8\omega + K = 0$$

Equating imaginary part to zero

$$-j\omega^3 + j8\omega = 0$$

$$-j\omega^3 = -j8\omega$$

$$\omega^2 = 8 \Rightarrow \omega = \pm\sqrt{8} = \pm 2.8$$

Equating real part to zero

$$-6\omega^2 + K = 0$$

$$K = 6\omega^2 = 6 \times 8 = 48$$

The crossing point of root locus is $\pm j2.8$. The value of K corresponding to this point is K = 48. (This is the limiting value of K for the stability of the system).

The complete root locus sketch is shown in fig 4.23.2. The root locus has three branches. One branch starts at the pole at s = –4 and travel through negative real axis to meet the zero at infinity. The other two root locus branches starts at s = 0 and s = –2 and travel through negative real axis, breakaway from real axis at s = –0.845, then crosses imaginary axis at s= ± j2.8 and travel parallel to asymptotes to meet the zeros at infinity

**(OR)**

9. a) Derive Transfer Function from State Model 5M CO4

**Note that as the system is time invariant, the coefficient of matrices A, B, C and D are constants. While the definition of transfer function is based on the assumption of zero initial conditions i.e. X(0) = 0.**

$$\therefore \quad s\,X(s) = A\,X(s) + B\,U(s)$$

$$\therefore \quad s\,X(s) - A\,X(s) = B\,U(s)$$

**Now s is an operator while A is matrix of order n × n hence to match the orders of two terms on left hand side, multiply 's' by identity matrix I of the order n × n.**

$$\therefore \quad sI\,X(s) - A\,X(s) = B\,U(s)$$

$$\therefore \quad [sI - A]\,X(s) = B\,U(s)$$

**Premultiplying both sides by $[sI - A]^{-1}$,**

$$[sI - A]^{-1}[sI - A]\,X(s) = [sI - A]^{-1}BU(s)$$

**Now** $$[sI - A]^{-1}[sI - A] = 1$$

$$\therefore \quad X(s) = [sI - A]^{-1}BU(s) \quad \ldots(3)$$

**Substituting in the equation (2b),**

$$Y(s) = C\,[sI - A]^{-1}B\,U(s) + D\,U(s)$$

$$\therefore \quad Y(s) = \{C\,[sI - A]^{-1}B + D\}\,U(s)$$

**Hence the transfer function is,**

Consider a standard state model derived for linear time invariant system as,

$$\dot{X}(t) = A\,X(t) + B\,U(t) \quad \dots (1a)$$

and
$$Y(t) = C\,X(t) + D\,U(t) \quad \dots (1b)$$

Taking Laplace transform of both sides,

$$[s\,X(s) - X(0)] = A\,X(s) + B\,U(s) \quad \dots (2a)$$

and
$$Y(s) = C\,X(s) + D\,U(s) \quad \dots (2b)$$

$$\boxed{T(s) = \frac{Y(s)}{U(s)} = C\,[sI - A]^{-1}\,B + D} \quad \dots (4a)$$

b) The state and output equations of a SISO system are 5M CO4

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = [1 \quad -1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Determine the controllability and observability.

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = [1 \quad -1]$$

**Solution : a)** For controllability, $Q_c = [B : AB]$ as $n = 2$

$$AB = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$\therefore \quad Q_c = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1 \text{ hence rank of } Q_c = 2 = n$$

Hence system is **completely controllable.**

For observability, $Q_o = [C^T : A^T C^T]$ as $n = 2$

$$C^T = [1 - 1]^T = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$$

$$\therefore \quad Q_o = \begin{bmatrix} 1 & -3 \\ -1 & 3 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & -3 \\ -1 & 3 \end{bmatrix} = 0$$

$\therefore$ Hence the rank of $Q_o = 1 < n$

Thus system is **not completely observable.**