18CS502

Hall Ticket Number:

III/IV B.Tech(Regular) DEGREE EXAMINATION

February, 2021 **CSE**

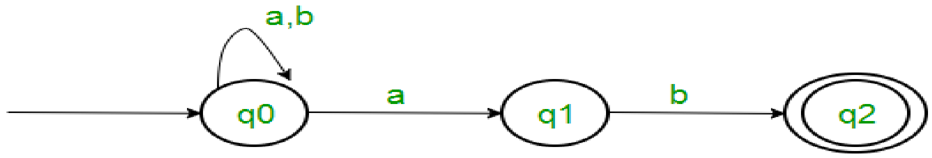
First Semester

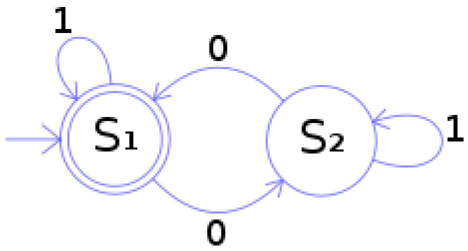
Automata Theory and Formal Languages

Time: Three Hours **Maximum:**50 Marks

Answer Question No.1 compulsorily. (10X1 = 10 Marks)

Answer ONE question from each unit. (4X10=40 Marks)

1.		Answer all questions		(10X1=10 Marks)
	a)	Give formal definition of DFA.	CLO-1	1M
	b)	Define epsilon closure of the state.	CLO-1	1M
	c)	What is the use of epsilon moves?	CLO-1	1M
	d)	Define regular language and regular expression.	CLO-2	1M
	e)	List out closure properties of regular languages.	CLO-2	1M
	f)	State pumping lemma for regular languages.	CLO-3	1M
	g)	Give formal definition of CFG.	CLO-3	1M
	h)	Give formal definition of PDA.	CLO-4	1M
	i)	Define CNF.	CLO-4	1M
	j)	Give formal definition of Turing machines.	CLO-4	1M
		UNIT I		
2.	a)	Construct DFA that accepts the language which contains any number of a's followed by at least 2 b's followed by exactly 3 c's followed by at most 2 d's.	CLO-1	5M
	b)	Prove that if L is accepted by NFA then there is DFA that accepts same language L.	CLO-1	5M
		(OR)		
3.	a)	Convert the following NFA to DFA  Figure 1	CLO-1	6M
	b)	Construct NFA that accepts the language which contains set of strings with 10^{th} symbol from the left end is 1 and 8^{th} symbol from the left end is 0.	CLO-1	4M
		UNIT II		
4.	a)	Find out the regular expression represented by the following DFA by using transitive closure method.	CLO-2	5M

			
	b)	Give epsilon NFA for the regular expression **(abb+ba)(a+b)*** . CLO-2	5M
		l(OR)	
5.	a)	Prove that **L={a^n b^n $\mid$ n≥0 }** is not regular. CLO-2	4M
	b)	Give regular expressions for the following languages. CLO-2 i. Set of strings contains any number of a's followed by at least one b followed by exactly 2 c's. ii. Set of strings contains even number of a's followed by odd number of b's iii. Set of strings end with 10.	6M
		UNIT III	
6.	a)	Give left most and right most derivations of the string "aabbabba" to the following grammar CLO-3 $S \rightarrow aB \mid bA$ $A \rightarrow a \mid aS \mid bAA$ $B \rightarrow b \mid bS \mid aBB$	5M
	b)	Convert the following grammar to PDA. CLO-3 $I \rightarrow a \mid b \mid Ia \mid Ib \mid I0 \mid I1$ $E \rightarrow E+E \mid E*E \mid (E) \mid I$	5M
		(OR)	
7.	a)	Construct CFG for the language **L = { $a^i b^j c^k$ $\mid$ i=j or j=k }** CLO-3	5M
	b)	Construct PDA to accept the language **L ={ wcw^R $\mid$ w∈{a,b}*}** CLO-3	5M
		UNIT IV	
8.		Convert the following grammar into CNF. CLO-3 $S \rightarrow AACD$ $A \rightarrow aAb \mid \in$ $C \rightarrow aC \mid a$ $D \rightarrow aDa \mid bDb \mid \in$	10 M
			5M
		(OR)	
9.	a)	Write short notes on closure properties and decision properties of CFLs. CLO-4	4M
	b)	Construct Turing machine to the language **L = { $a^n b^n c^n$ $\mid$ n>0}** CLO-4	6M

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Scheme

1\. (a). $M = (Q, \Sigma, \delta, q_0, F)$ Formal definition of DFA (1M)

Q – finite set of states
Σ – input symbols
δ – transition function
mapping from $\boxed{\delta : Q \times \Sigma \rightarrow Q}$
q_0 – initial state
F – set of final states

(b). <u>ϵ-closure(q)</u> It is defined as set of states p such that there is a path from state q to some state p with label ϵ. (1M)

(c). <u>Uses of ϵ-moves</u> For some languages it is difficult or impossible to draw DFA or NFA. At that time it is easy to go for NFA with ϵ-moves and then go for NFA without ϵ-moves. (1M)

(d). <u>Regular Languages & Regular expressions</u> Languages represented by finite automata are called regular languages. These languages represented by simple expressions called regular expressions. (1M).

e. properties of regular languages 1M.
1\. Union 2. Closure 3. Concatenation
4\. Intersection 5. Complement 6. difference.

f. pumping lemma for regular languages. (Con...)

Let L be the regular language, n is a constant such that $|z| \geq n$ for any string z in L. $z = uvw$ such z can be broken into three parts that

1. $|uv| \leq n$
2. $|v| \neq 0$
3. for all k $uv^kz \in L$.

g. Formal definition of CFG (1m)

$$G = (V, T, P, S)$$

V: set of variables representing set of strings

T: set of terminals or input symbols

P: set of recursive rules

S: start symbol of the grammar.

h. Give formal definition of PDA (1m)

$$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

Q - set of states

Σ - set of input symbols

Γ - set of stack symbols

δ - $\delta(q, a, X) = (p, \gamma)$

q_0 - initial state

Z_0 - initial stack symbol

F - set of final states

i. Define CNF. (1m)

A grammar is said to be in CNF if all of its productions are of the form

$$A \to BC \mid a$$

A, B, C are nonterminals

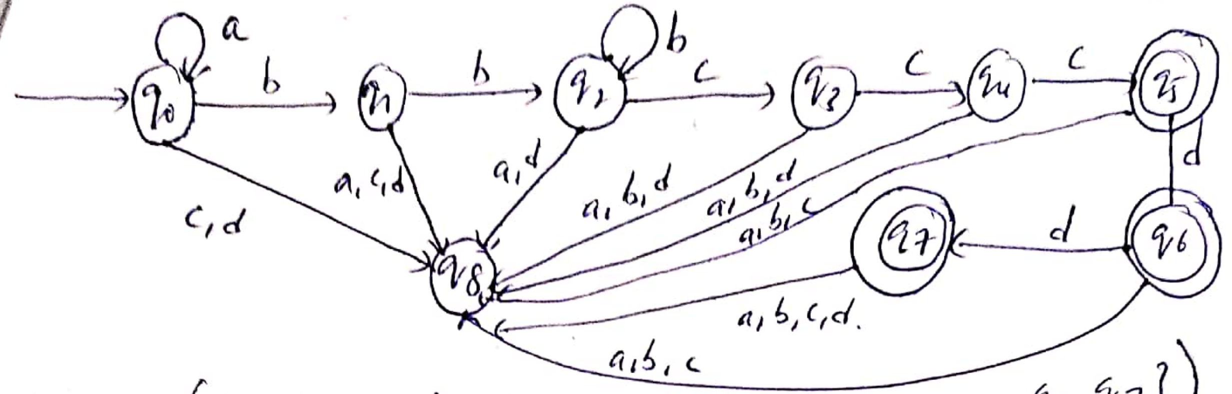
a is terminal

j. Formal definition of Turing Machine (1m)

$$M = (Q, \Sigma, \underset{\text{tape symbols}}{\Gamma}, \delta, q_0, \underset{\text{Blank symbol}}{B}, F) \quad \delta(q, X) = (p, Y, D)$$

Scanned with CamScanner

Construct DFA that accepts the language containing any number of a's followed by at least 2 b's followed by exactly 3 c's followed by at most 2 d's. (5M)



$M = (\{q_0, q_1 \cdots q_8\}, \{a,b,c,d\}, \delta, q_0, \{q_5, q_6, q_7\})$

δ is defined as

$\delta(q_0, a) = q_0$

$\delta(q_0, b) = q_1$

$\vdots$

δ	a	b	c	d
$\to q_0$	q_0	q_1	q_8	q_8
q_1	q_8	q_2	q_3	q_8
$\vdots$	$\vdots$			
q_7	q_8	q_8	q_8	q_8
q_8	q_8	q_8	q_8	q_8

2b. prove that L is accepted by NFA. then there is DFA that accepts L. (5M)

Let $M = (Q, \Sigma, \delta, q_0, F)$ be the given NFA

Let $M' = (Q', \Sigma, \delta', q_0', F')$ be the equivalent DFA

$Q' = 2^Q$, $\delta'([q_0], a) = \delta(\{q_0, q_1 \ldots q_i\}, a)$

$F' = \{[q_0 q_1 q_2 \ldots q_i] \text{ for some } q_i \in F\}$

$q_0' = [q_0]$

Basis of Induction :- $\delta'([q_0], \epsilon) = [q_0]$ iff $\hat{\delta}(q_0, \epsilon) = q_0$.

Inductive hypothesis : $\hat{\delta}'([q_0], w) = [P_1 P_2 \ldots P_k]$ iff $\hat{\delta}(q_0, w) = \{P_1 P_2 \ldots P_k\}$

Inductive step :- prove for claim length $n+1$ $\quad |w| = n \quad |x| = n+1$

$x = wa$

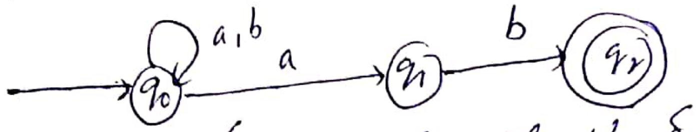
From Inductive hypothesis $\hat{\delta}'([q_0], w) = [P_1 P_2 \ldots P_k]$ iff $\hat{\delta}(q_0, w) = \{P_1 \ldots$

$\delta'([P_1 P_2 \ldots P_k], a) = [r_1 r_2 \ldots r_k]$ iff $\delta(\{P_1 P_2 \ldots P_k\}, a) = \{r_1 r_2 \ldots r_k$

$\delta'(\hat{\delta}'([q_0], w), a) = [r_1 r_2 \ldots r_k]$ iff $\delta(\hat{\delta}(q_0, w), a) = \{r_1 r_2 \ldots r_k$

$\hat{\delta}'(q_0, wa) = [r_1 r_2 \ldots r_k]$ iff $\hat{\delta}(q_0, w) = \{r_1 r_2 \ldots r_k\}$.

3(a) Convert the following NFA to DFA



NFA $M = (\{q_0, q_1, q_2\}, \{a, b\}, \delta, q_0, \{q_2\})$ where

is defined as

δ	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	ϕ	$\{q_2\}$
q_2	ϕ	ϕ

Let $M' = (Q', \{a, b\}, \delta', [q_0], F')$ be the equivalent DFA

where δ' is defined as

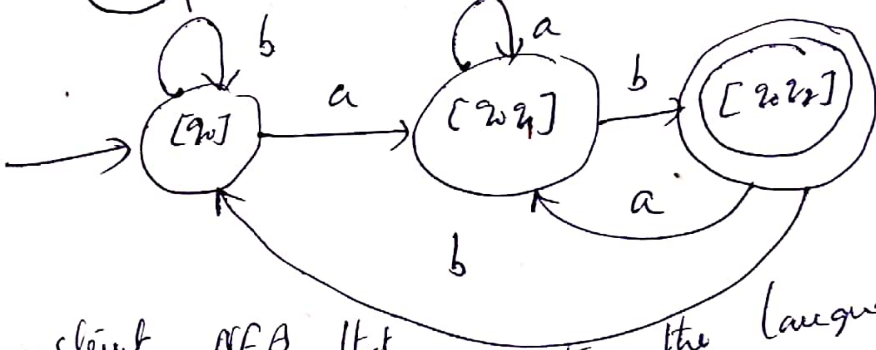
δ'	a	b
$\rightarrow [q_0]$	$[q_0 q_1]$	$[q_0]$
$[q_0 q_1]$	$[q_0 q_1]$	$[q_0 q_2]$
$[q_0 q_2]$	$[q_0 q_1]$	$[q_0]$

$\delta([q_0 q_1], a) = [\delta((q_0, q_1), a)]$

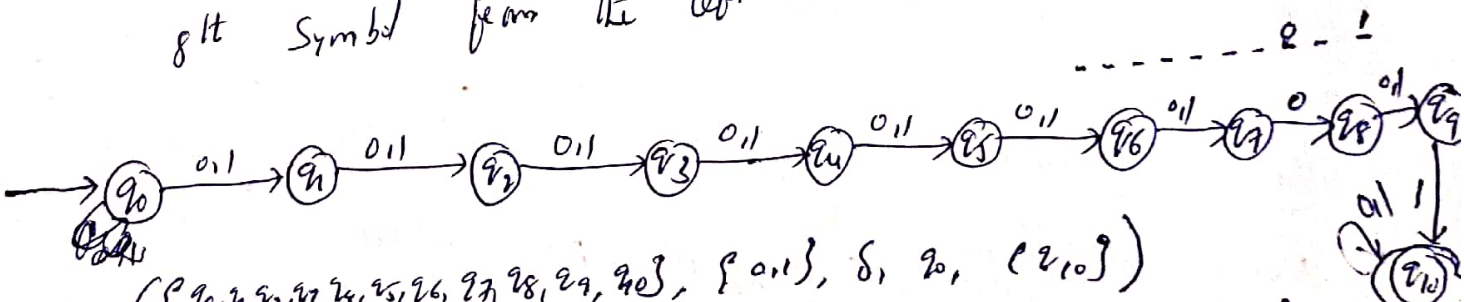
$= [q_0 q_1]$

$Q' = \{[q_0], [q_0 q_1], [q_0 q_2]\}$

$F' = \{[q_0 q_2]\}$



3(b). Construct NFA that accepts the language which contains set of string with 10^{th} symbol from the left end is 1 & 8^{th} symbol from the left end is 0. (4M)



$M = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}\}, \{0, 1\}, \delta, q_0, \{q_{10}\})$

δ	0	1
$\rightarrow q_0$	$\{q_1\}$	$\{q_1\}$
q_1	$\{q_2\}$	$\{q_2\}$
⋮		
q_8	$\{q_9$	$\{q_9\}$
q_9	ϕ	$\{q_{10}\}$
q_{10}	ϕ	ϕ

$\delta(q_0, 0) = \{q_1\}$

$\delta(q_0, 1) = \{q_1\}$

$\delta(q_1, 0) = \{q_2\}$

$\delta(q_1, 1) = \{q_2\}$

$\delta(q_2, 0) = \{q_3\}$

$\delta(q_2, 1) = q_3$

$\delta(q_3, 0) = \{q_4\}$

$\delta(q_3, 1) = \{q_4\}$

⋮

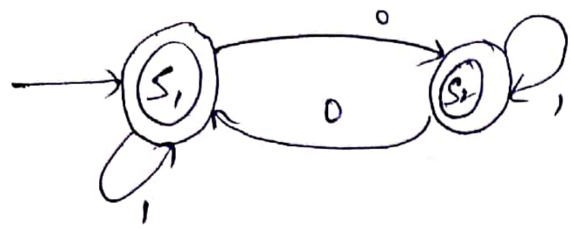
$\delta(q_7, 0) = \{q_8\}$

$\delta(q_8, 0) = \{q_9\}$

$\delta(q_8, 1) = \{q_9\}$

$\delta(q_9, 1) = q_{10}$

...d out the regular expression represented by the following DFA by using Kleene closure method (5M)

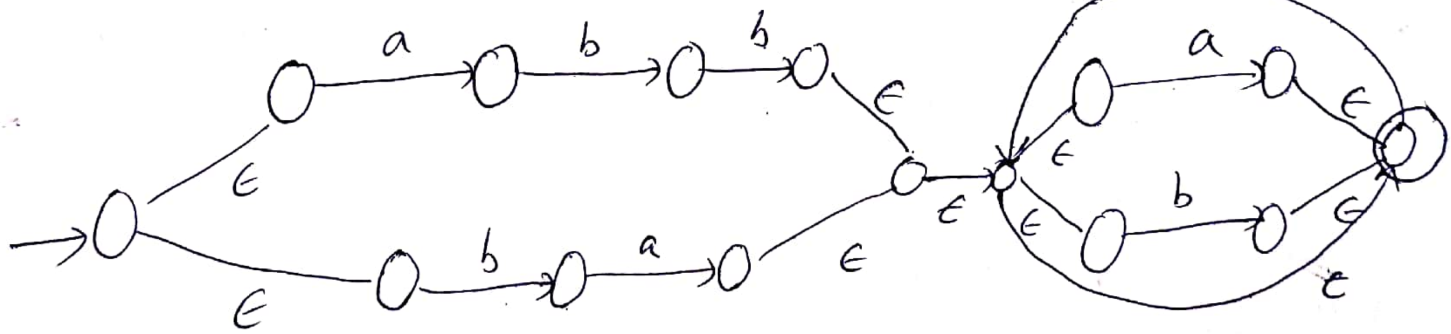


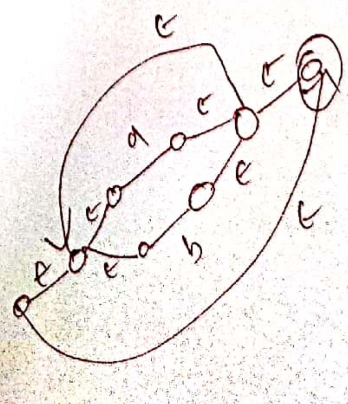
r_{ij}	$k=0$	$k=1$
r_{11}	$1+\epsilon$	1^*
r_{12}	0	1^*0
r_{21}	0	01^*
r_{22}	$1+\epsilon$	1^*

$$r_{11}^1 = r_{11}^0 + r_{11}^0 \cdot (r_{11}^0)^* (r_{11}^0)$$
$$= (\epsilon+1) + (1+\epsilon)(1+\epsilon)^*(1+\epsilon)$$
$$= (1+\epsilon)(\epsilon + (1+\epsilon)^*) = (1+\epsilon)^* = 1^*$$
$$r_{12}^1 = r_{12}^0 + r_{11}^0 \cdot (r_{11}^0)^* (r_{12}^0)$$
$$= 0 + 1 \cdot 1^* \cdot 0 = (\epsilon + 1^+)0$$
$$1^*0$$
$$r_{21}^1 = r_{21}^0 + (r_{21}^0)(r_{11}^0)^*(r_{11}^0)$$
$$= 0 + 0 \cdot 1^* \cdot 1$$
$$= 01^*$$
$$r_{11}^2 = r_{11}^1 + r_{12}^1 \cdot (r_{22}^1)^* (r_{21}^1)$$
$$= 1^* + (1^*0 \cdot 1^* \cdot 01^*)$$

Required regular expression $= 1^*[\epsilon + 01^*01^*) =$

4(b). Give ϵ-NFA for the regular expression $(abb+ba)(a+b)^*$ (5M)





5a. prove that $L = \{a^n b^n \mid n \geq 0\}$ is not regular

Assume L is regular

Let $z = a^n b^n$ is a string in L.

by pumping lemma we can break z into three parts uvw

$$z = \underbrace{a^{n-p}}_{u}\ \underbrace{a^{p}}_{v}\ \underbrace{b^{n}}_{w}$$

except for $p=0$ for all values of p

if we pump v any number of times or if we vomit v then the resulting string contains more or less number of a's than b's. The string is not there in L.

$\therefore$ L is not regular.

5(b). Give regular expressions for the following languages

$3 \times 2 = 6$

i. $(a^*)(bb^*)cc$

$a^* b^+ cc$

(ii) $(aa)^* b(bb)^*$

(iii) $(1+0)^* 10$

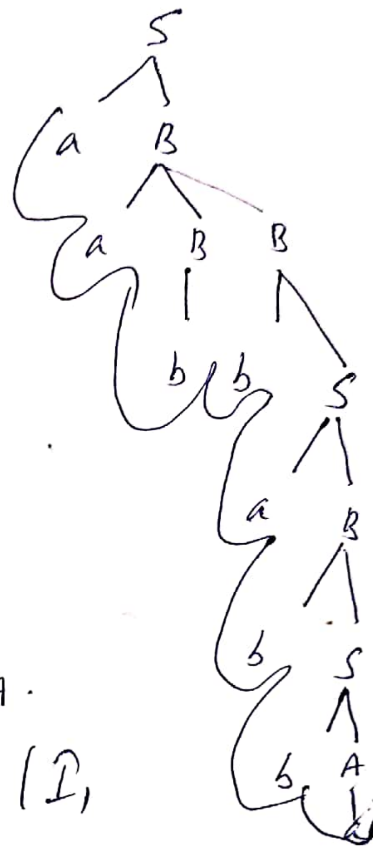
Give leftmost, rightmost derivations and parse trees of the string aabbabba to the following grammar (5M)

"aabbabba"

$S \rightarrow aB / bA$

$A \rightarrow a / aS / bAA$

$B \rightarrow b / bS / aBB$



LMD	RMD
$S \Rightarrow aB$	$S \Rightarrow aB$
$\Rightarrow aaBB$	$\Rightarrow aaBB$
$\Rightarrow aabB$	$\Rightarrow aaBbS$
$\Rightarrow aabbS$	$\Rightarrow aaBbbA$
$\Rightarrow aabbaB$	$\Rightarrow aaBbba$
$\Rightarrow aabbabS$	$\Rightarrow aabSbba$
$\Rightarrow aabbabbA$	$\Rightarrow aabbAbba$
$\Rightarrow aabbabba$	$\Rightarrow aabbabba$

6(a). Convert the following grammar to PDA.

$I \rightarrow a \mid b \mid Ia \mid Ib \mid I0 \mid I1$

$E \rightarrow E+E \mid E*E \mid (E) \mid I$

PDA $P = (q, \{a, b, 0, 1, *, +, (,)\}, \{I, E, *, +, (,), a, b, 0, 1\}, \delta, q, Z_0, \Phi)$

$\delta(q, \epsilon, E) = \{(q, E+E), (q, E*E), (q, (E)), (q, I)\}$

$\delta(q, \epsilon, I) = \{(q, a), \delta(q, b), \delta(q, Ia), (q, Ib), (q, I0), (q, I1)\}$

$\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

$\delta(q, *, *) = (q, \epsilon)$

$\delta(q, +, +) = (q, \epsilon)$

$\delta(q, (, () = (q, \epsilon)$

$\delta(q,),)) = (q, \epsilon)$

(5M)

7 (a) Construct CFG for the language $L = \{a^i b^j c^k \mid i, j\ldots$

productions P: $\{ S \rightarrow S_1 \mid S_2$

$S_1 \rightarrow AB$ $\qquad$ $S_2 \rightarrow CD$

$A \rightarrow aAb \mid \epsilon$ $\qquad$ $C \rightarrow aC \mid \epsilon$

$B \rightarrow cB \mid \epsilon$ $\qquad$ $D \rightarrow bDc \mid \epsilon \}$

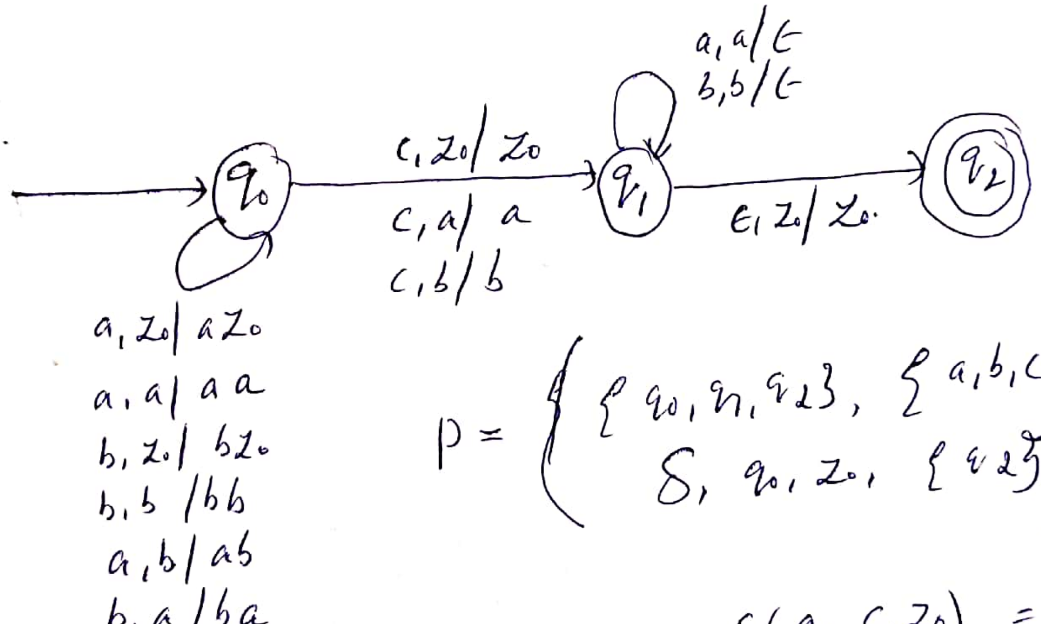
$V = \{S, S_1, S_2, A, B, C, D\}$

$T = \{a, b, c\}$

S is the start symbol of the grammar.

(5M)

7 (b). Construct PDA to accept the language $L = \{wcw^R \mid w \in \{a,b\}^*\}$



$$P = (\{q_0, q_1, q_2\}, \{a, b, c\}, \{a, b, Z_0\}, \delta, q_0, Z_0, \{q_2\})$$

1. $\delta(q_0, a, Z_0) = (q_0, aZ_0)$
2. $\delta(q_0, a, a) = (q_0, aa)$
3. $\delta(q_0, b, Z_0) = (q_0, bZ_0)$
4. $\delta(q_0, b, b) = (q_0, bb)$
5. $\delta(q_0, b, a) = (q_0, ba)$
6. $\delta(q_0, a, b) = (q_0, ab)$
7. $\delta(q_0, c, Z_0) = (q_1, Z_0)$
8. $\delta(q_0, c, a) = (q_1, a)$
9. $\delta(q_0, c, b) = (q_1, b)$
10. $\delta(q_1, a, a) = (q_1, \epsilon)$
11. $\delta(q_1, b, b) = (q_1, \epsilon)$
12. $\delta(q_1, \epsilon, Z_0) = (q_2, Z_0)$

Convert the following grammar into CNF. (10m).

Reduced grammar – 5m
CNF Conversion – 5m

$S \to AACD$
$A \to aAb \mid \epsilon$
$C \to aC \mid a$
$D \to aDa \mid bDb \mid \epsilon$

Reduced grammar

1. elimination of ϵ-productions
   Nullable variables $\{A, D\}$

   $S \to AACD \mid ACD \mid CD \mid AAC \mid AC \mid C$
   $A \to aAb \mid ab$
   $C \to aC \mid a$
   $D \to aDa \mid aa \mid bDb \mid bb$

2. eliminate Unit productions

   $S \to AACD \mid ACD \mid CD \mid AAC \mid AC \mid aC \mid a$
   $A \to aAb \mid ab$
   $C \to aC \mid a$
   $D \to aDa \mid aa \mid bDb \mid bb$

3. There are no useless symbols in the grammar.

CNF Conversion:

$S \to AACD \mid ACD \mid CD \mid AAC \mid AC \mid A_1C \mid a$
$A \to A_1AA_2 \mid A_1A_2$
$C \to A_1C \mid a$
$D \to A_1DA_1 \mid A_2DA_2 \mid A_1A_1 \mid A_2A_2$
$A_1 \to a$
$A_2 \to b$

CNF gra[mmar]

$S \to CD \mid AC$
$S \to AA_3$
$A_3 \to AA_4$
$A_4 \to CD$
$S \to AA_4$
$S \to AA_5$
$A_5 \to AC$
$A \to A_1A_2$
$A \to A_1A_6$
$A_6 \to AA_2$

$D \to A_1A_7$ $\quad D \to A_2A_8$
$A_7 \to DA_1$ $\quad A_8 \to DA_2$
$D \to A_1A_1 \mid A_2A_2$ $\quad C \to A_1C \mid a$

9 (a). Write short notes on Closure properties and decision properties of CFL's. (4m)

(3 m)

<u>Closure properties</u> <u>Explanation</u>

1. Union 2. Concatenation 3. Closure

4. Reversal 5. Homomorphism 6. Inverse homomorphism

CFL's are not closed under (1 m)

1. intersection 2. Complement 3. difference.

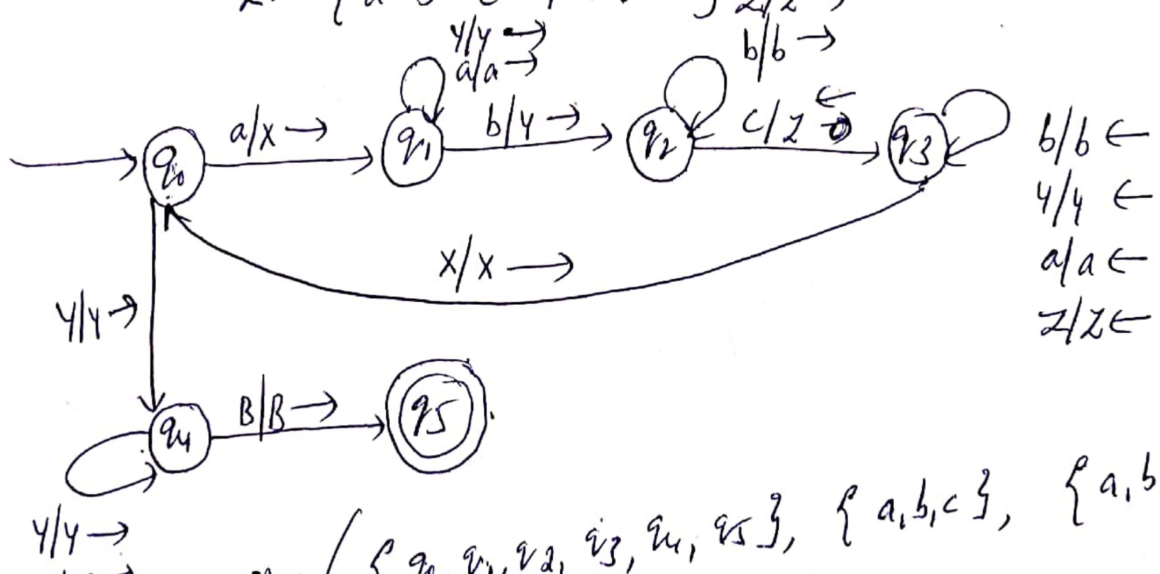
<u>Decision properties</u> <u>Explanation</u> (1 m)

1. emptiness
2. finiteness
3. Membership testing

9 (b). Construct turing machine to the language

$L = \{a^n b^n c^n \mid n \geq 0\}$

(3m)



$M = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{a, b, c\}, \{a, b, c, X, Y, Z, B\}, \delta, q_0, B, \{q_5\})$ (3m)

$\delta(q_0, a) = (q_1, X, R)$

$\delta(q_1, a) = (q_1, a, R)$

$\delta(q_1, b) = (q_2, Y, R)$

$\delta(q_4, B) = (q_5, B, R)$

--|B|a|a|b|b|c|c|B|---- — read 'a' replace it by X move right

--|B|X|a|b|b|c|c|B|--- — read a, skip it move right upto b.

--|B|X|a|a|b|b|c|c|B|— — read b, replace it by Y move right ~~upto~~

--|B|X|a|Y|b|c|c|B|— . . . —