III/IV B.Tech (Regular) DEGREE EXAMINATION
OPERATIONS RESEARCH
18MED11

----Scheme of Evaluation----

PART-A

Each question carries 1 mark **(1 X 10=10M)**

1.

a.

Operations Research approach comprises the following seven sequential steps: (1) Orientation, (2) Problem Definition, (3) Data Collection, (4) Model Formulation, (5) Solution, (6) Model Validation and Output Analysis, and (7) Implementation and Monitoring.

b.

Objective Function – In a problem, the objective function should be specified in a quantitative way. Linearity – The relationship between two or more variables in the function must be linear. ... Non-negativity – The variable value should be positive or zero.

c.

In order to use the simplex method on problems with mixed constraints, we turn to a device called an artificial variable. This variable has no physical meaning in the original problem and is introduced solely for the purpose of obtaining a basic feasible solution so that we can apply the simplex method.

d.

Balanced transportation problem : In this problem supply equal to total demand. Unbalanced transportation problem: In this problem supply is not equal to total demand. It can may be less or greater.

e.

In a standard transportation problem with m sources of supply and n demand destinations, the test of optimality of any feasible solution requires allocations in m + n - 1 independent cells. If the number of allocations is short of the required number, then the solution is said to be degenerate.

f.

Reneging occurs when customers in a queueing system choose to leave the system prior to receiving service.

g.

There are three parts to a service line system: the number of servers, arrival time of customers, and the waiting line rules. The waiting line can be finite, where only a certain number of customers can be waiting at any given time, or infinite, where it doesn't matter.

h.

In a zero-sum matrix game, an outcome is a saddle point if the outcome is a minimum in its row and maximum in its column.

i.

Simulation is a numerical technique for conducting experiments that involve certain types of mathematical and logical relationships necessary to describe the behaviour and structure of a complex real world system over extended period of time.

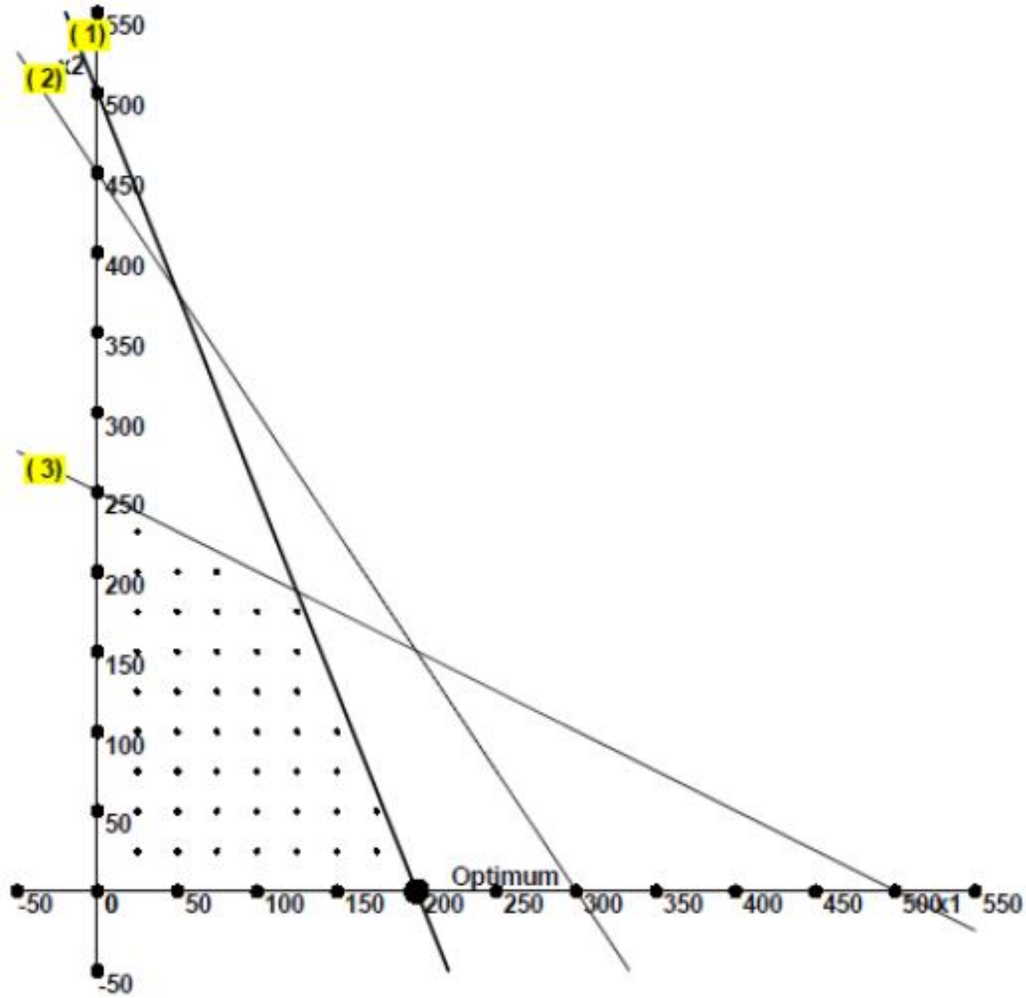
j.

Congruential method is described by the expression: r_i+1= ar_i+b ($modulo$,)
Where a, b and m are constants. r_i and r_i+1 are the i th and (i+1)th random numbers.
The expression implies multiplication of a by $ri$and addition of b and then division by m. Then r_i+1 is the remainder or residue. To begin the process of random number generation in addition to a, b and m, the value of r_0 is also required. It may be any random number and is called *seed*.
The above expression is for a mixed type congruential method as it comprises both multiplication of a and ri and addition of ar_i and b.

PART-B

2.

	x1	x2		
Maximize	100.00	40.00		
Subject to				
(1)	5.00	2.00	<=	1000.00
(2)	3.00	2.00	<=	900.00
(3)	1.00	2.00	<=	500.00
Lower Bound	0.00	0.00		
Upper Bound	infinity	infinity		
Unrestr'd (y/n)?	n	n		



Problem solving & Graphical Presentation= 5M+5M

3.

Solution:

	x1	x2		
Maximize	3.00	-1.00		
Subject to				
(1)	2.00	1.00	>=	2.00
(2)	1.00	3.00	<=	3.00
(3)	0.00	1.00	<=	4.00
Lower Bound	0.00	0.00		
Upper Bound	infinity	infinity		
Unrestr'd (y/n)?	n	n		

Iteration 1						
Basic	x1	x2	Sx3	Rx4	sx5	sx6
z (max)	-203.00	-99.00	100.00	0.00	0.00	0.00
Rx4	2.00	1.00	-1.00	1.00	0.00	0.00
sx5	1.00	3.00	0.00	0.00	1.00	0.00
sx6	0.00	1.00	0.00	0.00	0.00	1.00
Lower Bound	0.00	0.00				
Upper Bound	infinity	infinity				
Unrestr'd (y/n)?	n	n				

Basic	Solution
z (max)	-200.00
Rx4	2.00
sx5	3.00
sx6	4.00

Iteration 2						
Basic	x1	x2	Sx3	Rx4	sx5	sx6
z (max)	0.00	2.50	-1.50	101.50	0.00	0.00
x1	1.00	0.50	-0.50	0.50	0.00	0.00
sx5	0.00	2.50	0.50	-0.50	1.00	0.00
sx6	0.00	1.00	0.00	0.00	0.00	1.00
Lower Bound	0.00	0.00				
Upper Bound	infinity	infinity				
Unrestr'd (y/n)?	n	n				

Basic	Solution
z (max)	3.00
x1	1.00
sx5	2.00
sx6	4.00

Iteration 3							
Basic	x1	x2	Sx3	Rx4	sx5	sx6
z (max)	0.00	10.00	0.00	100.00	3.00	0.00
x1	1.00	3.00	0.00	0.00	1.00	0.00
Sx3	0.00	5.00	1.00	-1.00	2.00	0.00
sx6	0.00	1.00	0.00	0.00	0.00	1.00
Lower Bound	0.00	0.00				
Upper Bound	infinity	infinity				
Unrestr'd (y/n)?	n	n				

Basic	Solution
z (max)	9.00
x1	3.00
Sx3	4.00
sx6	4.00

Ans:

Max Z= 9; x1=3, x2=0

Problem solving & Answer= 8M+2M

4.

Since the given problem is of maximization type, first convert the problem in to a minimization problem by subtracting the cost elements (entries or cij) from the highest cost element (cij=44) in the given TP.

Then the given problem becomes;

FACTORY					CAPACITY (SUPPLY)
	A	B	C	D	
1	4	19	22	11	100
2	0	9	14	14	30
3	6	6	16	14	70
REQUIREMENT (DEMAND)	40	20	60	30	

	Name	D1 A	D2 B	D3 C	D4 D	D5 DummyD	**Supply**
S1	1	4.00	19.00	22.00	11.00	0.00	**100.00**
S2	2	0.00	9.00	14.00	14.00	0.00	**30.00**
S3	3	6.00	6.00	16.00	14.00	0.00	**70.00**
Demand		**40.00**	**20.00**	**60.00**	**30.00**	**50.00**	

ObjVal **1810.00**

	Name		D1 A v1=4.00	D2 B v2=19.00	D3 C v3=22.00	D4 D v4=11.00	D5 DummyD v5=13.00	**Supply**
S1	1	u1=0.00	4.00 **10** 0.00	19.00 **0** 0.00	22.00 **60** 0.00	11.00 **30** 0.00	0.00 13.00	**100**
S2	2	u2=-4.00	0.00 **30** 0.00	9.00 6.00	14.00 4.00	14.00 -7.00	0.00 9.00	**30**
S3	3	u3=-13.00	6.00 -15.00	6.00 **20** 0.00	16.00 -7.00	14.00 -16.00	0.00 **50** 0.00	**70**
	Demand		**40**	**20**	**60**	**30**	**50**	

ObjVal **1810.00**

	Name		D1 A v1=4.00	D2 B v2=6.00	D3 C v3=22.00	D4 D v4=11.00	D5 DummyD v5=0.00	**Supply**
S1	1	u1=0.00	4.00 **10** 0.00	19.00 -13.00	22.00 **60** 0.00	11.00 **30** 0.00	0.00 **0** 0.00	**100**
S2	2	u2=-4.00	0.00 **30** 0.00	9.00 -7.00	14.00 4.00	14.00 -7.00	0.00 -4.00	**30**
S3	3	u3=0.00	6.00 -2.00	6.00 **20** 0.00	16.00 6.00	14.00 -3.00	0.00 **50** 0.00	**70**
	Demand		**40**	**20**	**60**	**30**	**50**	

:	ObjVal	1510.00						
	Name		D1 A v1=4.00	D2 B v2=12.00	D3 C v3=22.00	D4 D v4=11.00	D5 DummyD v5=0.00	**Supply**
S1	1	u1=0.00	4.00 **10** 0.00	19.00 -7.00	22.00 **10** 0.00	11.00 **30** 0.00	0.00 **50** 0.00	**100**
S2	2	u2=-4.00	0.00 **30** 0.00	9.00 -1.00	14.00 4.00	14.00 -7.00	0.00 -4.00	**30**
S3	3	u3=-6.00	6.00 -8.00	6.00 **20** 0.00	16.00 **50** 0.00	14.00 -9.00	0.00 -6.00	**70**
	Demand		**40**	**20**	**60**	**30**	**50**	

:	ObjVal	1470.00						
	Name		D1 A v1=4.00	D2 B v2=8.00	D3 C v3=18.00	D4 D v4=11.00	D5 DummyD v5=0.00	**Supply**
S1	1	u1=0.00	4.00 **20** 0.00	19.00 -11.00	22.00 -4.00	11.00 **30** 0.00	0.00 **50** 0.00	**100**
S2	2	u2=-4.00	0.00 **20** 0.00	9.00 -5.00	14.00 **10** 0.00	14.00 -7.00	0.00 -4.00	**30**
S3	3	u3=-2.00	6.00 -4.00	6.00 **20** 0.00	16.00 **50** 0.00	14.00 -5.00	0.00 -2.00	**70**
	Demand		**40**	**20**	**60**	**30**	**50**	

The current solution is optimal and unique. The optimum allocation schedule is given by:

$$x_{11} = 20, \quad x_{14} = 30, \qquad x_{15} = 50, x_{21} = 20, x_{23} = 10, x_{32} = 20, x_{33} = 50$$

The optimum profit is given by

: (20x40)+(30x33)+(50x0)+(20x44)+(10x30)+(20x38)+(50x28)=Rs 5130.

Procedure, Problem solving & Answer= 4M+4M+2M

5.

Step 1 Subtract the least element of each row from all the elements of the respective row. We get

	J_1	J_2	J_3	J_4
W_1	0	5	14	20
W_2	6	10	18	0
W_3	0	6	18	4
W_4	0	15	23	9

Subtract the least element of each column from all the elements of the respective column. We have

	J_1	J_2	J_3	J_4
W_1	0	0	0	20
W_2	6	5	4	0
W_3	0	1	4	4
W_4	0	10	9	9

Step 2
All the zeroes can be covered by a minimum of 3 lines as shown below.

	J_1	J_2	J_3	J_4
W_1	0	0	0	20
W_2	6	5	4	0
W_3	0	1	4	4
W_4	0	10	9	9

Since the number of lines is 3 < 4 we go to step 3.

Step 3
The least element not covered by any line is 1. Subtract 1 from all the elements not on the

lines and add 1 to the elements at the points of intersection of the lines. We obtain

	J_1	J_2	J_3	J_4
W_1	1	0	0	20
W_2	7	5	4	0
W_3	0	0	3	3
W_4	0	9	8	8

Now we find that the minimum number of lines required to cover all the zeroes is 4 (n = 4). Hence an optimal solution is reached.

Step 4

In the first row there are two zeroes. Therefore take the second row and there is only one zero in (2, 4) cell. Squaring it we have

	J_1	J_2	J_3	J_4
W_1	1	0	0	20
W_2	7	5	4	$\boxed{0}$
W_3	0	0	3	3
W_4	0	9	8	8

The third row has two zeroes. Go to the fourth row. Square the only zero in (4, 1) cell and cancel the zero in the first column (3, 1) cell. We have

	J_1	J_2	J_3	J_4
W_1	1	0	0	20
W_2	7	5	4	$\boxed{0}$
W_3	$\cancel{0}$	0	3	3
W_4	$\boxed{0}$	9	8	8

Now examine the columns. In the first column one zero has been squared already. In the second column there are two zeroes. Go to the third column. There is only one zero in the third column (1, 3) cell. Square it and cancel the zero in (1, 2) cell. This gives

	J_1	J_2	J_3	J_4
W_1	1	$\cancel{0}$	$\boxed{0}$	20
W_2	7	5	4	$\boxed{0}$
W_3	$\cancel{0}$	0	3	3
W_4	$\boxed{0}$	9	8	8

Now there is only one zero in the third row (or second column) (3, 2) cell. Squaring that zero also we get the optimal solution table.

	J_1	J_2	J_3	J_4
W_1	1	$\cancel{0}$	$\boxed{0}$	20
W_2	7	5	4	$\boxed{0}$
W_3	$\cancel{0}$	$\boxed{0}$	3	3
W_4	$\boxed{0}$	9	8	8

The optimal assignment is

$W_1 \rightarrow J_3$: 24

$W_2 \rightarrow J_4$: 10

$W_3 \rightarrow J_2$: 18

$W_4 \rightarrow J_1$: 9

Total = 61 Optimum cost = 61

Procedural steps, Problem solving & Answer= 4M+4M+2M

6.

Here $\lambda = \frac{96}{24} = 4\, patients/hr$

$\mu = \frac{1}{10} \times 60 = 6\, patients/hr$

Average number of patients in the queue

$$L_q = \frac{\lambda}{\mu} . \frac{\lambda}{\mu - \lambda} = \frac{4}{6} . \frac{4}{(6-4)} = \frac{4}{3} = 1\frac{1}{3}$$

This number is to be reduced from $1\frac{1}{3}$ to $\frac{1}{2}$ (L_q'). This can be achieved by increasing the service rate to, say μ'.

$$\text{Now, } L_q' = \frac{\lambda}{\mu'} . \frac{\lambda}{(\mu' - \lambda)}$$

$$\frac{1}{2} = \frac{4}{\mu'} . \frac{4}{\mu' - 4}$$

$$\text{Or } \mu'^2 - 4\mu' - 32 = 0 \; or \; (\mu' - 8)(\mu' + 4) = 0$$

$$\mu' = 8\, patients\, /\, hr$$

Therefore, average time required by each patient = 1/8 hr = 15/2 minutes.

Decrease in the time required to attend a patient = $10 - \frac{15}{2} = \frac{5}{2}\, minutes$

Therefore, the budget required for each patient = Rs $(100 + \frac{5}{2} \text{x} 10)$ = Rs 125.

Determination of Lq, ρ, Average time, time reqd and budget reqd=2M+2M+2M+2M+2M

7.a)

Solution:

	B_1	B_2	B_3	B_4
A_1	1	7	3	4
A_2	5	6	4	5
A_3	7	2	0	3

$$\begin{array}{c|cccc|c} & B_1 & B_2 & B_3 & B_4 & \text{Row minimum} \\ A_1 & 1 & 7 & 3 & 4 & 1 \\ A_2 & 5 & 6 & 4 & 5 & 4 \\ A_3 & 7 & 2 & 0 & 3 & 0 \\ \text{Col. max} & 7 & 7 & 4 & 5 & \end{array}$$

Maximin value = 4 = Minimax value = $\underline{v} = v = \overline{v}$

The value of the game is 4. Saddle point is (2, 3). Optimal strategy of A is A_2 and that of B is B_3.

Saddle point identification & game value: 3M+2M

7 b)

Solution:

Arrival rate $\lambda = \frac{15}{8\times60} = \frac{1}{32}$ units /minute

Service rate $\mu = \frac{1}{20}$ units/minute

Number of jobs ahead of the cell phone brought in = average number of jobs in the system

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{1/32}{1/20 - 1/32} = \frac{5}{3}$$

Number of hours for which the repairman remains busy in an 8-hour day

$$= 8\frac{\lambda}{\mu} = 8 \times \frac{1/32}{1/20} = 8 \times \frac{20}{32} = 5\ hours.$$

Therefore, Time for which repairman remains idle in an 8-hour day= 8-5 = 3 hours.

Determination of ρ, L_s No. of busy hours and idle time=1M+1M+1M+1M+1M

8.

Here we have two stages for two variables.

Stage 1

$$f_1(B_{11}\ B_{21}) = Max\ (c_1 x_1)$$

$$= Max(3x_1) = 3\ Max(x_1)$$

From the constraints we find that

$$x_1 \leq \frac{120 - 5x_2}{2} \quad and$$

$$x_1 \leq \frac{40 - x_2}{2}$$

In order to satisfy both the constraints

$$x_1 \leq min\left\{\frac{120 - 5x_2}{2}, \qquad (40 - x_2)/2\right\}$$

$$\therefore f_1(B_{11}\ B_{21}) = 3min\left\{\frac{120 - 5x_2}{2}, \qquad (40 - x_2)/2\right\}$$

Stage 2

$$f_2(B_{12}\ B_{22}) = Max\,\{f_1(B_{11}\ B_{21}) + 4x_2\}$$

$$= Max\left[3\min\left\{\frac{120-5x_2}{2},\frac{(40-x_2)}{2}\right\} + 4x_2\right]$$

$$= max\begin{cases}\dfrac{3(120-5x_2)}{2} + 4x_2\ \ if\ \dfrac{120-5x_2}{2} \le (40-x_2)/2 \\ \dfrac{3(40-x_2)}{2} + 4x_2\ \ if\ \dfrac{120-5x_2}{2} \ge (40-x_2)/2\end{cases}$$

Now $\frac{120-5x_2}{2} \le \frac{40-x_2}{2}$

$$\Rightarrow 120 - 5x_2 \le 40 - x_2$$

$$\Rightarrow 80 \le 4x_2\ \ \Rightarrow x_2 \ge 20$$

$$\frac{120-5x_2}{2} \ge \frac{40-x_2}{2}$$

$$120 - 5x_2 \ge 40 - x_2$$

$$\Rightarrow x_2 \le 20$$

When $x_2 = 20$

$$\frac{3(120-5x_2)}{2} + 4x_2\ = \frac{3(40-x_2)}{2} + 4x_2 = 110$$

$$\therefore f_2(120, 40) = 110$$

$$x_2{}^* = 20\ \ and\ x_1{}^* = (40 - x_2{}^*)/2 =\ 10$$

Solution is $x_1 = 10, x_2 = 20\ \ \ and\ Z^* = 110$

Procedure (Stage 1 stage 2) and solution: 3M+4M+3M

9.

Solution:

Demand	*Probability*	*Cumulative Probability*	*Random number Interval (tag no)*
0	0.01	0.01	00-00
15	0.15	0.16	01-15
25	0.20	0.36	16-35
35	0.50	0.86	36-85
45	0.12	0.98	86-97
50	0.02	1.00	98-99

Day	*Random Number*	*Demand*
1	48	35
2	78	35
3	09	15
4	51	35
5	56	35
6	77	35
7	15	15
8	14	15
9	68	35
10	09	15

The number of cakes demanded in the next 10 days is given by 35, 35, 15, 35, 35, 35, 15, 15, 35 and 15.

The stock situation for various days, if the decision is made to make 35 cakes every day is given by the table below:

Day	*Demand*	*No of cakes made*	*Stock situation*
1	35	35	-
2	35	35	-
3	15	35	20
4	35	35	20
5	35	35	20
6	35	35	20
7	15	35	40
8	15	35	60
9	35	35	60
10	15	35	80

The average daily demand$= \frac{35+35+15+35+35+35+15+15+35+15}{10} = \frac{270}{10} = 27\ cakes.$

Determination of 10 days demand:4M
Stock situation: 3M
Determination of daily demand: 3M

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