18ME501

Hall Ticket Number:

III/IV B. Tech (Regular) DEGREE EXAMINATION

February, 2021 — **Mechanical Engineering**

Fifth Semester — **Machine Dynamics**

Time: Three Hours — **Maximum: 50 Marks**

Answer Question No.1 compulsorily. **(1X10 = 10 Marks)**

1. Answer all questions. **10 M**

a) State D'Alembert's principle.
b) What do you mean by piston effort?
c) Differentiate between a flywheel and governor.
d) When do you say that a governor is isochronous?
e) What do you mean by angle of heel?
f) Differentiate between static and dynamic balancing.
g) What do you mean by resonance?
h) Define damping ratio.
i) What is a seismic unit?
j) Define whirling or whipping speed of a shaft.

Answer any FOUR questions out of the given EIGHT Questions. **(4X10 = 40 Marks)**

2. a) Derive expressions for velocity and acceleration of a piston in slider crank mechanism. **4 M**

b) A vertical petrol engine 100 mm diameter and 125 mm stroke has a connecting rod 250 mm long. The weight of the piston is 12 N. The speed is 2000 rpm. On the expansion stroke, with the crank 20° from the top dead centre, the gas pressure is 700 kN/m^2. Determine:

i) net force on the piston
ii) resultant load on the gudgeon pin
iii) thrust on the cylinder walls
iv) crank pin effort **6 M**

3. a) What is a controlling force curve? How is it useful in analyzing the performance of a governor? **4 M**

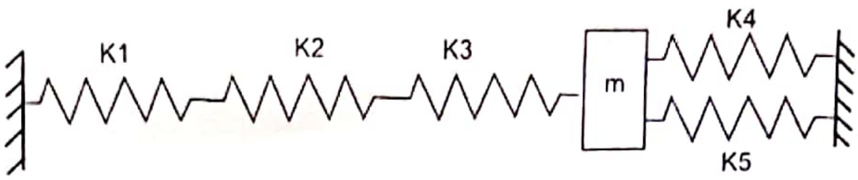
b) The upper arms of a porter governor are pivoted on the axis of rotation and the lower arms are pivoted to the sleeve at a distance of 30 mm from the axis of rotation. The length of each arm is 300 mm, and the mass of each ball is 6 kg. If the equilibrium speed is 200 rpm, when the radius of rotation is 200 mm, find the required mass on the sleeve. If the friction is equivalent to a force of 40 N at the sleeve, find the coefficient of insensitiveness at 200 mm radius. **6 M**

4. a) What is gyroscopic couple? Derive an expression for the same. **4 M**

b) The turbine rotor of a ship of mass 6000 kg has a radius of gyration of 500 mm. It rotates at 1820 rpm, clockwise when viewed from the stern. Determine the gyroscopic effect on the ship when, the ship is pitching, and the bow is descending with maximum velocity. The pitching is simple harmonic, periodic time being 20 seconds, and the total angular movement between extreme positions is 10°. **6 M**

5. A shaft has three eccentrics, each 75 mm diameter and 25 mm thick, machined in one piece with the shaft. The central planes of the eccentric are 60 mm apart. The distance of the centers from the axis of rotation are 12 mm, 18 mm and 12 mm and their angular positions are 120° apart. The density of the metal is 7000 kg/m^3. If the shaft is balanced by adding two masses at a radius 75 mm and at distances of 100 mm from the central plane of the middle eccentric, find the amount of masses and their angular positions. **10 M**

6. a) Derive the differential equation of motion of a spring-mass system and find out its natural frequency. **4M**
b) Find the mass of the given spring mass system shown in Fig. Where k_1=2000N/m, k_2=1500 N/m, k_3=3000 N/m and k_4=k_5=500N/m. The natural frequency of the system is of 10 Hz.



6 M

7. a) What is logarithmic decrement? Derive an expression for the same. **4 M**
b) A spring –mass-damper system has a mass of 3 kg with stiffness 100 N/m and damping constant of 3 N-s/m. Determine i) Damping factor, ii) Natural frequency of damped vibration, iii) Logarithmic decrement, iv) Ratio of two consecutive amplitudes **6 M**

8. a) A vehicle has a mass of 490 kg and the total spring constant of its suspension system is 58800 N/m. The profile of the road may be approximated to a sine wave of amplitude 40 mm and wave length 4.0 m. Determine i) the critical speed of the vehicle, ii) the amplitude of steady state motion of the mass when the vehicle is driven at critical speed and the damping factor is 0.5, and iii) the amplitude of steady state motion of the mass when the vehicle is driven at 57 km/hr and the damping factor is 0.5. **5 M**
b) A machine of 100 kg mass is supported on springs of total stiffness 700 kN/m and has an unbalanced rotating element, which results in a disturbing force of 350 N at a speed of 3000 rpm. Assuming a damping factor of 0.20, determine i) its amplitude of motion due to unbalance, ii) the transmissibility, and iii) the transmitted force. **5 M**

9. a) Derive an expression for transmissibility ratio. **5 M**
b) Explain the working principle of vibration measuring instruments. **5 M**

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Scheme of Evaluation

1. a) If a body under the action of forces possess an acceleration, to that body if we add an inertia force, mass times its acceleration at its centre of mass in the opposite side of acceleration, then the body is said to be in dynamic equilibrium – (1M)

b) The net force acting on the piston along its line of stroke which tends to move it is known as piston effort – (1M)

c) Fly wheel maintains a constant speed over a cycle, whereas a Governor maintains a constant speed over a number of cycles. (1M)

e) The angle made by a two wheeler along with the rider with the vertical, to balance the centrifugal and Gyroscopic couples is known as angle of heel – (1M)

d) For all configurations of the Governor, if there is only one equilibrium speed, then the governor is said to be isochronous – (1M)

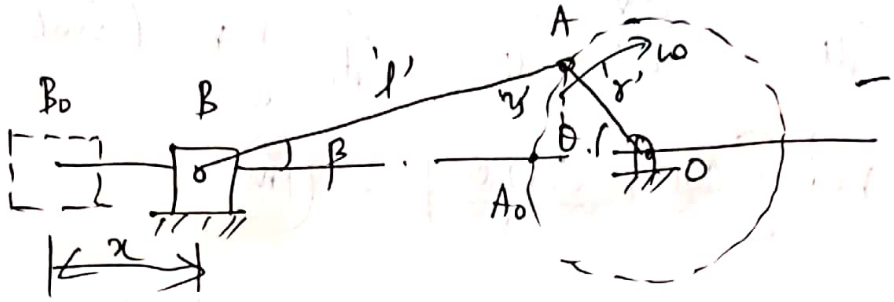
f) If the resultant force of several mass rotating in different planes is zero then the system is said to be in static balance If both resultant force and Couple are zero then the system is said to be in dynamic balance - 1M

g) If the excitation frequency coincides with the natural frequency of the system then resonance occurs ($\omega = \omega_n$) - 1M

h) The ratio between damping Constant to the critical damping constant is known as damping ratio - 1M

i) Seismic unit is a spring-mass-damper system used to measure vibrations - 1M

j) whirling or whipping speed is the speed of rotation of the plane formed by bearing axis and the bent shaft axis about the bearing axis. - 1M a)



— 1M

Displacement of the piston, $x = B_0B$

$$x = OB_0 - OB = (l+r) - (l\cos\beta + r\cos\theta)$$

taking $n = \frac{l}{r}$

$$x = (nr + r) - (nr\cos\beta + r\cos\theta)$$

$$x = r[(n+1) - (n\cos\beta + \cos\theta)]$$

$$\frac{\sin\beta}{r} = \frac{\sin\theta}{l} \Rightarrow \sin\beta = \frac{\sin\theta}{n}.$$

$$\cos\beta = \sqrt{1-\sin^2\beta} = \frac{1}{n}\sqrt{n^2-\sin^2\theta}.$$

$$x = r\left[(n+1) - \left(\sqrt{n^2-\sin^2\theta} + \cos\theta\right)\right]$$

$$x = r\left[(1-\cos\theta) + \left(n - \sqrt{n^2-\sin^2\theta}\right)\right]$$ — 1M

Velocity of the piston

$$V = \frac{dx}{dt} = \frac{dx}{d\theta}\cdot\frac{d\theta}{dt} \quad \left[\frac{d\theta}{dt} = \omega\right]$$

$$V = \frac{d}{d\theta}\left[r(1-\cos\theta) + \left(n - \sqrt{n^2-\sin^2\theta}\right)\right]\omega$$

$$V = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right] \Rightarrow V = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$$ 1M

Since 'n' is large.

Acceleration of the piston

$$a = \frac{dv}{dt} = \frac{dv}{d\theta} \cdot \frac{d\theta}{dt} = \frac{d}{d\theta}\left[r\omega\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)\right] \cdot \omega$$

$$a = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right] \quad - \text{(1M)}$$

b) $D = 100\,mm$; $2r = 125\,mm$; $l = 250\,mm$.

$W = 12\,N$; $N = 2000\,rpm$; $\theta = 20°$; $P_1 = 700\,\frac{kN}{m^2}$

To find

(i) $F = ?$ (ii) $F_c = ?$ (iii) $F_N = ?$ (iv) $F_t = ?$

$$F = F_G + W - F_I = \frac{\pi}{4}(100\times10^{-3})^2 \times 700\times10^{3} + 12 - \frac{12}{9.81}\times 62.5\times10^{-3}\times\left(\frac{2\pi\times2000}{60}\right)^2\left[\cos 20 + \frac{\cos 40}{4.17}\right]$$

$F_G = \frac{\pi}{4}D^2 \times P_1$ $\quad F_I = mr\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

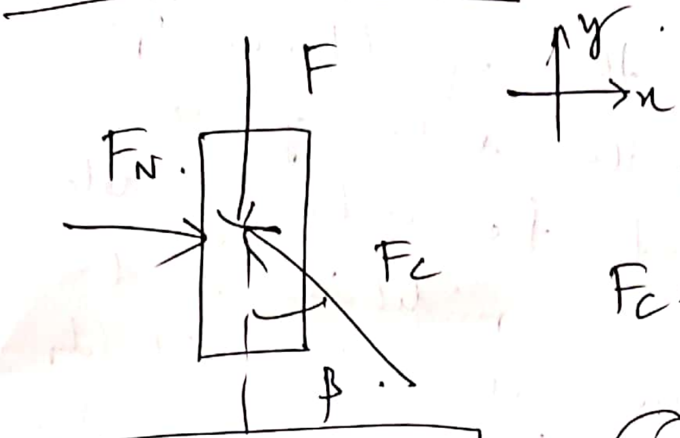
$n = \frac{l}{r} = 4.17$ $\quad \beta = \sin^{-1}\left[\frac{\sin\theta}{n}\right] = 4.7°$

$F_G = 5.5\times10^{3}\,N$; $F_I = 3.19\times10^{3}\,N$.

$F = 5.5\times10^{3} + 12 - 3.19\times10^{3} = 2.3\times10^{3}\,N$.

$\boxed{F = 2.3\,kN}$ – (3M)

FBD of the piston.



$\Sigma F_y = 0$

$F_c \cos\beta = F$

$F_c = \dfrac{F}{\cos\beta} = 2.31\times10^{3}\ N.$

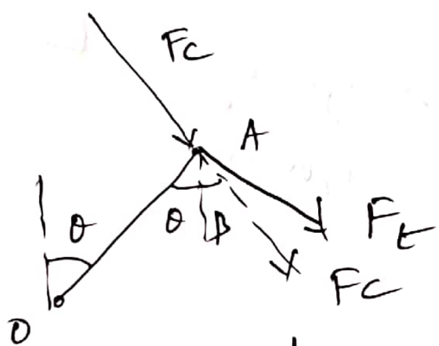
$\boxed{F_c = 2.31\ kN}$ — 1M

$\Sigma F_x = 0 \Rightarrow F_N = F_c \sin\beta.$

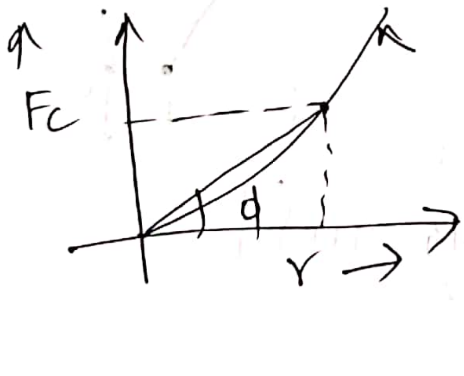
$\boxed{F_N = 189.28\ N}$ — 1M

$F_t = F_c \sin(\theta+\beta)$

$\boxed{F_t = 965\ N.}$ — 1M



3. a) A curve drawn by taking radius of rotation of the governor fly balls as the horizontal axis and the controlling force as the vertical axis is known as a controlling force curve. — 1M



Performance of the Governor. — (3M) (6)

(i) We can find the equilibrium speed corresponding to the radius of rotation (or) we can find the radius of rotation corresponding to a equilibrium speed.

(ii) We can discuss about the stability. As 'r' increases if '$\tan\phi$' increases the governor is said to be stable. If $\tan\phi$ decreases as 'r' increases the governor is said to be unstable.

(iii) Sensitiveness: If the variation of $\tan\phi$ is small over a range of radii of rotation, then the governor is said to be sensitive.

b) $BC = 30$ mm

$OA = AB = 300$ mm

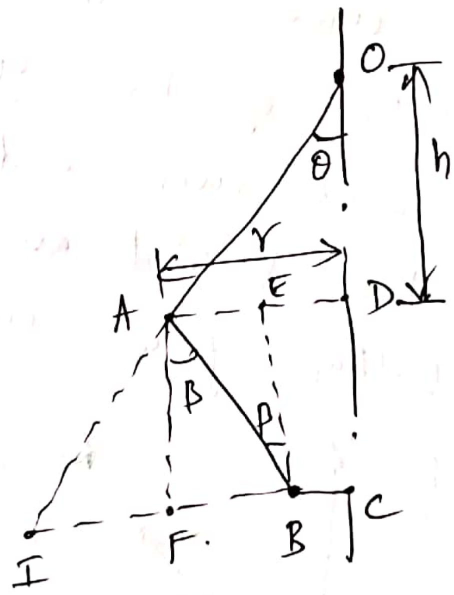
$m = 6$ kg

$N = 200$ rpm.

$r = 200$ mm.

$M = ?$

$f = 40N$.



— (1M)

$$mr\omega^2 = \tan\theta\left[mg + \frac{Mg}{2}(1+k)\right]$$

$$\omega = \frac{2\pi \times 200}{60} = 20.94 \text{ r/s}$$

$$\sin\theta = \frac{AD}{OA} = \frac{200}{300} \Rightarrow \theta = 41.81^\circ$$

$$\sin\beta = \frac{AE}{AB} = \frac{200-30}{300} \Rightarrow \beta = 34.52^\circ$$

$$k = \frac{\tan\beta}{\tan\theta} = 0.77.$$

$$6 \times 200 \times 10^{-3} \times 20.94^2 = \tan 41.81 \left[6 \times 9.81 + \frac{Mg}{2}(1+0.77)\right]$$

$$588.29 = 58.86 + 8.68M$$

$$\boxed{M = 61 \text{ kg}} \text{ — } 2M$$

Co-efficient of insensitiveness $= \frac{N'' - N'}{N}$

where N'' = Equilibrium speed where the sleeve is just about to raise considering friction

N' = Equilibrium speed where the sleeve is just about to come down considering friction

$$mr(\omega'')^2 = \tan\theta \left[mg + \left(\frac{Mg+f}{2}\right)(1+k)\right]$$

$$6 \times 200 \times 10^{-3} (\omega'')^2 = \tan 41.81 \left[6 \times 9.81 + \left(\frac{61 \times 9.81 + 40}{2}\right)(1+0.77)\right]$$

$$\omega'' = 21.56 \text{ r/s} \Rightarrow \boxed{N'' = 205.9 \text{ rpm}} \text{ — } 1M$$

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$$mr(\omega')^2 = \tan\theta\left[mg + \frac{Mg-f}{2}(1+k)\right]$$

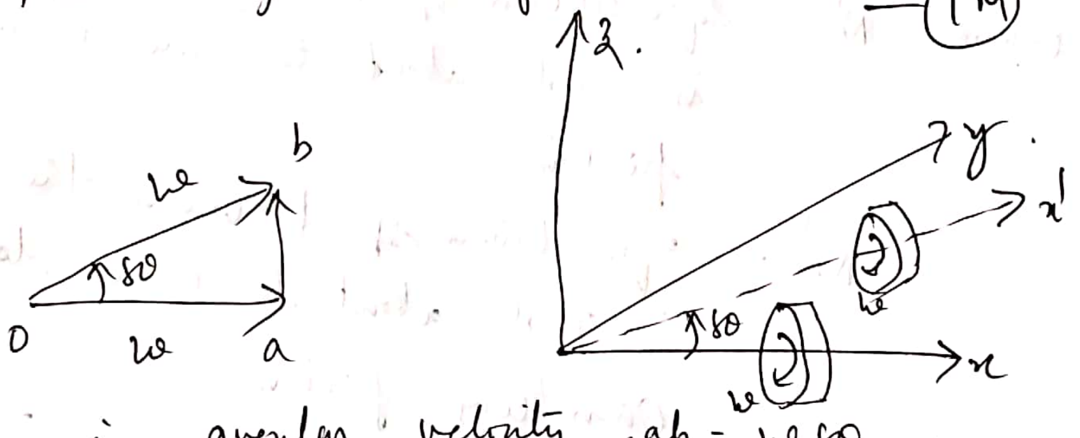
$$6\times200\times10^{-3}\times(\omega')^2 = \tan 41.81\left[6\times9.81 + \left(\frac{11\times9.81-40}{2}\right)(1+0.77)\right]$$

$\omega' = 20.30$ $\Rightarrow$ $\boxed{N' = 193.88 \text{ rpm}}$ — 1M

Co-efficient of insensitiveness $= \frac{N'' - N'}{N} = \frac{205.9-193.88}{200}$.

$\boxed{\text{Co-efficient of insensitiveness} = 0.06}$. — 1M

4. a) Gyroscopic couple is the couple acting on a spinning body, which possess angular acceleration due to change in direction of angular velocity, without any change in the magnitude of the angular velocity. — 1M



change in angular velocity, $ab = \omega\,\delta\theta$.

$$\alpha_p = \lim_{\delta t\to 0} \frac{\omega\,\delta\theta}{\delta t} = \omega\frac{d\theta}{dt} = \omega\cdot\omega_p.$$

Gyroscopic torque, $C = I\alpha_p = I\omega\omega_p$. — 3M

I = M.I of the spinning body ω = angular velocity of rotation ω_p = angular velocity of precession.

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b) $m = 6000\ kg \quad K = 500mm \quad N = 1820\ rpm$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1820}{60} = 190.59\ r/s \left(\xrightarrow{\omega}\right) - (1M)$$

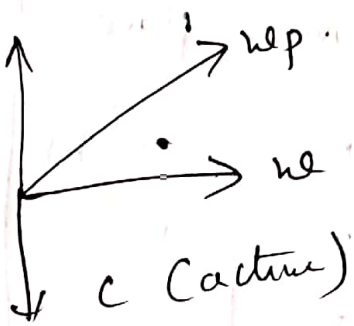
$\nearrow \omega_p \qquad \tau = 20s \qquad \phi = 10^\circ \quad ; \omega_0 = \frac{2\pi}{\tau} = 0.31\frac{r}{s}$

$$(\omega_p)_{max} = \phi\, \omega_0 \Rightarrow (\omega_p)_{max} = 10 \times \frac{\pi}{180} \times 0.31 = 0.054\ r/s. \quad - (1M)$$

$$C = I\omega(\omega_p)_{max} = 6000 \times (0.5)^2 \times 190.59 \times 0.054$$

$$C = 15.44 \times 10^3\ N\text{-}m.$$

$$\boxed{C = 15.44\ KN\text{-}m} - (2M)$$



the gyroscopic effect is to push the ship towards the port. — (2M)

5) $d = 75mm; \quad t = 25mm; \quad \rho = 7000\ kg/m^3$

$$m_1 = m_2 = m_3 = m = \rho \times v = \rho \times \pi/4\, d^2 \times t$$

$$m = 7000 \times \pi/4\, (75 \times 10^{-3})^2 \times 25 \times 10^{-3} = 0.77\ kg.$$

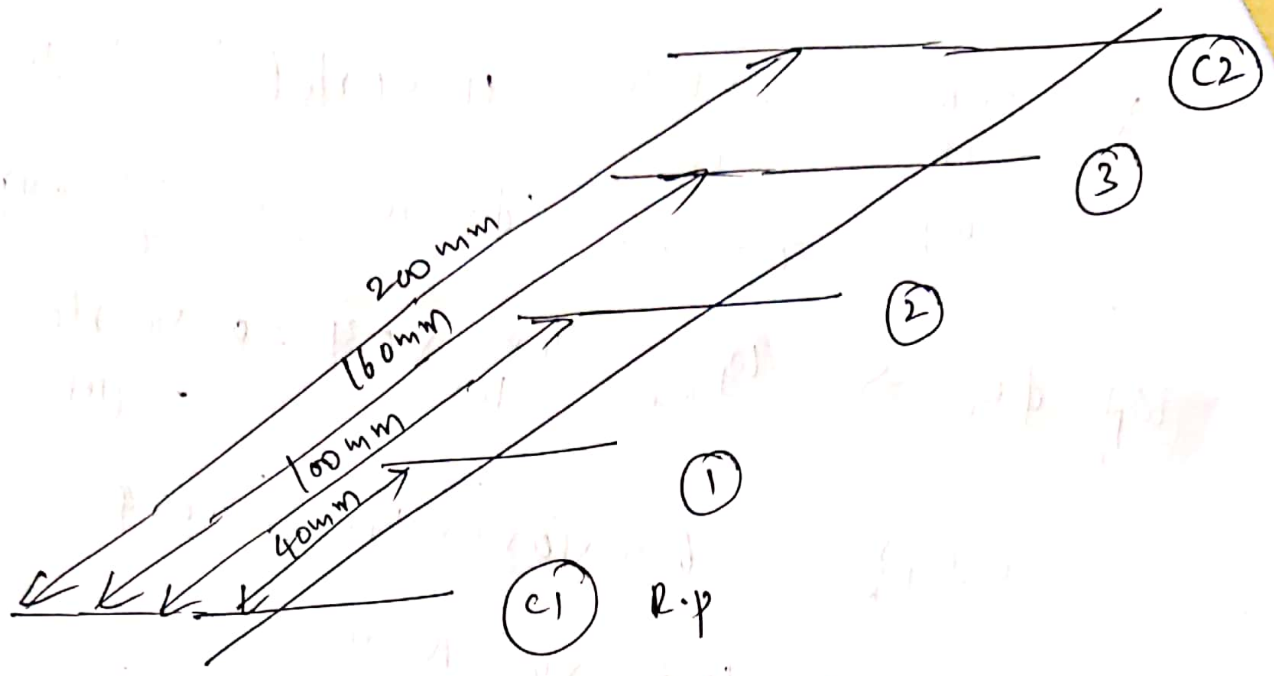
$r_1 = 12mm; \quad r_2 = 18mm; \quad r_3 = 12mm.$

$\theta_1 = 0^\circ \quad \theta_2 = 120^\circ \quad \theta_3 = 240^\circ$

~~$\omega = c2800$~~

$m_{c1} = ? \quad r_{c1} = 75mm; \quad \theta_{c1} = ?$

$m_{c2} = ? \quad r_{c2} = 75mm; \quad \theta_{c2} = ?$ — (2M)



S.NO	plane	m (kg)	r (mm)	θ (deg)	mr	l	mrl
1	C1 RP	m_{c1}	75	θ_{c1}	$75m_{c1}$	—	—
2	1	0.77	12	0	9.24	40	369.6
3	2	0.77	18	120	13.86	100	1386
4	3	0.77	12	240	9.24	160	1478.4
5	C2	m_{c2}	75	θ_{c2}	$75m_{c2}$	200	$15000m_{c2}$

2M

Couple balance

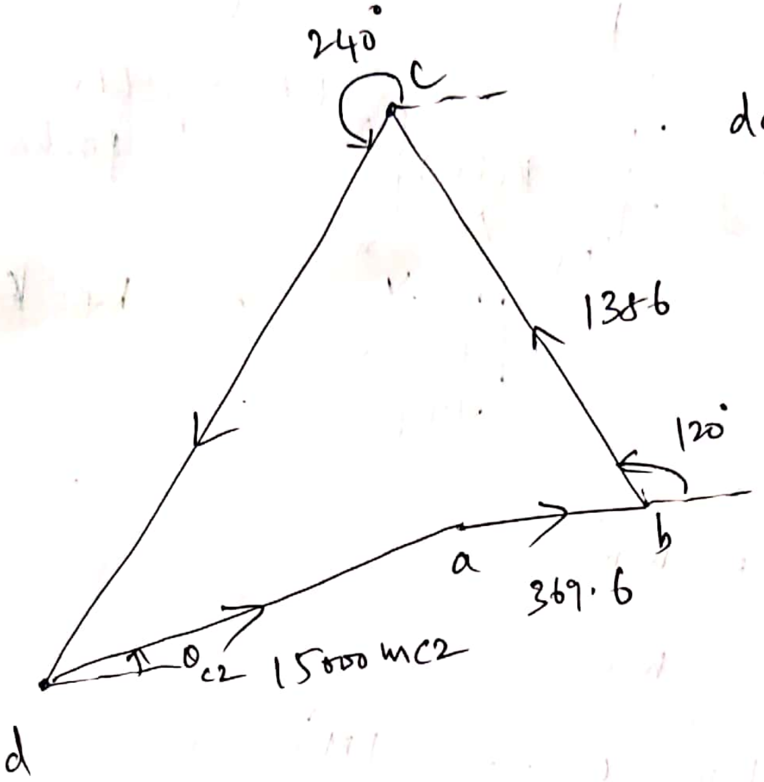
$$\Sigma m\vec{r}l + m_{c2}\vec{r}_{c2}l_{c2} = 0$$

Force balance

$$\Sigma m\vec{r} + m_{c2}\vec{r}_{c2} + m_{c1}\vec{r}_{c1} = 0$$

(1.1)

Couple polygon.



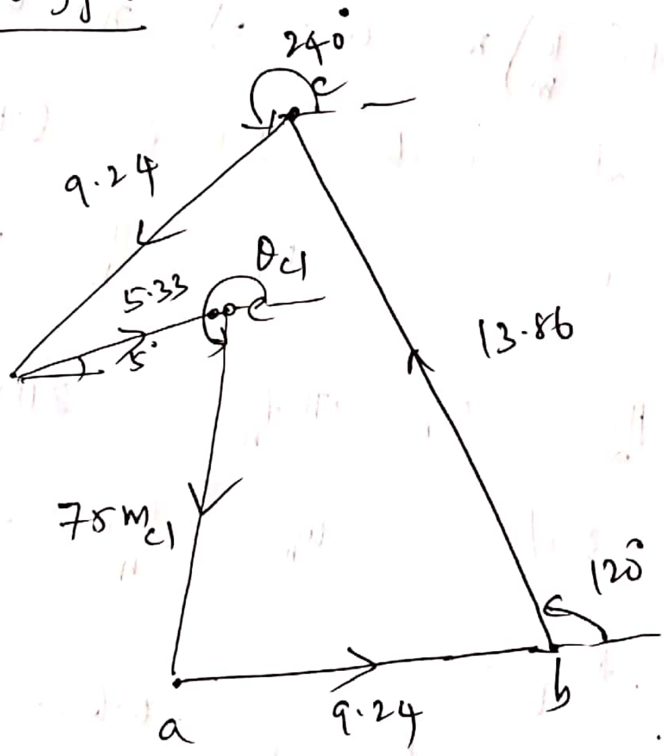
$da = 15000\, m_{c2}$

$\Rightarrow \boxed{m_{c2} = 0.071\,kg}$

$\boxed{\theta_{c2} = 5°}$

— 3M

Force polygon.

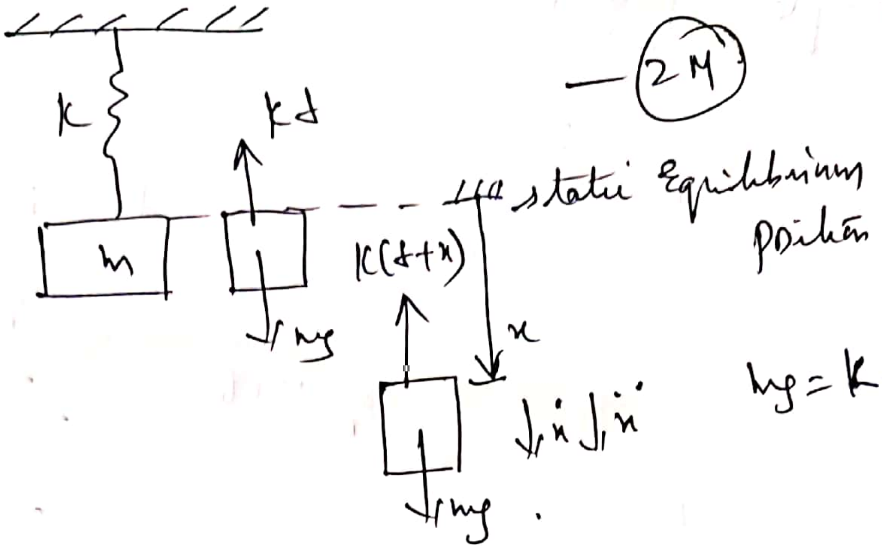


$ea = 75\, m_{c1}$

$\Rightarrow \boxed{m_{c1} = 0.069\,kg}$. $\boxed{\theta_{c1} = 236°}$

— 3M

6 a) (12)



— (2M)

$mg = k\delta$.

$\Sigma F_x = ma_x$

$mg - k(\delta + x) = m\ddot{x}$

$\cancel{mg} - \cancel{k\delta} - kx = m\ddot{x}$

$\Rightarrow \quad m\ddot{x} + kx = 0$. — (1M)

$\ddot{x} + \left(\sqrt{\frac{k}{m}}\right)^2 x = 0. \Rightarrow \ddot{x} + \omega_n^2 x = 0$.

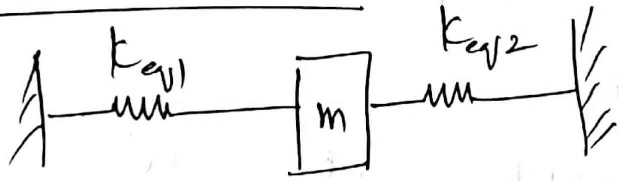
Comparing it with D.E of S.H.M.

$\ddot{x} + \omega^2 x = 0 \quad \Rightarrow \quad \omega_n = \sqrt{\frac{k}{m}}$. — (1M)

b) $f_n = 10\,Hz$ $\quad \omega_n = 2\pi f_n = 2\pi \times 10 = 62.83\ r/s$

$\omega_n = 62.83\ r/s$. $\quad \omega_n = \sqrt{\frac{K_{eq}}{m}}$ — (1M)

To find K_{eq}.



— (2M)

where K_{eq1} is the equivalent stiffness of springs in series (K_1, K_2, K_3)

K_{eq2} is the equivalent stiffness of springs in parallel (K_4, K_5)

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$k_1 = 2000\ N/m \quad k_2 = 1500\ N/m \quad k_3 = 3000\ N/m$

$$\frac{1}{k_{eq1}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \Rightarrow k_{eq1} = 666.67\ \frac{N}{m}.$$

— (1M)

$$k_{eq2} = k_4 + k_5 \Rightarrow k_{eq2} = 500 + 500 = 1000\ \frac{N}{m}.$$

— (1M)

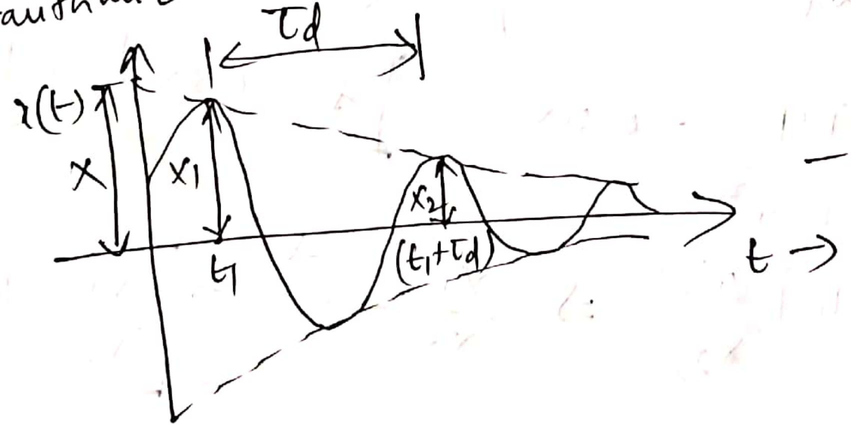
k_{eq} is the equivalent spring stiffness of k_{eq1} & k_{eq2} in parallel.

$$k_{eq} = k_{eq1} + k_{eq2} = 1666.67\ \frac{N}{m}.$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} \Rightarrow (62.83)^2 = \frac{1666.67}{m}$$

$$\Rightarrow \boxed{m = 0.42\ kg}$$ — (1M)

7. a) The natural logarithm of the ratio between any two successive amplitudes of a under damped vibratory system is known as logarithmic decrement. — (1M)



— (1M)

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$$\delta = \ln \frac{X_1}{X_2} = \ln \left(\frac{X e^{-\xi \omega_n t_1} \sin(\omega_n \sqrt{1-\xi^2}\, t_1) + \phi)}{X e^{-\xi \omega_n (t_1 + \tau_d)} \sin(\omega_n \sqrt{1-\xi^2}\, t_1 + \tau_d) + \phi} \right)$$

Since the value of sin is equal after one cycle (τ_d)

$$\delta = \ln \left(\frac{e^{-\xi \omega_n t_1}}{e^{-\xi \omega_n (t_1 + \tau_d)}} \right) = \ln \left(e^{\xi \omega_n \tau_d} \right)$$

$$\delta = \xi \omega_n \frac{2\pi}{\omega_d} = \xi \omega_n \frac{2\pi}{\omega_n \sqrt{1-\xi^2}} = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$\boxed{\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}}$$ — (2M) for small values of ξ, $\delta = 2\pi\xi$.

b) $m = 3\,kg$; $k = 100\, N/m$ $c = 3\, \frac{N\text{-}s}{m}$

$\xi = \frac{c}{C_c}$ where $C_c = 2\sqrt{km} = 34.64$.

$$\xi = \frac{3}{34.64} = 86.6 \times 10^{-3}$$

$\boxed{\xi = 86.6 \times 10^{-3}}$ — $1\frac{1}{2}$M

$$\omega_d = \omega_n \sqrt{1-\xi^2} = \sqrt{\frac{k}{m}} \sqrt{1-\xi^2} = 5.75\, r/s$$

$\boxed{\omega_d = 5.75\, r/s}$ — $1\frac{1}{2}$M

$\boxed{\delta = 0.55}$ — $1\frac{1}{2}$M

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = 0.55$$

$$\delta = \ln \left(\frac{X_1}{X_2} \right) \Rightarrow \frac{X_1}{X_2} = e^{\delta} \Rightarrow \boxed{\frac{X_1}{X_2} = 1.73}$$ — $1\frac{1}{2}$M

a) $m = 490$ kg ; $k = 58800 \frac{N}{m}$

$Y = 40$ mm $\lambda = 4.0$ m

~~R-effect of~~ Critical speed = ?

$\omega_n = \sqrt{\frac{k}{m}} = 10.95$ r/s $f_n = \frac{\omega_n}{2\pi} = 1.74$ Hz.

when excitation frequency coincides with the natural frequency, the speed of the vehicle is critical.

$V_c = f_n \lambda \Rightarrow V_c = 1.74 \times 4 = 6.96$ m/s.

$V_c = 6.96 \times \frac{18}{5} = 25$ kmph. $\boxed{V_c = 25 \text{ kmph}}$ — 2M

$\frac{X}{Y} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$ here $r = \frac{\omega}{\omega_n} = 1$; $\xi = 0.5$

$\frac{X}{40} = \frac{\sqrt{1+(2\xi)^2}}{2\xi} \Rightarrow \boxed{X = 56.57 \text{ mm}}$ — 1M

$f = \frac{V}{\lambda} = \frac{57 \times \frac{5}{18}}{4} = 3.96$ Hz $\Rightarrow \omega = 2\pi f$.

$\omega = 24.88$ r/s.

$r = \frac{\omega}{\omega_n} = \frac{24.88}{10.95} = 2.27$.

$\frac{X}{Y} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}} = \frac{2.48}{4.73} \Rightarrow \boxed{X = 20.97 \text{ mm}}$.

— 2M

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b) $m = 100$ kg ; $k = 700 \times 10^3$ $\frac{N}{m}$

$F_0 = 350N$ $N = 3000$ rpm , $\zeta = 0.20$.

$$\frac{mX}{m_0 e} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$\omega = \frac{2\pi \times 3000}{60} = 314.16 \text{ r/s}.$$

$F_0 = m_0 e \omega^2 \Rightarrow m_0 e = 3.55 \times 10^{-3}$ kg-m .

$\omega_n = \sqrt{\frac{k}{m}} = 83.67$ r/s , $r = \frac{\omega}{\omega_n} = 3.75$

$$\frac{100 X}{3.55 \times 10^{-3}} = \frac{3.75^2}{\sqrt{(1-3.75^2)^2 + (2 \times 0.2 \times 3.75)^2}}$$

$X = 3.8 \times 10^{-5}$ m $\Rightarrow$ $\boxed{X = 0.038 \text{ mm}}$ — 2M

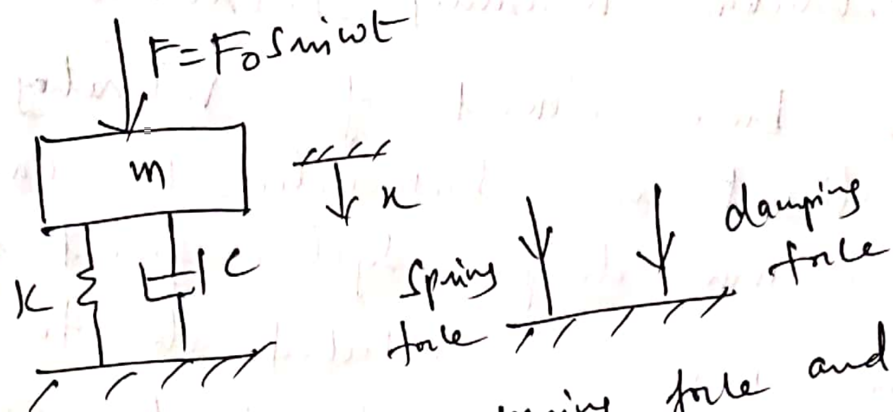
$$T.R = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{1.8}{13.15} \Rightarrow \boxed{T.R = 136.88 \times 10^{-3}}$$ — 2M

$T.R = \frac{F_{tr}}{F_0} \Rightarrow F_{tr} = T.R \times F_0 = 47.9$ N .

$\boxed{F_{tr} = 47.9 \text{ N}}$ — 1M

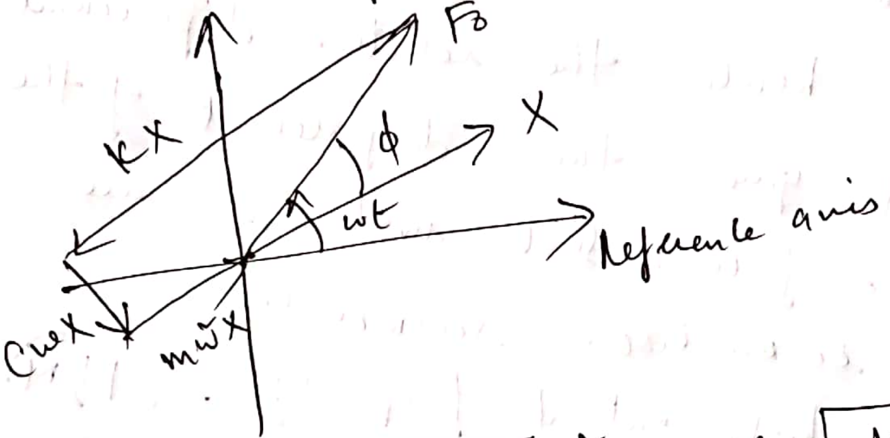
Scanned with CamScanner

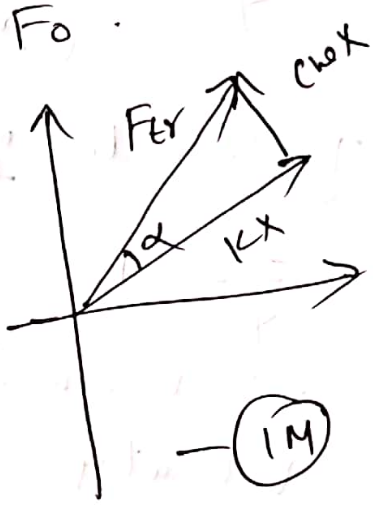
a)



The resultant of the spring force and damping force transmitted to the support is the transmitted force.

$$T.R = \frac{\text{transmitted force}}{\text{impressed force}} = \frac{F_{tr}}{F_o}. \quad \text{— (2M)}$$





— (1M)

$$F_{tr} = \sqrt{(KX)^2 + (CwX)^2} = X\sqrt{K^2 + (Cw)^2}$$

we know that $X = \dfrac{F_o}{\sqrt{(K - m\omega^2)^2 + (C\omega)^2}}$

$$F_{tr} = \frac{F_o\sqrt{K^2 + (C\omega)^2}}{\sqrt{(K - m\omega^2)^2 + (C\omega)^2}} \Rightarrow \frac{F_{tr}}{F_o} = T.R = \frac{\sqrt{1 + \left(\frac{C\omega}{K}\right)^2}}{\sqrt{\left(1 - \frac{m\omega^2}{K}\right)^2 + \left(\frac{C\omega}{K}\right)^2}}$$

$$T.R = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1 - r^2)^2 + (2\xi r)^2}} \quad \text{— (2M)}$$

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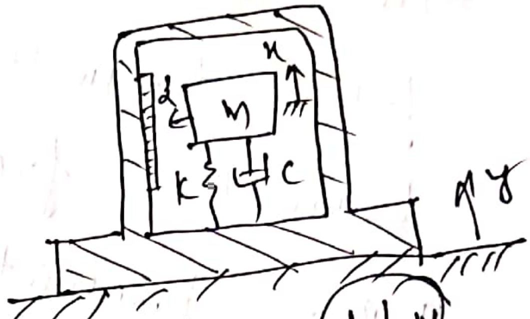
b) Vibration measuring Instruments

- The basic element of a vibration measuring instrument is a Seismometer. It consists of a spring-mass-damper system placed in a housing.
- The housing is attached to the surface whose vibrational characteristics are to be measured.



$$\frac{z}{Y} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \quad \text{— (2M)}$$

Instruments of low natural frequency — $(1\frac{1}{2}M)$

$\frac{z}{Y} \to 1$; hence the relative displacement 'z' measured gives the displacement of the vibrating surface. Such instruments are known as ~~seismoteers~~. seismometers.

Instruments of high natural frequency — $(1\frac{1}{2}M)$

$\frac{z}{Y} = \frac{\omega}{\omega_n^2}$ since $\sqrt{(1-r^2)^2 + (2\zeta r)^2} \to 1$

$z = \frac{\omega^2 Y}{\omega_n^2}$. Thus z becomes proportional to the acceleration of the motion to be measured with a factor $\frac{1}{\omega_n^2}$.

Prepared by

(C.D. Harayana chandary)