III/IV B.Tech(Regular) DEGREE EXAMINATION

Scheme of Evalueation

FEB, 2021 **Electronics & Communication Engineering**

Fifth Semester **Digital Signal Processing**

Time: Three Hours **Maximum:** 50 Marks

Answer Question No.I compulsorily. (10X1 = 10 Marks)

Answer any four question from Part A. (4X10=40 Marks)

1.	Answer all questions		(10X1=10 Marks)
	a)	The signals that are discrete in time and quantized in amplitude are called______________ signals. Ans:-Digital	1M
	b)	A system is said to be____________ *if Added signals pass through it* without interacting. Ans:-Additive	1M
	c)	For a time-invariant system, its___________ do not change with time. Ans:- Characteristics	1M
	d)	What is one-sided Z-transform? Ans:- one-sided z-transform does not contain information about the signal x(n) for negative. values of time (i.e., for n<0)	1M
	e)	Why is FFT called so? Ans:- FFT is called so because using this algorithm DFT is computed in a faster way. This is achieved by utilizing the symmetry and periodicity properties of W_N^K.	1M
	f)	What is radix-2 FFT? ***Ans.*** The radix-2 FFT is an efficient algorithm for computing *N*-point DFT of an *N*-point sequence. In radix-2 FFT, the *N*-point sequence is decimated into 2-point sequences and the 2-point DFT for each decimated sequence is computed. From the results of 2-point DFTs, the 4-point DFTs are computed. From the results of 4- point DFTs, the 8-point DFTs are computed and so on until we get *N*-point DFT.	1M
	g)	What is the disadvantage of impulse invariant method? Ans:-The disadvantage of the impulse invariance method is the unavoidable frequency-domain aliasing.	1M
	h)	What is the relation between analog and digital frequencies in impulse invariant transformation? Ans:-	1M
	i)	What is the necessary condition for Linear phase realization of FIR systems? Ans:-h(n)=h(N-n-1)	1M
	j)	Name basic design elements of discrete time system. Ans:-Adder,Multiplier,delay element and Advance aelemeni..	1M
		Part A	
2.	a)	How are discrete-time signals classified? Differentiate between them. discrete-time signals are classified as follows: 1. Deterministic and random signals 2. Periodic and non-periodic signals 3. Energy and power signals 4. Causal and non-causal signals 5. Even and odd signals <u>Deterministic and random signals:</u> A signal exhibiting no uncertainty of its magnitude and phase at any given instant of time is called deterministic signal. A deterministic signal can be completely represented by mathematical equation at any time and its nature and amplitude at any time can be predicted. *Examples:* Sinusoidal sequence $x(n) = \cos wn$, Exponential sequence $x(n) = e^{jwn}$, ramp sequence $x(n) = \alpha n$. A signal characterized by uncertainty about its occurrence is called a non-deterministic	5M

or random signal. A random signal cannot be represented by any mathematical equation. example of a non-deterministic signal is thermal noise.

<u>Periodic and non-periodic signals</u>

A signal which has a definite pattern and repeats itself at regular intervals of time is called a periodic signal, and a signal which does not repeat at regular intervals of time is called a non-periodic or aperiodic signal.

A discrete-time signal $x(n)$ is said to be periodic if it satisfies the condition $x(n) = x(n + N)$ for all integers n.

The smallest value of N which satisfies the above condition is known as fundamental period.

If the above condition is not satisfied even for one value of n, then the discrete-time signal is aperiodic. Sometimes aperiodic signals are said to have a period equal to infinity.

<u>Energy and power signals:</u>

A signal is said to be an energy signal if and only if its total energy E *over the interval* $(-\alpha,\alpha)$ is finite (i.e., $0 < E < \alpha$ *). For an energy signal, average power* $P = 0$.

Non-periodic signals which are defined over a finite time (also called time limited signals) are the examples of energy signals.

Since the energy of a periodic signal is always either zero or infinite, any periodic signal cannot be an energy signal.

A signal is said to be a power signal, if its average power P *is finite (i.e.,* $0 < P < \alpha$ *). For* a power signal, total energy $E = \alpha$. *Periodic signals are the examples of power signals.*

<u>Causal and Non Causal Signals</u>:

A discrete-time signal x(n) is said to be causal if x(n) = 0 for n < 0, otherwise the signal is non-causal. A discrete-time signal x(n) is said to be anti-causal if x(n) = 0 for n > 0.

A causal signal does not exist for negative time and an anti-causal signal does not exist for positive time.

A signal which exists in positive as well as negative time is called a non-casual signal.

<u>EVEN and odd signals</u>

Any signal x(n) can be expressed as sum of even and odd components. That is $x(n) = x_e(n) + x_o(n)$.

A general signal neither even nor odd, but have both even and odd components

A discrete-time signal x(n) is said to be an even (symmetric) signal if it satisfies the condition: x(n) = x(–n) for all n.

Even signals are symmetrical about the vertical axis or time origin. Hence they are also called symmetric signals:

an odd (anti-symmetric) signal if it satisfies the condition:

x(–n) = –x(n) for all n

Odd signals are anti-symmetrical about the vertical axis. Hence they are called antisymmetric signals.

Sinusoidal sequence is an example of an odd signal.

For an odd signal x (0) = 0.

	b)	Find the even and odd components of the signal $x(n) = \{-3, 1, \underset{\uparrow}{2}, -4, 2\}$	5M

		Given $x(n)=\{-3,1,\underset{\uparrow}{2},-4,2\}$ $\therefore$ $x(-n)=\{2,-4,\underset{\uparrow}{2},1,-3\}$ $x_e(n)=\frac{1}{2}[x(n)+x(-n)]$ $=\frac{1}{2}[-3+2,1-4,2+2,-4+1,2-3]$ $=\{-0.5,-1.5,\underset{\uparrow}{2},-1.5,-0.5\}$ $x_o(n)=\frac{1}{2}[x(n)-x(-n)]$ $=\frac{1}{2}[-3-2,1+4,2-2,-4-1,2+3]$ $=\{-2.5,2.5,\underset{\uparrow}{0},-2.5,2.5\}$	
3.	a)	Obtain the relation between DFT and Z Transform. There is a close relationship between Z transform and Fourier transform. If we replace the complex variable z by $e^{-j\omega}$, then z transform is reduced to Fourier transform. Z transform of sequence x(n) is given by $X(z)=\sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$ **(Definition of z-Transform)** Fourier transform of sequence x(n) is given by $X(\omega)=\sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$ **(Definition of Fourier Transform)** Complex variable z is expressed in polar form as $Z=re^{j\omega}$ where $r=\|z\|$ and ω is $\angle z$. Thus we can be written as $X(z)=\sum_{n=-\infty}^{\infty} [x(n)\, r^{-n}]\, e^{-j\omega n}$	5M

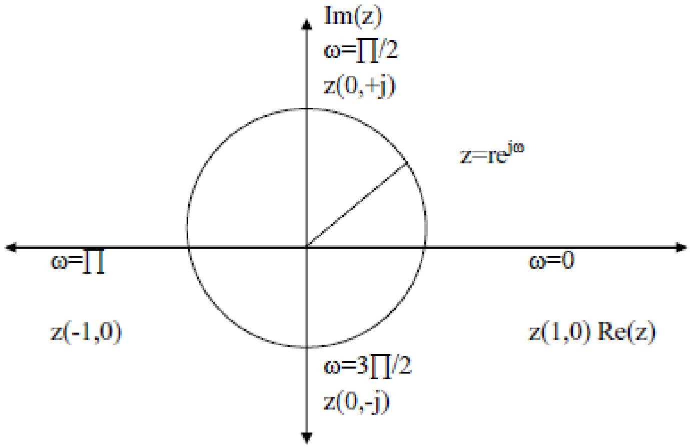
$$X(z)\big|_{z=e^{jw}} = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$X(z)\big|_{z=e^{jw}} = x(\omega) \qquad \text{at } |z| = \text{unit circle.}$$

Thus, X(z) can be interpreted as Fourier Transform of signal sequence ($x(n)\, r^{-n}$). Here r^{-n} grows with n if r<1 and decays with n if r>1. X(z) converges for |r|= 1. hence Fourier transform may be viewed as Z transform of the sequence evaluated on unit circle. Thus The relationship between DFT and Z transform is given by

$$X(z)\big|_{z=e^{j2\prod kn}} = x(k)$$

The frequency ω=0 is along the positive Re(z) axis and the frequency ∏/2 is along the positive Im(z) axis. Frequency ∏ is along the negative Re(z) axis and 3∏/2 is along the negative Im(z) axis.



Frequency scale on unit circle X(z)= X(ω) on unit circle

b) Find Z Transform including the region of convergence of **5M**

$$x(n) = \begin{cases} a^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

By the definition

$$Z[x(n)] = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = X(z)$$

$$X(z) = Z[a^n\, u(n)] = \sum_{n=-\infty}^{\infty} a^n\, u(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} (az^{-1})^n \qquad |az^{-1}| < 1$$

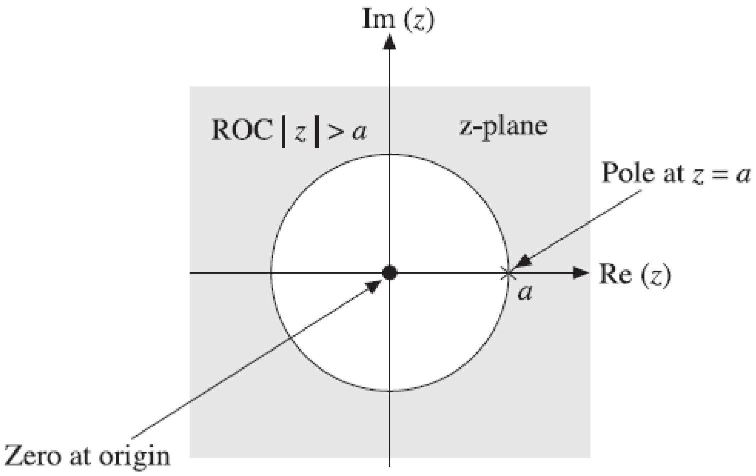
With geometric progression formula

$$X(z) = \frac{1}{1 - az^{-1}} = \frac{z}{z-a}$$

Converges for $|az^{-1}| < 1$

Equivalent to $|z| > |a|$, ROC

Values of z for which X(z)=0 are called zeros of X(z).
Values of z for which X(z)= are called poles of X(z).
Poles are indicated with X and zeros with o.



4.	a)	Find 4 point DFT of sequence x(n)={1,-1,2,-2} directly. Sol: Given sequence is $x(n) = \{1, -1, 2, -2\}$. Here the DFT $X(k)$ to be found is $N = 4$-point and length of the sequence $L = 4$. So no padding of zeros is required. We know that the DFT $\{x(n)\}$ is given by	5M

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{nk} = \sum_{n=0}^{N-1} x(n)\, e^{-j(2\pi/N)nk} = \sum_{n=0}^{3} x(n)\, e^{-j(\pi/2)nk}, \quad k = 0, 1, 2, 3$$

$$X(0) = \sum_{n=0}^{3} x(n)\, e^{0} = x(0) + x(1) + x(2) + x(3) = 1 - 1 + 2 - 2 = 0$$

$$X(1) = \sum_{n=0}^{3} x(n) e^{-j(\pi/2)n} = x(0) + x(1)\, e^{-(j\pi/2)} + x(2)\, e^{-j\pi} + x(3)\, e^{-j(3\pi/2)}$$

$$= 1 + (-1)(0 - j) + 2(-1 - j0) - 2(0 + j)$$

$$= -1 - j$$

$$X(2) = \sum_{n=0}^{3} x(n) e^{-j\pi n} = x(0) + x(1) e^{-j\pi} + x(2) e^{-j2\pi} + x(3) e^{-j3\pi}$$

$$= 1 - 1(-1 - j0) + 2(1 - j0) - 2(-1 - j0) = 6$$

$$X(3) = \sum_{n=0}^{3} x(n) e^{-j(3\pi/2)n} = x(0) + x(1) e^{-j(3\pi/2)} + x(2) e^{-j3\pi} + x(3) e^{-j(9\pi/2)}$$

$$= 1 - 1(0 + j) + 2(-1 - j0) - 2(0 - j) = -1 + j$$

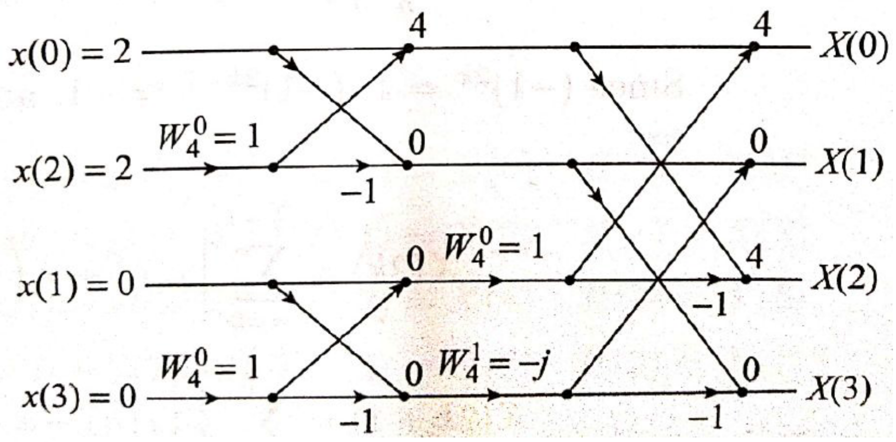
$$X(k) = \{0, -1 - j, 6, -1 + j\}$$

	b)	Find 4 point DFT of sequence x(n)={2,0,2,1} using DIT-FFT.	5M

Given that $N = 4$. We know that $W_N^k = e^{-j\frac{2\pi}{N}k}$. Therefore,

$$W_4^0 = e^{-j\frac{2\pi}{4}0} = 1,$$
$$W_4^1 = e^{-j\frac{2\pi}{4}} = e^{-j\frac{\pi}{2}} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} = -j$$

The butterfly diagram of a 4-point DIT-FFT algorithm is shown in Fig.



$X(k) = \{\underset{\uparrow}{4}, 0, 4, 0\}$

5.	a)	Derive 8 point DIT-FFT radix-2 algorithm and draw signal flow graph.	10M

<u>1. DECIMATION IN TIME (DITFFT)</u>

There are three properties of twiddle factor W_N

1) $W_N^{k+N} = W_N^K$ (Periodicity Property)
2) $W_N^{k+N/2} = -W_N^K$ (Symmetry Property)
3) $W_N^2 = W_{N/2}$.

N point sequence x(n) be splitted into two N/2 point data sequences f1(n) and f2(n). f1(n) contains even numbered samples of x(n) and f2(n) contains odd numbered samples of x(n). This splitted operation is called decimation. Since it is done on time domain sequence it is called "**Decimation in Time**". Thus

f1(m)=x(2m) where n=0,1,…………N/2-1
f2(m)=x(2m+1) where n=0,1,…………N/2-1

N point DFT is given as

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn} \qquad (1)$$

Since the sequence x(n) is splitted into even numbered and odd numbered samples, thus

$$X(k) = \sum_{m=0}^{N/2-1} x(2m)\, W_N^{2mk} + \sum_{m=0}^{N/2-1} x(2m+1)\, W_N^{k(2m+1)} \qquad (2)$$

$$X(k) = F1(k) + W_N^{k}\, F2(k) \qquad (3)$$

$$X(k+N/2) = F1(k) - W_N^{k}\, F2(k) \quad \text{(Symmetry property)} \qquad (4)$$

Fig 1 shows that 8-point DFT can be computed directly and hence no reduction in computation.

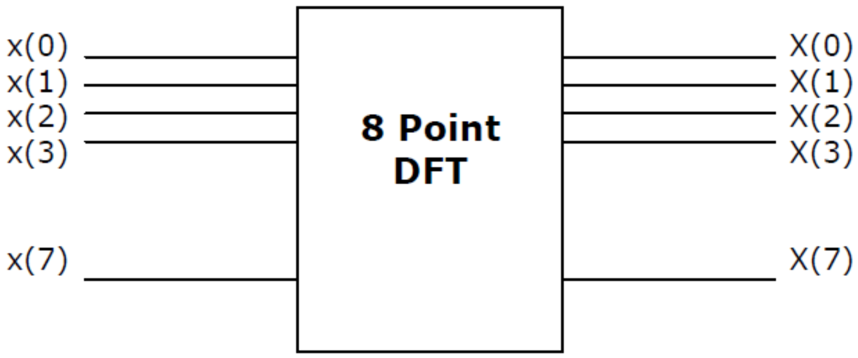


Fig . DIRECT COMPUTATION FOR N=8

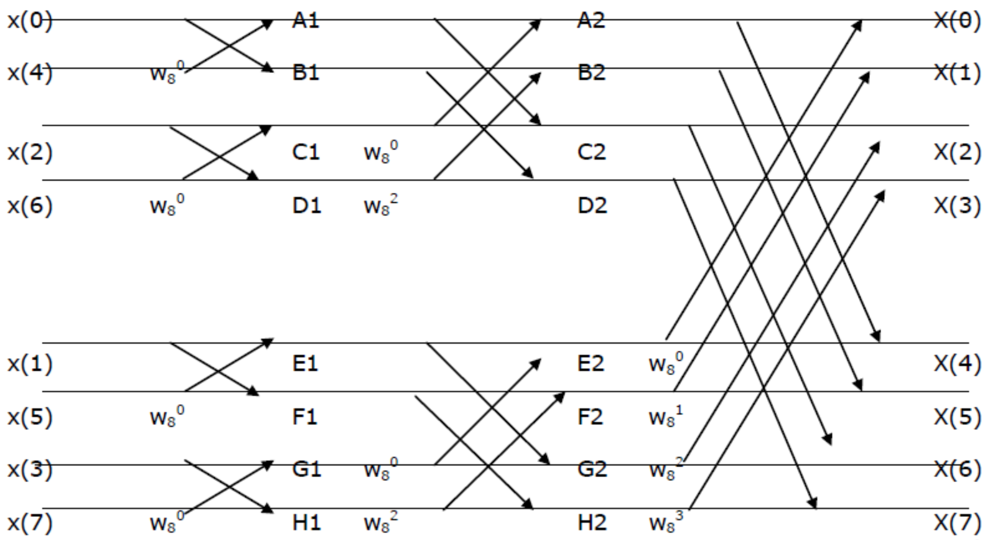


Fig . SIGNAL FLOW GRAPH FOR RADIX- DIT FFT N=8

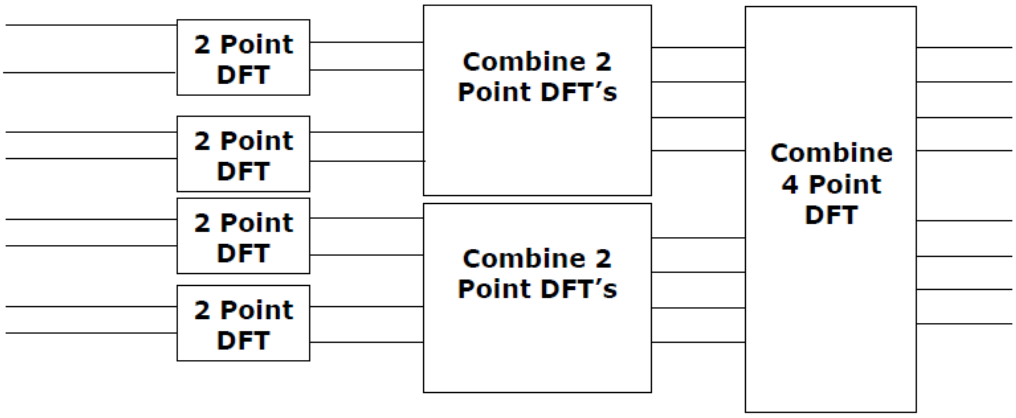


Fig . BLOCK DIAGRAM FOR RADIX- DIT FFT N=8

6. a) Obtain $H(z)$ from $Ha(s)$ when $T = 1$ s and $H_a(s) = \frac{3s}{s^2+0.5s+2}$ *using bilenear transformation.* 5M

Solution: Given $H_a(s) = \dfrac{3s}{s^2 + 0.5s + 2}$ and $T = 1$ s.

To get $H(z)$ using the bilinear transformation, put $s = \dfrac{2}{T}\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right)$ in $H_a(s)$.

$$\therefore \quad H(z) = H_a(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} = \frac{3s}{s^2 + 0.5s + 2}\Bigg|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{3 \times 2\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right)}{\left[2\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + 0.5\left[2\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right)\right] + 2}$$

$$= \frac{6\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right)}{\dfrac{4(1-z^{-1})^2 + (1-z^{-1})(1+z^{-1}) + 2(1+z^{-1})^2}{(1+z^{-1})^2}}$$

$$= \frac{6(1+z^{-1})}{4(1-2z^{-1}+z^{-2}) + (1-z^{-2}) + 2(1+2z^{-1}+z^{-2})}$$

$$= \frac{6+6z^{-1}}{7-4z^{-1}+5z^{-2}}$$

	b)	Compare bilinear and Impulse invariance.	5M

Ans. The impulse invariant and bilinear transformations are compared as follows:

Impulse invariant transformation	*Bilinear transformation*
(i) It is many-to-one mapping.	(i) It is one-to-one mapping.
(ii) The relation between analog and digital frequency is linear.	(ii) The relation between analog and digital frequency is nonlinear.
(iii) To prevent the problem of aliasing, the analog filters should be band limited.	(iii) There is no problem of aliasing and so the analog filter need not be band limited.
(iv) The magnitude and phase responses of analog filter can be preserved by choosing low sampling time or high sampling frequency.	(iv) Due to the effect of warping, the phase response of analog filter cannot be preserved. But the magnitude response can be preserved by prewarping.

7.		Design a Type-1 Chebyshev filter to meet following specifications: αp=3dB;αs=16dB; fp=1KHz and fs= 2KHz.	10M

From the given data we can find

$$\Omega_p = 2\pi \times 1000\,\text{Hz} = 2000\,\pi\ \text{rad/sec}$$
$$\Omega_s = 2\pi \times 2000\,\text{Hz} = 4000\,\pi\ \text{rad/sec}$$

and $\alpha_p = 3\,\text{dB}; \alpha_s = 16\,\text{dB}$.

Step 1:

$$N \geq \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\dfrac{\Omega_s}{\Omega_p}} = \frac{\cosh^{-1}\sqrt{\dfrac{10^{1.6}-1}{10^{0.3}-1}}}{\cosh^{-1}\dfrac{4000\pi}{2000\pi}}$$
$$= 1.91$$

Step 2: Rounding N to next higher value we get $N = 2$.

For N even, the oscillatory curve starts from $\dfrac{1}{\sqrt{1+\varepsilon^2}}$.

Step 3: The values of minor axis and major axis can be found as below

$$\varepsilon = (10^{0.1\alpha_p}-1)^{0.5} = (10^{0.3}-1)^{0.5} = 1$$
$$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 2.414$$
$$a = \Omega_p\frac{[\mu^{1/N} - \mu^{-1/N}]}{2} = 2000\pi\frac{[(2.414)^{1/2} - (2.414)^{-1/2}]}{2} = 910\pi$$
$$b = \Omega_p\frac{[\mu^{1/N} + \mu^{-1/N}]}{2} = 2000\pi\frac{[(2.414)^{1/2} + (2.414)^{-1/2}]}{2} = 2197\pi,$$

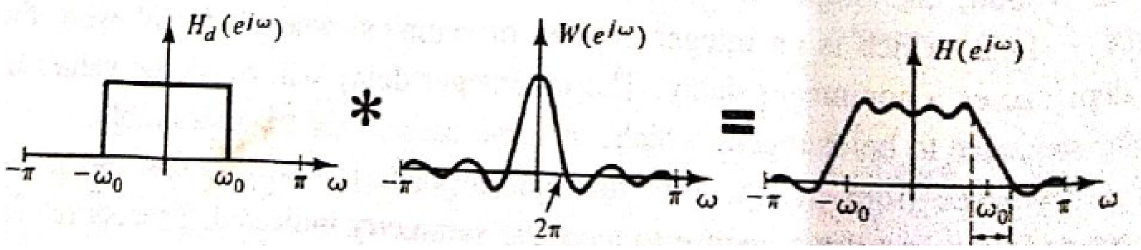
		Step 4: The poles are given by $$s_k = a\cos\phi_k + jb\sin\phi_k, \quad k = 1,2$$ $$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad k = 1,2$$ $$\phi_1 = \frac{\pi}{2} + \frac{\pi}{4} = 135^\circ$$ $$\phi_2 = \frac{\pi}{2} + \frac{3\pi}{4} = 225^\circ$$ $$s_1 = a\cos\phi_1 + j\,b\sin\phi_1 = -643.46\pi + j1554\pi$$ $$s_2 = a\cos\phi_2 + j\,b\sin\phi_2 = -643.46\pi - j1554\pi$$ **Step 5:** The denominator of $H(s) = (s + 643.46\pi)^2 + (1554\pi)^2$ **Step 6:** The numerator of $H(s) = \frac{(643.46\pi)^2 + (1554\pi)^2}{\sqrt{1+\varepsilon^2}} = (1414.38)^2\pi^2$ The transfer function $H(s) = \frac{(1414.38)^2\pi^2}{s^2 + 1287\pi s + (1682)^2\pi^2}$.	
8.		Illustrate Frequency response of FIR filter with rectangular window. **Design of FIR filters Using Windows** The easiest way to obtain an FIR filter is to simply truncate the impulse response of an IIR filter. If $h_d(n)$ represents the impulse response of a desired IIR filter, then an FIR filter with impulse response $h(n)$ can be obtained as follows: $$h(n) = \begin{cases} h_d(n), & N_1 \le n \le N_2 \\ 0, & \text{Otherwise} \end{cases}$$ In general, $h(n)$ can be thought of as being formed by the product of $h_d(n)$ and a "window function," $w(n)$, as follows: $$h(n) = h_d(n) \cdot w(n)$$ For the $h(n)$ of (4.43), $w(n)$ is said to be a rectangular window and is given by $$w(n) = \begin{cases} 1, & N_1 \le n \le N_2 \\ 0, & \text{otherwise} \end{cases}$$	5M

If we let $H(e^{j\omega})$, $H_d(e^{j\omega})$, and $W(e^{j\omega})$ represent the Fourier transforms of $h(n)$, $h_d(n)$, and $w(n)$, respectively, the frequency response $H(e^{j\omega})$ of the resulting filter is the convolution of $H_d(e^{j\omega})$ and $W(e^{j\omega})$ given by

$$H(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\theta})W(e^{j(\omega-\theta)})\,d\theta = H_d(e^{j\omega}) * W(e^{j\omega})$$

For example, if $H_d(e^{j\omega})$ represents an ideal low-pass filter with cutoff frequency ω_0 and $w(n)$ is a rectangular window positioned about the origin, the $H(e^{j\omega})$ is as shown in Fig. 4.13.

Therefore, it is seen that the convolution produces a smeared version of the ideal low-pass frequency response $H_d(e^{j\omega})$. In general, the wider the main



8	b)	Explain the steps in the design of FIR filters by frequency sampling technique	5M

Design Steps of FIR Filter Using Frequency Sampling Technique

Step 1: Calculation of desired frequency response:-

Length of filter (M) = Given

Cutoff frequency (ω_c) = Given (rad/sample).

$$H_d(\omega) = \begin{cases} e^{-j\omega\left(\frac{M-1}{2}\right)} & \text{for } 0 \le \omega \le \omega_c \\ 0 & \text{for } \omega_c \le \omega \le \pi \end{cases}$$

Step 2: Calculation of H(k)

- $\omega = \frac{2\pi k}{M}$; $k = 0, 1, 2 \ldots M-1$
- $H(k) = H_d(\omega)\Big|_{\omega = \frac{2\pi k}{M}}$
- Modify Range of k

Step 3: Calculation of h(n)

$$\boxed{h(n) = \frac{1}{M}\left\{H(0) + 2\sum_{k=1}^{P} \mathrm{Re}\left[H(k)\, e^{j2\pi \frac{nk}{M}}\right]\right\}}$$

Here $P = \begin{cases} \frac{M-1}{2} & \text{if } M \text{ is odd} \\ \frac{M}{2} - 1 & \text{if } M \text{ is even.} \end{cases}$

Merits of frequency sampling technique

(i) Unlike the window method, this technique can be used for any given magnitude response.
(ii) This method is useful for the design of non-prototype filters where the desired magnitude response can take any irregular shape.

There are some disadvantages with this method i.e the frequency response obtained by interpolation is equal to the desired frequency response only at the sampled points. At the other points, there will be a finite error present.

9. a) Draw the direct form I, and direct form II, structures for the system given by: **5M**

$$H(z) = \frac{1 + 4z^{-1} + 3z^{-2}}{1 + \left(\frac{3}{12}\right) z^{-1} + \left(\frac{9}{24}\right) z^{-2} + \left(\frac{1}{24}\right) z^{-3}}.$$

Therefore, the transfer function of the system is

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 4z^{-1} + 3z^{-2}}{1 + (13/12)z^{-1} + (9/24)z^{-2} + (1/24)z^{-3}}$$

Let $$\frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \frac{W(z)}{X(z)}$$

where $$\frac{W(z)}{X(z)} = \frac{1}{1 + (13/12)z^{-1} + (9/24)z^{-2} + (1/24)z^{-3}}$$

Sol:

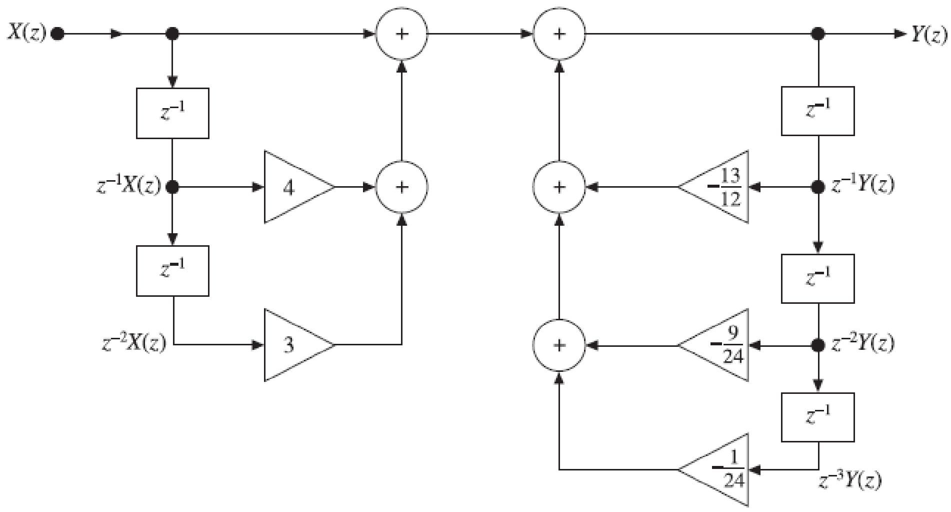
and $$\frac{Y(z)}{W(z)} = 1 + 4z^{-1} + 3z^{-2}$$

On cross multiplying the above equations, we get

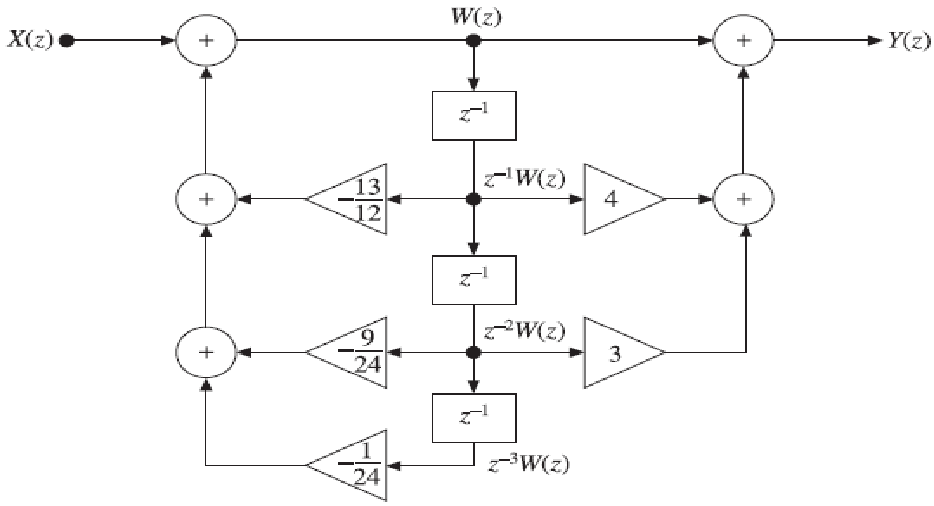
$$W(z)\left[1 + \frac{13}{12}z^{-1} + \frac{9}{24}z^{-2} + \frac{1}{24}z^{-3}\right] = X(z)$$

i.e. $$W(z) = X(z) - \frac{13}{12}z^{-1}W(z) - \frac{9}{24}z^{-2}W(z) - \frac{1}{24}z^{-3}W(z)$$

and $$Y(z) = W(z) + 4z^{-1}W(z) + 3z^{-2}W(z)$$



Direct form-I realization structure



Direct form-II realization structure

	b)	Realize FIR linear phase filter for N, even.	5M

$$h(n) = h(N-1-n)$$

It will now be shown that this symmetry allows the transfer function to be rewritten so that approximately half the number of multiplications are required for the resulting realization. The transfer function $H(z)$ is the Z transform of the impulse response as follows:

$$H(z) = Z[h(n)] = \sum_{n=0}^{N-1} h(n)z^{-n}$$

Using (5.70) and (5.71), $H(z)$ can be easily written as follows for odd and even N.

for even N:

$$H(z) = \sum_{n=0}^{N/2-1} h(n)[z^{-n} + z^{-(N-1-n)}]$$

impulse response as follows:

$$H(z) = Z[h(n)] = \sum_{n=0}^{N-1} h(n)z^{-n}$$

for odd N:

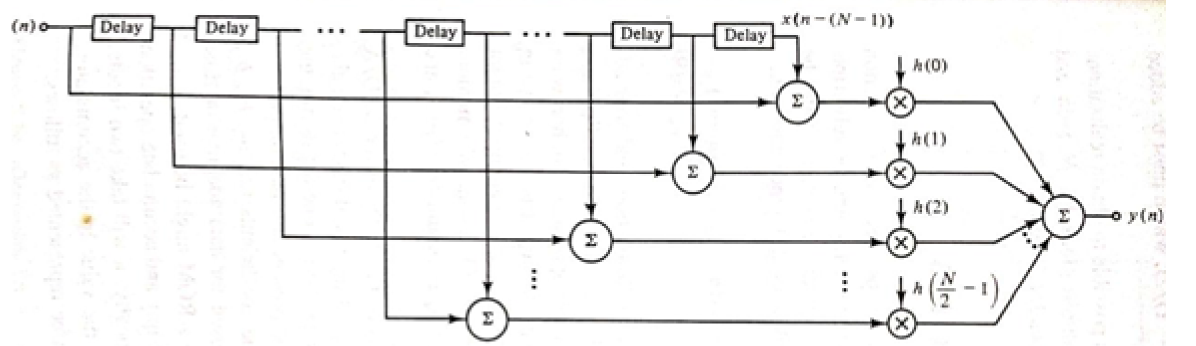
$$H(z) = \sum_{n=0}^{(N-3)/2} h(n)[z^{-n} + z^{-(N-1-n)}] + h((N-1)/2)z^{-(N-1)/2}$$

The output transform, $Y(z)$, can be obtained by multiplying the input transform, $X(z)$, where N is even, the following:

$$Y(z) = \sum_{n=0}^{N/2-1} h(n)[z^{-n} + z^{-(N-1-n)}]X(z)$$

$$= h(0)[1 + z^{-(N-1)}]X(z) + h(1)[z^{-1} + z^{-(N-2)}]X(z)$$

$$+ \cdots + h(N/2-1)[z^{-(N/2-1)} + z^{-N/2}]X(z)$$



(n)
Delay
Delay
Delay
Delay
Delay
x(n − (N − 1))
h(0)
h(1)
h(2)
h(N/2 − 1)
y(n)