Hall Ticket Number:

III/IV B.Tech (Supplementary) DEGREE EXAMINATION

October,2020 **Common to ECE & EIE**

Fifth Semester **Linear Control Systems**

Time: Three Hours **Maximum :** 60 Marks

Answer Question No.1 compulsorily. (1X12 = 12 Marks)

Answer ONE question from each unit. (4X12=48 Marks)

1 Answer all questions (1X12=12 Marks)

a) What is the effect of feedback on sensitivity.
b) What is meant by takeoff point or branch point?
c) What does the time constant of a system indicate.
d) Define steady state error.
e) The closed loop transfer function of second order system is $\frac{C(S)}{R(S)} = \frac{10}{S^2+6S+10}$. What is the type of damping in the system.
f) Draw the appropriate polar plot for a Type 0 second order system.
g) Give the advantages and limitations of Nyquist stability criterion.
h) Define gain crossover frequency.
i) When do you say that a system is completely state controllable.
j) What are the drawbacks of transfer function?
k) What does a gain margin close to unity or phase margin close to zero indicate?
l) Explain peak overshoot.

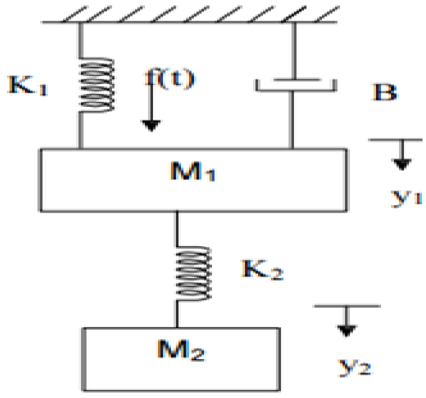
UNIT I

2 a) With the help of suitable examples differentiate between (i) Open and closed loop systems (ii) Linear and Non linear systems 6M

b) Discuss the mathematical modeling of fundamental components of mechanical translational system. 6M

(OR)

3 a) Write the differential equations governing the behavior of the mechanical system shown in fig below and obtain $Y_2(s)/F(s)$. Also obtain an analogous electrical circuit based on force – current analogy



12M

UNIT II

4 a) Damping factor and natural frequency of the second order system are 0.12 and 84.2 rad/sec respectively. 6M
Determine the rise time, peak time, maximum peak overshoot and sttling time.

b) Determine the stability of the closed loop system whose open loop transfer is $\frac{5(2s+1)}{s(s+1)(1+3s)(1+0.5s)}$ using Routh-Hurwitz criterion.

(OR)

5 a) Define the steady state error and error constants of different types of inputs. 6M

b) Find K_P, K_v, K_a and steady state error for a system with open loop transfer function as $G(s)H(s) = \frac{10(s+2)(s+3)}{s(s+1)(s+5)(s+4)}$. 6M

UNIT III

6 a) Given the open loop transfer function $G(s) = \frac{5}{(1+2s+s^2)(1+3s)}$. Sketch the Nyquist plot and investigate the open loop and closed loop systems stability. 6M

b) Explain how phase margin and gain margin are obtained from polar plot. 6M

(OR)

7 a) A unity feedback control system has $G(s) = \frac{k}{s(s+4)(s+10)}$. Draw the bode plot. Find 'K' when phase margin $=30^0$. 12M

UNIT IV

8 a) A unity feedback system has an open loop transfer function $G(s) = \frac{k}{s(s^2+3s+10)}$ make a rough sketch of root locus plot by determine the following (i) Centroid, number and angle of symptotes (ii) angle of departure of root loci from the poles, (iii) Break away points if any, (iv) points of intersection with jω axis and (v) maximum value of k for stability. 12M

(OR)

9 a) Determine the state transition matrix for the system $\dot{X} = AX$, where

$$A=\begin{bmatrix} -2 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 0 & -1 \end{bmatrix}$$ 6M

b) Determine the state model of the system characterized by the differential equation $(S^4+2S^2+8S^3+4S+3)Y(s)=10U(s)$. 6M

1 **Answer all questions** **(1X12=12 Marks)**

a) What is the effect of feedback on sensitivity.

Ans: we got the sensitivity of the overall gain of closed loop control system as the reciprocal of (1+GH). So, Sensitivity may increase or decrease depending on the value of (1+GH).If the value of (1+GH) is less than 1, then sensitivity increases. In this case, 'GH' value is negative because the gain of feedback path is negative.If the value of (1+GH) is greater than 1, then sensitivity decreases. In this case, 'GH' value is positive because the gain of feedback path is positive.

In general, 'G' and 'H' are functions of frequency. So, feedback will increase the sensitivity of the system gain in one frequency range and decrease in the other frequency range. Therefore, we have to choose the values of 'GH' in such a way that the system is insensitive or less sensitive to parameter variations.

b) What is meant by takeoff point or branch point?

Ans:The take-off point is a point from which the same input signal can be passed through more than one branch. That means with the help of take-off point, we can apply the same input to one or more blocks, summing points.

c) What does the time constant of a system indicate.

Ans:Time Constant is the “how fast” variable. It describes the speed with which the measured Process Variable (PV) responds to changes in the Controller Output (CO).

d) Define steady state error.

Ans:

Steady-**state error** is **defined** as the difference between the input (comman
output of a **system** in the limit as time goes to infinity (i.e. when the response
reached **steady state**). The **steady**-**state error** will depend on the type of in
etc.) as well as the **system** type (0, I, or II).

e) The closed loop transfer function of second order system is $\frac{C(S)}{R(S)} = \frac{10}{S^2+6S+10}$. What is the type of damping in the system.

Ans:

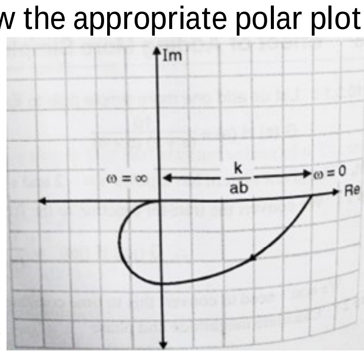
$2\delta\omega_n=6$

$\omega_n=3.16$

$\delta=0.949$

Since $\delta<1$ it is under damped system.

f) Draw the appropriate polar plot for a Type 0 second order system.

Ans: 

g) Give the advantages and limitations of Nyquist stability criterion.

Ans:

Following are some of the advantages of nyquist plot

- In practice most of the real systems experience delay such systems will have loop transfer functions involving exponentials. Such systems cannot be treated with Routh Hurwitz criterion and are difficult to treat with Root-Locus method. The stability of such systems can be estimated using Nyquist plot.
- The nyquist plot is easy to obtain especially with the aid of computer.
- Nyquist plot in addition to providing absolute stability, also gives information on relative stability of stable systems and degree of instability of unstable system.
- It also gives information on the frequency characteristics such as peak resonant amplitude, peak resonant frequency, bandwidth, gain margin, phase margin e.t.c

The Nyquist plot has some limitations :

1.The frequency is not clearly shown on the plot and it is not possible to determine, for a specific point, the frequency used to the record that point;

2.The ohmic and polarization resistances can be directly determined from the plot but the electrode capacitance can be only calculated if the frequency information is known.

3.If there are high and low impedance components in the circuit, the larger impedance controls plot scaling and distinguishing the low impedance semicircle would probably be impossible.

h) Define gain crossover frequency.

Ans:

The frequency at which the Nyquist plot is having the magnitude of one is known as the gain c

denoted by ωgc.

i) When do you say that a system is completely state controllable.

Ans:

Controllability: In order to be able to do whatever we want with the given dynamic system under control input, the system must be controllable.

State Controllability:

1. The state equation (1) or the pair (A, B) is state controllable if and only if the state controllability matrix

$$U_C = \begin{bmatrix} B & AB & A^2B & \ldots\ldots & A^{n-1}B \end{bmatrix}$$

has rank n, i.e., full row rank.

j) What are the drawbacks of transfer function?

Ans:

Disadvantages of Transfer function

1. Transfer function does not take into account the initial conditions.
2. The transfer function can be defined for linear systems only.
3. No inferences can be drawn about the physical structure of the system.

k) What does a gain margin close to unity or phase margin close to zero indicate?

Ans:

A gain margin close to unity or phase margin close to zero indicates the system is highly oscillatory. In the oscillating system, the force acts in an opposite direction to the displacement of the particle from the equilibrium point. This force tends to change with time or it can be constant

l) Explain peak overshoot.

Ans:

Peak Overshoot

Peak overshoot Mp is defined as the deviation of the response at peak time from the final value of response. It is also called the maximum overshoot.

Mathematically, we can write it as

Mp=c(tp)−c(∞)

UNIT-I

2.(a).

Comparison open loop system and closed loop system - 6M

Open loop system	**Closed loop system**
Output measurement is not required for operation of the system.	Output measurement is necessary.
Highly affected by non-linearities	Reduced effect of non-linearities.
Highly sensitive to the disturbances and environmental changes	Less sensitive to disturbances and environmental changes.
Feedback element and error detector are absent	Feedback element and error detector are absent
Generally stable in nature	Stability is the major consideration while designing.

Linear control system

The system is said to be linear, if it satisfies the following two properties:

• Additivity property that is for any x and y belonging to the domain of the function f, we have

F(x+y) = f(x) +f(y)

• Homogeneous property that is for any x belonging to the domain of the function f and for any scalar constant We have

F(αx) = α f(x)

These two properties together constitute a principle of superposition.

A system is said to be non linear .if it does not satisfy the principle of super position .

- Most of the systems are non-linear in nature because of different non-linearities such as saturation, friction, dead zone etc. present in the system.

2.(b).

Electrical analogous elements- 2M

Explanation- 4M

1.9 ELECTRICAL ANALOGOUS OF MECHANICAL TRANSLATIONAL SYSTEMS

Systems remain ***analogous*** as long as the differential equations governing the systems or transfer functions are in identical form. The electric analogue of any other kind of system is of greater importance since it is easier to construct electrical models and analyse them.

The three basic elements mass, dash-pot and spring that are used in modelling mechanical translational systems are analogous to resistance, inductance and capacitance of electrical systems.

The input force in mechanical system is analogous to either voltage source or current source in electrical systems. The output velocity (first derivative of displacement) in mechanical system is analogous to either current or voltage in an element in electrical system.

Since the electrical systems has two types of inputs either voltage or current source, there are two types of analogies : ***force-voltage analogy*** and ***force-current analogy***.

FORCE-VOLTAGE ANALOGY

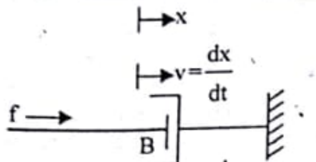
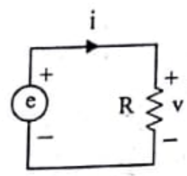
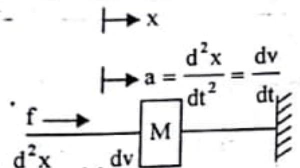
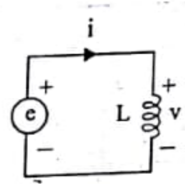
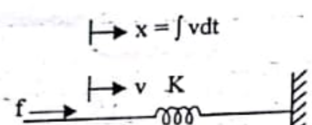
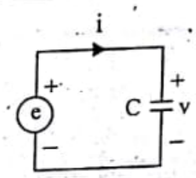
The force balance equations of mechanical elements and their analogous electrical elements in force-voltage analogy are shown in table-2.The table-3 shows the list of analogous quantities in force-voltage analogy.

The following points serve as guidelines to obtain electrical analogous of mechanical systems based on force-voltage analogy.

1. In electrical systems the elements in series will have same current, likewise in mechanical systems, the elements having same velocity are said to be in series.
2. The elements having same velocity in mechanical system should have analogous same current in electrical analogous system.
3. Each node (meeting point of elements) in the mechanical system corresponds to a closed loop in electrical system. A mass is considered as a node.
4. The number of meshes in electrical analogous is same as that of the number of nodes (masses) in mechanical system. Hence the number of mesh currents and system equations ... of nodes (masses) in mechanical system.

will be same as that of the number of velocities of nodes (masses) in mechanical systems

LE- 1.2 : Analogous Elements in Force-Voltage Analogy

Mechanical system	Electrical system
Input : Force Output : Velocity	Input : Voltage source Output : Current through the element
 $f = B\frac{dx}{dt} = Bv$	 $e = v$ and $v = Ri$ $\therefore e = Ri$
 $f = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$	 $e = v$ and $v = L\frac{di}{dt}$ $\therefore e = L\frac{di}{dt}$
 $f = Kx = K\int v dt$	 $e = v$ and $v = \frac{1}{C}\int i dt$ $\therefore e = \frac{1}{C}\int i dt$

E -1.3 : Analogous Quantities in Force-Voltage Analogy

Item	Mechanical system	Electrical system (mesh basis system)
Independent variable (input)	Force, f	Voltage, e, v
Dependent variable (output)	Velocity, v	Current, i
	Displacement, x	Charge, q
Dissipative element Dash pot	Frictional coefficient of dashpot, B	Resistance, R
Storage element Mass	Mass, M	Inductance, L
Spring	Stiffness of spring, K	Inverse of capacitance, 1/C
Physical law	Newton's second law $\Sigma f = 0$	Kirchoff's voltage law $\Sigma v = 0$
Changing the level of independent variable	Lever $\frac{f_1}{f_2} = \frac{l_1}{l_2}$	Transformer $\frac{e_1}{e_2} = \frac{N_1}{N_2}$

TABLE-1.4 : Analogous Elements in Force-Current Analogy

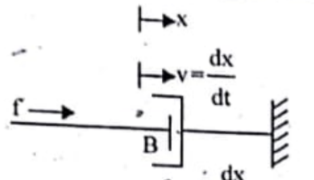
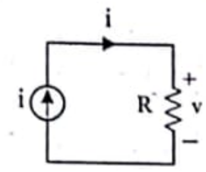
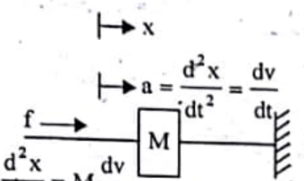
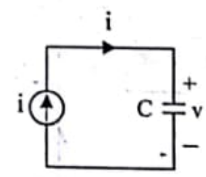
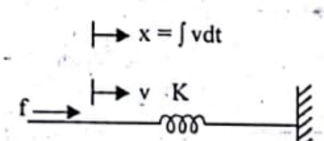
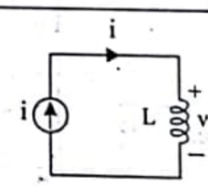
Mechanical system	Electrical system
Input : Force Output : Velocity	Input : Current source Output : Voltage across the element
 $f = B\frac{dx}{dt} = Bv$	 $i = \frac{1}{R}v$
 $f = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$	 $i = C\frac{dv}{dt}$
 $f = Kx = K\int vdt$	 $i = \frac{1}{L}\int vdt$

TABLE-1.5 : Analogous Quantities in Force-Current Analogy

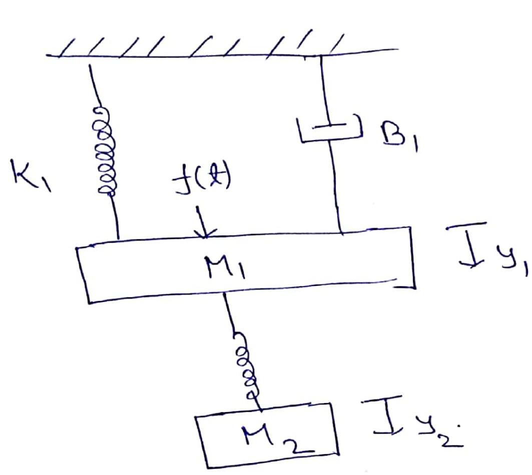
Item	Mechanical system	Electrical system (node basis system)
Independent variable (input)	Force, f	Current, i
Dependent variable (output)	Velocity, v Displacement, x	Voltage, v Flux, ϕ
Dissipative element Dash pot	Frictional coefficient of dashpot, B	Conductance G=1/R
Storage element Mass Spring	Mass, M Stiffness of spring, K	Capacitance, C Inverse of inductance, 1/L
Physical law	Newton's second law $\Sigma f = 0$	Kirchoff 's current law $\Sigma i = 0$
Changing the level of independent variable	Lever $\frac{f_1}{\ } = \frac{l_1}{\ }$	Transformer $\frac{i_1}{\ } = \frac{N_2}{\ }$

(OR)

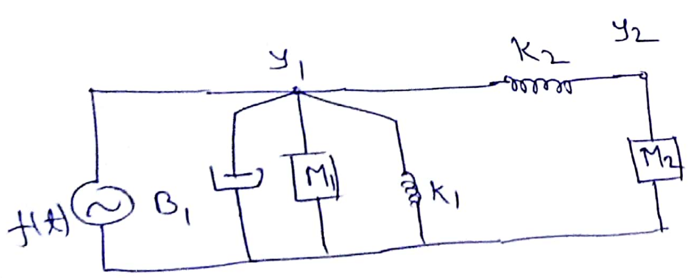
3.(a)

Free body diagram- **2M**
Transfer function- **4M**
Analogous circuit and equations- **6M**

3. (a). Write the differential equations governing the behaviour of the mechanical system shown in figure below and obtain $\frac{Y_2(S)}{F(S)}$. Also obtain an analogous electrical circuit based on force-current analogy.



Sol:- Free body diagram of given mechanical system



The differential equations governing the system are.

$$M_1 \frac{d^2 y_1}{dt^2} + B_1 \frac{dy_1}{dt} + K_1 y_1 + K_2(y_1 - y_2) = f(t). \quad \rightarrow (1)$$

Take Laplace transform of the above equation

$$M_1 s^2 Y_1(s) + B_1 s Y_1(s) + K_1 Y_1(s) + K_2[Y_1(s) - Y_2(s)] = F(s).$$

$$Y_1(s)\,[M_1 s^2 + Bs + (K_1+K_2)] - Y_2(s) K_2 = F(s) \quad \rightarrow (2)$$

$$M_2 \frac{d^2 y_2}{dt^2} + k_2 (y_2 - y_1) = 0 \quad \rightarrow (3)$$

Take Laplace Transform of the above equation

$$M_2 s^2 Y_2(s) + K_2[Y_2(s) - Y_1(s)] = 0$$

$$Y_2(s)\,[M_2 s^2 + K_2] - Y_1(s) K_2 = 0$$

$$Y_1(s) = Y_2(s) \frac{M_2 s^2 + K_2}{K_2} \quad \rightarrow (4)$$

Substituting for $Y_1(s)$ from equation (4) in equation (3) we get.

$$Y_2(s)\left[\frac{M_2 s^2 + K_2}{K_2}\right][M_1 s^2 + Bs + (K_1+K_2)] - Y_2(s) K_2 = F(s)$$

$$Y_2(s)\left[\frac{(M_2 s^2 + K_2)[M_1 s^2 + Bs + (K_1+K_2)] - K_2^2}{K_2}\right] = F(s).$$

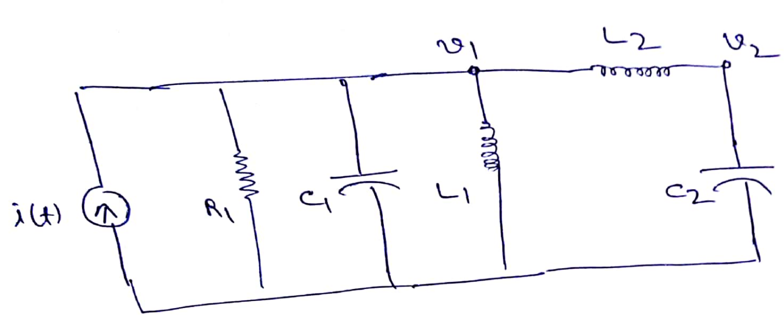
$$\frac{Y_2(s)}{F(s)} = \frac{K_2}{[M_1 s^2 + Bs + (K_1+K_2)][M_2 s^2 + K_2] - K_2^2}.$$

Force- current Analogous Circuit:-

The list of electrical analogous elements.

$f(t) \rightarrow i(t)$; $M_1 \rightarrow C_1$ $\quad K_1 \rightarrow 1/L_1$ $\quad B_1 \rightarrow 1/R_1$

$M_2 \rightarrow C_2$ $\quad K_2 \rightarrow 1/L_2$



The Nodal equations of the above equations are.

$$C_1 \frac{dv_1}{dt} + \frac{1}{R_1} v_1 + \frac{1}{L_1}\int v_1\, dt + \frac{1}{L_2}\int (v_1 - v_2)\, dt = i(t)$$

$$C_2 \frac{dv_2}{dt} + \frac{1}{L_2}\int (v_2 - v_1)\, dt = 0.$$

UNIT-II

4.(a).

Rise time & peak time -------3 M

Peak over shoot & settling time ----- 3M

Ans:

Damping factor =0.12

Natural frequency=84.2 rad/sec

Rise time= 0.0188 sec

Peak time=0.037 sec

Maximum peak over shoot=2

Settling time= 0.39 sec

4.(b).

Charester stic equation -2M

Routh Array -4M

Characteristic equation of the given system is

$1.5s^4+5s^3+4.5s^2+11s+5=0$

Routh array

S^4	1.5	4.5	5
S^3	5	11	0
S^2	1.2	5	
S^1	9.833	0	
S^0	5		

The given system is a stable system.

(OR)

5.(a).

Definitions of steady state error constants -3M

Steady state error for various input signals ---- 3M

STATIC ERROR CONSTANTS

When a control system is excited with standard input signal, the steady state error may be zero, constant or infinity. The value of steady state error depends on the type number and the input signal. Type – 0 system will have a constant steady state error when the input is step signal. Type –1 system ~~will have a constant~~ steady state error when the input is ramp signal or velocity signal. Type–2 system will have a constant steady state error when the input is parabolic signal or acceleration signal. For the three cases mentioned above the steady state error is associated with one of the constants defined as follows,

Positional error constant, $K_p = \underset{s\to 0}{Lt}\ G(s)\,H(s)$ .....(3.58)

Velocity error constant, $K_v = \underset{s\to 0}{Lt}\ s.G(s)\,H(s)$ .....(3.59)

Acceleration error constant, $K_a = \underset{s\to 0}{Lt}\ s^2G(s)\,H(s)$ .....(3.60)

The K_p, K_v and K_a are in general called static error constants.

3.2 : The steady state error for various types of inputs

ype number	Steady state error when the input signal is		
	Unit step	Unit ramp	unit parabolic
0	$\frac{1}{1+K_p}$	∞	∞
1	0	$\frac{1}{K_v}$	∞
2	0	0	$\frac{1}{K_a}$
3	0	0	0

5.(b).

Kp & Kv -----   --3M

Ka & steady state error ----- 3M

Example 7.2 : *Find K_p, K_v, K_a and steady state error for a system with open loop transfer function as :*

$$G(s)H(s) = \frac{10(s+2)(s+3)}{s(s+1)(s+5)(s+4)}$$

where the input is

$$r(t) = 3+t+t^2$$

Solution : Express the given open loop transfer function in time constant form.

$$G(s)H(s) = \frac{10\,(s+2)\,(s+3)}{s\,(s+1)\,(s+5)\,(s+4)} = \frac{10.2.3 \times \left(1+\frac{s}{2}\right)\left(1+\frac{s}{3}\right)}{s\times 1\times 5\times 4\times (1+s)\left(1+\frac{s}{5}\right)\left(1+\frac{s}{4}\right)}$$

$$= \frac{3 \times \left(1+\frac{s}{2}\right)\left(1+\frac{s}{3}\right)}{s\,(1+s)\left(1+\frac{s}{5}\right)\left(1+\frac{s}{4}\right)}$$

$K_p = \underset{s\to 0}{Lt}\; G(s)H(s) = \infty$; $K_v = \underset{s\to 0}{Lt}\; sG(s)H(s) = 3.$

$$K_a = \lim_{s \to 0} s^2 G(s)H(s) = 0$$

Now input is, $r(t) = 3+t+t^2 = 3+t+2\cdot\frac{t^2}{2}$

The input is combination of three standard inputs.

$$A_1 = 3, \text{ step of } 3$$

$$A_2 = 1, \text{ ramp of } 1$$

$$A_3 = 2, \text{ parabolic input of } 2$$

Note that parabolic input must be expressed as $\frac{A}{2}t^2$.

a) For step of 3 the error is,

$$e_{ss1} = \frac{A_1}{1+K_p} = \frac{3}{1+\infty} = 0$$

b) For ramp of 1 the error is,

$$e_{ss2} = \frac{A_2}{K_v} = \frac{1}{3}$$

c) For parabolic of 2, the error is

$$e_{ss3} = \frac{A_3}{K_a} = \frac{2}{0} = \infty$$

Hence steady state error is,

$$e_{ss} = e_{ss1} + e_{ss2} + e_{ss3} = 0 + \frac{1}{3} + \infty$$

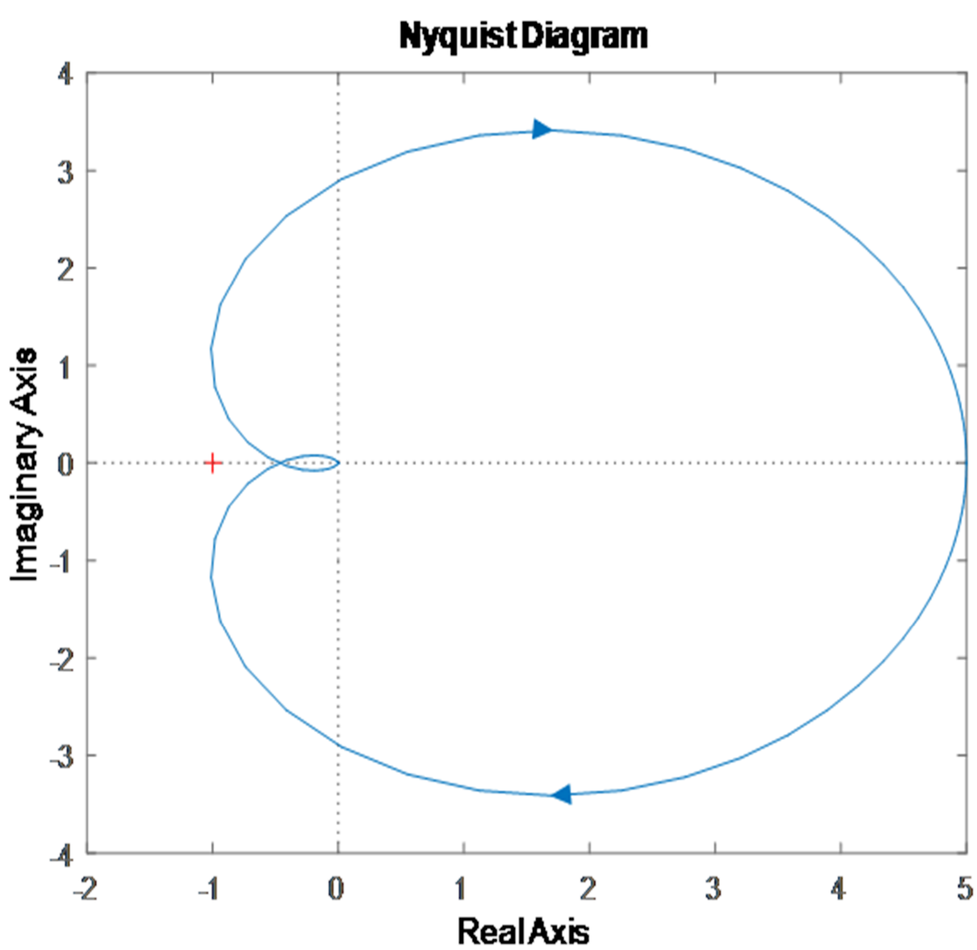
$$= \infty$$

UNIT-III

6.(a).

Nyquist plot procedure-----3M

Nyquist plot ------3M



6.(b).

Rules for drawing polar plot ----- 3M

Gain margin & Phase margin ----3M

The Polar plot is a plot, which can be drawn between the magnitude and the phase angle of G(jω)H(jω) by varying ω from zero to ∞. The polar graph sheet is shown in the following figure.

This graph sheet consists of concentric circles and radial lines. The concentric circles and the radial lines represent the magnitudes and phase angles respectively. These angles are represented by positive values in anti-clock wise direction. Similarly, we can represent angles with negative values in clockwise direction. For example, the angle 2700 in anti-clock wise direction is equal to the angle −900 in clockwise direction.

Rules for Drawing Polar Plots

Follow these rules for plotting the polar plots.

Substitute, s=jω in the open loop transfer function.

Write the expressions for magnitude and the phase of G(jω)H(jω).

Find the starting magnitude and the phase of G(jω)H(jω) by substituting ω=0. So, the polar plot starts with this magnitude and the phase angle.

Find the ending magnitude and the phase of G(jω)H(jω) by substituting ω=∞. So, the polar plot ends with this magnitude and the phase angle.

Check whether the polar plot intersects the real axis, by making the imaginary term of G(jω)H(jω) equal to zero and find the value(s) of ω.

Check whether the polar plot intersects the imaginary axis, by making real term of G(jω)H(jω) equal to zero and find the value(s) of ω.

For drawing polar plot more clearly, find the magnitude and phase of G(jω)H(jω) by considering the other value(s) of ω.

Gain Cross over Frequency

The frequency at which the Nyquist plot is having the magnitude of one is known as the gain cross over frequency. It is denoted by ωgc.

The stability of the control system based on the relation between phase cross over frequency and gain cross over frequency is listed below.

If the phase cross over frequency ωpc is greater than the gain cross over frequency ωgc, then the control system is stable.

If the phase cross over frequency ωpc is equal to the gain cross over frequency ωgc, then the control system is marginally stable.

If phase cross over frequency ωpc is less than gain cross over frequency ωgc, then the control system is unstable.

Gain Margin

The gain margin GM is equal to the reciprocal of the magnitude of the Nyquist plot at the phase cross over frequency.

GM=1/Mpc

Where, Mpc is the magnitude in normal scale at the phase cross over frequency.

Phase Margin

The phase margin PM is equal to the sum of 1800 and the phase angle at the gain cross over frequency.

PM=1800+φgc

Where, φgc is the phase angle at the gain cross over frequency.

The stability of the control system based on the relation between the gain margin and the phase margin is listed below.

If the gain margin GM is greater than one and the phase margin PM is positive, then the control system is stable.

(OR)

7.(a).
Bode plot procedure -------6M
Determination of K value ---3M
Bode plot ----- 3M

7.(a).

given system $G(s) = \frac{K}{s(s+4)(s+10)}$.

Time constant form $G(s) = \frac{(K/40)}{s(1+0.25s)(1+0.1s)}$

$$G(j\omega) = \frac{(K/40)}{j\omega(j\omega+1)(0.1j\omega+1)}$$

Factor	Corner freq	Slope dB/dec	Net slope
$\frac{(K/40)}{s}$	None	−20	−20
$\frac{1}{(1+0.25s)}$	4	−20	−40
$\frac{1}{1+0.1s}$	10	−20	−60

* The lower and upper limits of frequency scales are 0.1 rad/sec and 100 rad/sec respectively.

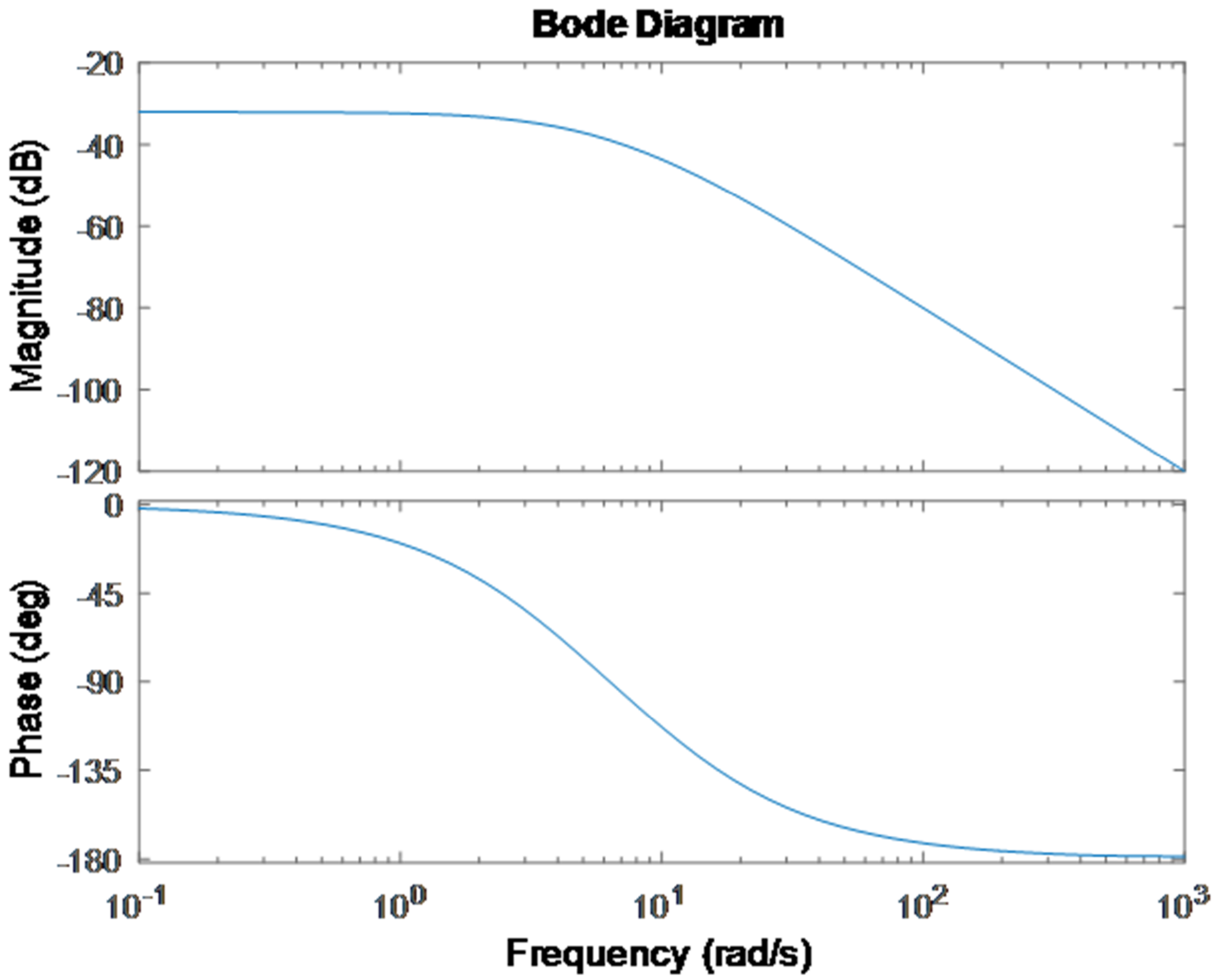
The lower and upper limits of magnitude in dB are −80 dB and 40 dB respectively.

* Phase response is given by

$$\angle G(j\omega) = -90^\circ - \tan^{-1}(0.1\omega) - \tan^{-1}(0.25\omega).$$

*

ω	0	0.1	0.2	0.5	1	5	10
$\angle G(j\omega)$	−90	−96.2	−102	119.43	−140.7	−195.25	−219



* To find the value of open loop gain k for a phase margin 30° : Required phase angle of the system for a phase margin 30°.

$$= -180 + 30 = -150°$$

Gain margin at $\angle G(j\omega) = -150°$ is -16 dB.

The magnitude plot has to be shifted up by 16dB. so that the phase margin is 30° \$.

$$20 \log k = 16 \Rightarrow k = (10)^{(16/20)} = 6.30$$

∴ The required open loop gain is 6.30.

UNIT-IV

8.(a).

Centroid , Angle of asymptodes ------3M

Point of intersection with imaginary axis -----6M

Graph ------ 3M

12M

8.(a).

given that $G(s) = \frac{K}{s(s^2+3s+10)}$.

Open loop poles are the roots of denominator polynomial

$s(s^2+3s+10) = 0 \Rightarrow s = 0$; $s^2+3s+10 = 0$;

$P_1 = 0$; $P_{2,3} = \frac{-3 \pm \sqrt{9-40}}{2} = -1.5 \pm j\,5.56$.

* All the points on the real axis between $-\infty$ and 0 lie on the root locus, since there is one pole to the right of these points

* The three branches that terminate at infinity do so along the asymptotes with angles.

$\phi_A = \frac{(2q+1)\,180^\circ}{n-m}$; $q = 0, 1, 2, \ldots (n-m-1)$

$q = 0$; $\phi_{A_1} = \frac{180}{3} = 60^\circ$

$q = 1$; $\phi_{A_2} = \frac{3(180^\circ)}{3} = 180^\circ$

$q = 2$; $\phi_{A_3} = \frac{5(180^\circ)}{3} = 300^\circ$.

* centroid $\sigma_A = \frac{0-1.5-1.5}{3} = -1$.

Break away point of root locus are the solution

of $\frac{dk}{ds} = 0$;

$$G(s)\,H(s) = \frac{k}{s(s^2+3s+10)} \quad ; \text{ and } H(s) = 1.$$

we know that $1 + G(s)\,H(s) = 0$.

$$k = -s(s^2+3s+10) = -(s^3+3s^2+10s).$$

$$\frac{dk}{ds} = -(3s^2+6s+10)$$

$$\frac{dk}{ds} = 0 \Rightarrow 3s^2+6s+10 = 0$$

Roots are $= \frac{-6 \pm \sqrt{36-120}}{6} = -1 \pm j\,1.5$.

This point is not on the root locus therefore there is no break away point.

* Angle of departure $\phi_p = 180° + \phi$.

$$\phi_{p_1} = 180 - \tan^{-1}\left(\frac{+1.5}{5.56}\right) = 180 - 15 = 165°$$

$$\phi = 180 - (165 + 90°) = -75°.$$

Similarly Angle of departure at P_3 is $+75°$.

* point of intersection with imaginary axis

$$s^3 + 3s^2 + 10s + k = 0.$$

s^3 :	1	10
s^2 :	3	k
s^1 :	$\frac{30-k}{3}$	
s^0 :	k	

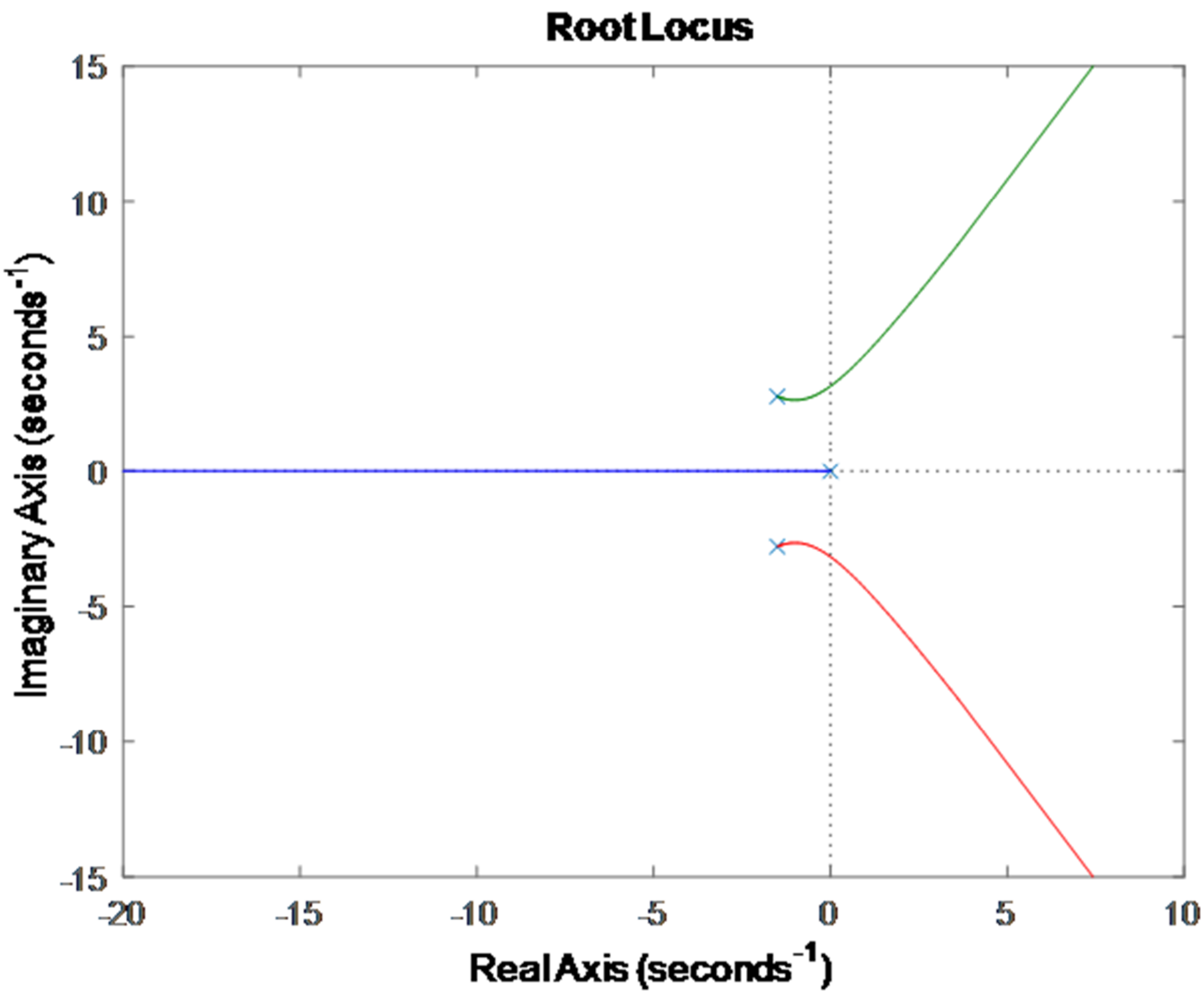
for stable system ; $k > 0$ & $\frac{30-k}{3} > 0$; $k < 30$.

(ie) $0 < k < 30$.

$$3s^2 + k = 0 \Rightarrow 3s^2 + 30 = 0 \Rightarrow s^2 + 10 = 0$$

$$s = \pm j\,3.16.$$

Root locus crosses the imaginary axis at $s = \pm j\,3.16$.



(OR)

9.(a).

Inverse of (SI-A) ------- 3M

Inverse Lapalace transform -------3M

$(sI-A)^{-1}$=

$$\begin{pmatrix} s+2 & 0 & -1 \\ 0 & s+1 & -1 \\ -2 & 0 & s+1 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{s+1}{s(s+3)} & 0 & \frac{1}{s^2+3s} \\ \frac{2}{(s+1)(s^2+3s)} & \frac{1}{s+1} & \frac{s+2}{(s+1)(s^2+3s)} \\ \frac{2}{s^2+3s} & 0 & \frac{s+2}{s^2+3s} \end{pmatrix}$$

$L^{-1}(SI-A)^{-1}$=

1/3 H(t) $e^{(-3t)}$	0	1/3H(t)(1/3)e^{-3t}
-$e^{(-t)}$+2/3H(t)+1/3$e^{(-3t)}$	e^{-t}	-$e^{(-t)}$+2/3 H(t)+1/3$e^{(-3t)}$
2/3 H(t)-2/3$e^{(-3t)}$	0	2/3 H(t)+1/3e(-3t)

9.(b).

State variables -----3M

State Model ------3M

9(b).

$$\frac{Y(s)}{U(s)} = \frac{10}{s^4 + 8s^3 + 2s^2 + 4s + 3}$$

$$s^4 Y(s) + 8s^3 Y(s) + 2s^2 Y(s) + 4s\, Y(s) + 3Y(s) = 10U(s)$$

By taking I.L.T.

$$s = \frac{d}{dt} \; ; \quad s^2 = \frac{d^2}{dt^2} \; ; \quad s^3 = \frac{d^3}{dt^3} \; ; \quad s^4 = \frac{d^4}{dt^4}$$

$$\ddddot{y} + 8\dddot{y} + 2\ddot{y} + 4\dot{y} + y = 10u \quad \rightarrow (1)$$

State variables:-

$y = x_1$

$\dot{y} = \dot{x}_1 = x_2$

$\ddot{y} = \dot{x}_2 = x_3$

$\dddot{y} = \dot{x}_3 = x_4$

$\ddddot{y} = \dot{x}_4 = x_5$

substituting in equation (1)

$$\dot{x}_4 + 8\dot{x}_3 + 2\dot{x}_2 + 4\dot{x}_1 + x_1 = 10u$$

$$\dot{x}_4 + 8x_4 + 2x_3 + 4x_2 + x_1 = 10u$$

$$\dot{x}_4 = -x_1 - 4x_2 - 2x_3 - 8x_4 + 10u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -4 & -2 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$