a) A three hinged parabolic arch of span 'l' m and central rise y_c is subjected to a UDL of w/m run over the entire span. The vertical reaction at the abutment is ...

$\frac{WL}{2}$

b.) A three hinged arch has a span of 24 m and a central rise of 8 m. The body of the arch is fabricated from rolled steel sections. Find the change in central rise due to an increase in temperature of 30°C (Take $\alpha = 12 \times 10^{-6}/°C$).

Increase in central rise y_c due to increase in temperature is

$$= \frac{l^2 + 4y_c^2}{4y_c}\alpha T = \frac{24^2 + 4 \times 8^2}{4 \times 8} \times 12 \times 10^{-6} \times 30$$

$$= 9.36 \text{ mm (UP)}$$

c) What is the nature of forces in cables?

Cables have only tension and no compression ~~or~~ bending

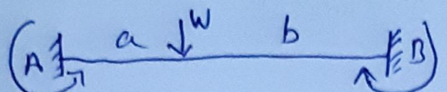
d) What is the prop reaction of a propped cantilever beam of span 'L' and subjected to a UDL of W/m?

$\frac{3WL}{8}$

e) Write the theorem of three moments equation for a two-span continuous beam.

$$M_a l_1 + 2M_b(l_1 + l_2) + M_c l_2 = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2}$$

f) What are the Fixed End moments of a Fixed beam carrying eccentric point load.



$$\bar{M}_{ab} = -\frac{Wab^2}{l^2}$$

$$\bar{M}_{ba} = +\frac{Wa^2b}{l^2}$$

g.) Define carryover moment.

The distributed moments in the ends of members meeting at a joint cause moments in the other Ends, which are assumed to be fixed. These induced moments at the other Ends are called Carryover Moments.

h.) Write down the general slope-deflection equation and state what each term represents

$$M_{ab} = \bar{M}_{ab} + \frac{2EI}{l}\left(2i_a + i_b - \frac{3\delta}{l}\right)$$

$$M_{ba} = \bar{M}_{ba} + \frac{2EI}{l}\left(2i_b + i_a - \frac{3\delta}{l}\right)$$

M_{ab} & M_{ba} are Final moments

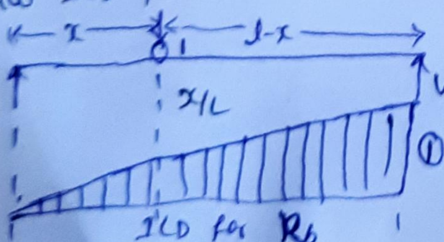
$\bar{M}_{ab}$ & $\bar{M}_{ba}$ are Fixed end moments

i_a & i_b are the slopes at the support

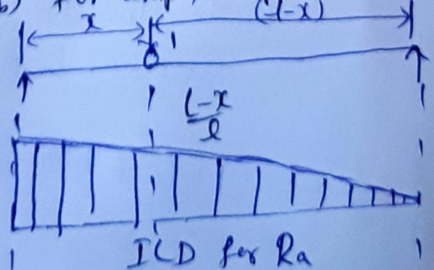
i.) For drawing ILD, what value of test load is assumed and why?

1 unit load is assumed as calculations are easy then, actual loads are multiplied with the results obtained to calculate further.

j.) Draw ILD for vertical reactions (R_a & R_b) for simply supported beam.

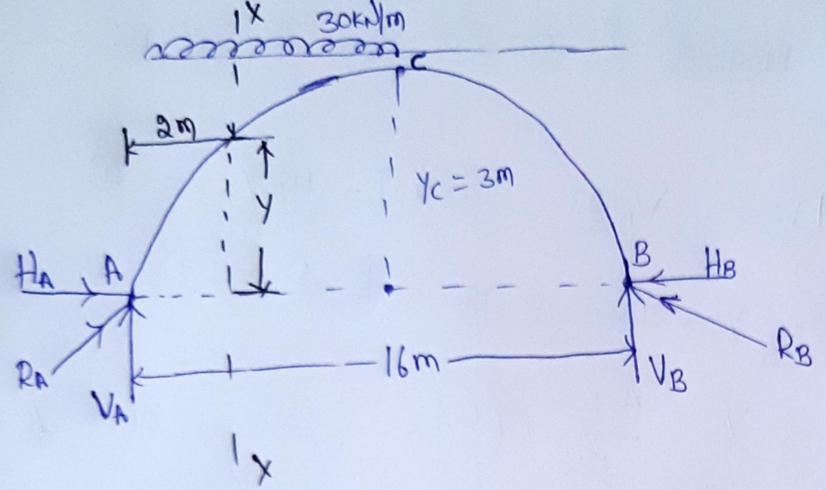


ILD for R_b



ILD for R_a

2.) A three hinged parabolic arch carries a UDL of 30 kN/m on the left half of the span. It has a span of 16 m and a central rise of 3 m. Determine the reasultant reactions at the supports. Also find the normal thrust, radial shear and bending moment at 2 m from the left end A.



Reactions at A and B:

i) Taking moments about A,

$$M_A = -V_B \times 16 + 30 \times 8 \times \frac{8}{2} = 0$$

$\therefore$ $V_B = 60$ kN

$V_A + V_B$ = Total load

$\therefore$ $V_A = 30 \times 8 - 60 = 180$ kN

ii) Taking Moments about crown point 'c'

$$M_C = V_A \times 8 - 30 \times 8 \times \frac{8}{2} - H_A \times Y_C = 0$$

$\therefore$ $H_A = 160$ kN, $H_A = H_B$ since $\Sigma H = 0$

$\therefore$ $H_B = 160$ kN

(ii) Reasultant Reaction at A and B:

$$R_A = \sqrt{V_A^2 + H_A^2} = \sqrt{(180)^2 + (160)^2} = 240.83\ kN$$

$$R_B = \sqrt{V_B^2 + H_B^2} = \sqrt{(60)^2 + (160)^2} = 170.88\ kN$$

Bending moment at $x = 2m$ from Support A:

$$B.m\ M_x = V_A(2) - 30 \times 2 \times 1 - H_A(y) \quad \text{--- (1)}$$

where, y = Rise of the arch at $x = 2m$ from 'A'

$$y = \frac{4y_c}{l^2} \times x(l-x) \qquad [\text{since } x = 2m \text{ \& } y_c = 3m]$$

$$y = \frac{4 \times 3}{(16)^2} \times 2 \times (16-2) = 1.3125\ m$$

Substitute y value in Equation (1)

B.m at 2m from left support 'A' is

$$M_x = 180(2) - 30 \times 2 \times 1 - 160 \times 1.3125$$

$$= 90\ kN\text{-}m$$

Radial Shear at $x = 2m$ from support 'A'

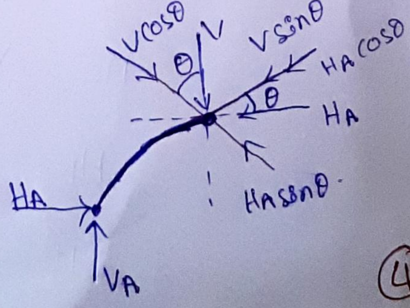
$$\text{Radial Shear } R = V\cos\theta - H_A\sin\theta \rightarrow (2)$$

where

$$V = V_A - W(2)$$

$$V = 180 - 30 \times 2$$

$$V = 120\ kN$$



Slope of the arch is

$$\frac{dy}{dx} = \tan\theta = \frac{4y_c}{l^2}(l-2x)$$

$$\theta = \tan^{-1}\left[\frac{4y_c}{l^2}(l-2x)\right]$$

$$\theta = 29^\circ 21'$$

Substitute values in Eq ②

∴ Radial Shear $R = V\cos\theta - H_A\sin\theta$

$$= 120\cos 29^\circ 21' - 160\sin 29^\circ 21'$$

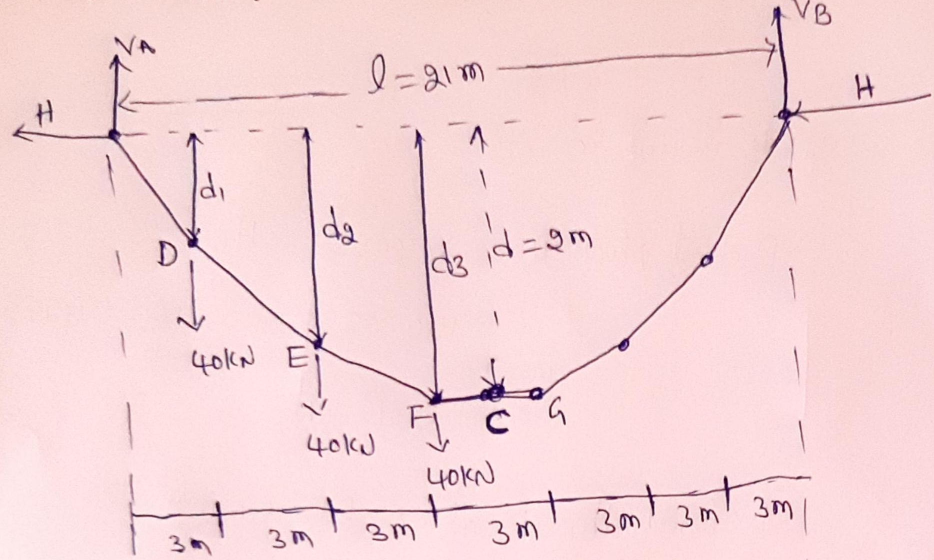
$$= 26.15\ kN$$

Normal thrust at $x = 2m$ from A

$$N = V\sin\theta + H_A\cos\theta$$

$$= 198.28\ kN.$$

3) A suspension cable of horizontal span 21 m is to be used to support six equal loads of 40 kN each at 3 m spacing. The central dip of the cable is limited to 2 m. Find the length of the cable required and also its sectional area if the safe tensile stress is 750 N/mm².



$l = 21\,m; \quad d = 2\,m; \quad \sigma = 750\ N/mm^2$

$$V_A = V_B = \frac{\text{Total load}}{2} = \frac{6 \times 40}{2} = 120\,kN.$$

ⓐ To find the horizontal pull:

Equating moments about C of all the forces one side of 'C' to zero.

$$V_A \times 10.5 - 40 \times 7.5 - 40 \times 4.5 - 40 \times 1.5 - H \times 2 = 0$$

$\therefore \quad H = 360\ kN$

Equating moments about 'D' to zero,

$$120 \times 3 - 360 \times d_1 = 0$$

$$d_1 = \frac{120 \times 3}{360} = 1\,m$$

$$\therefore AD = \sqrt{3^2+1^2} = 3.162\,m.$$

Taking Moments about E:

$$120 \times 6 - 40 \times 3 - 360 \times d_2 = 0$$

$$d_2 = 1.667\,m$$

$$\therefore DE = \sqrt{3^2+(1.667-1)^2} = 3.073\,m$$

Taking moments about F'

$$120 \times 9 - 360 \times d_3 - 40 \times 6 - 40 \times 3 = 0$$

$$d_3 = 2\,m$$

$$\therefore EF = \sqrt{3^2+(2-1.667)^2} = 3.018\,m$$

$$FC = 1.5\,m$$

Therefore length of cable,

$$S = 2(AD+DE+EF+FC)$$

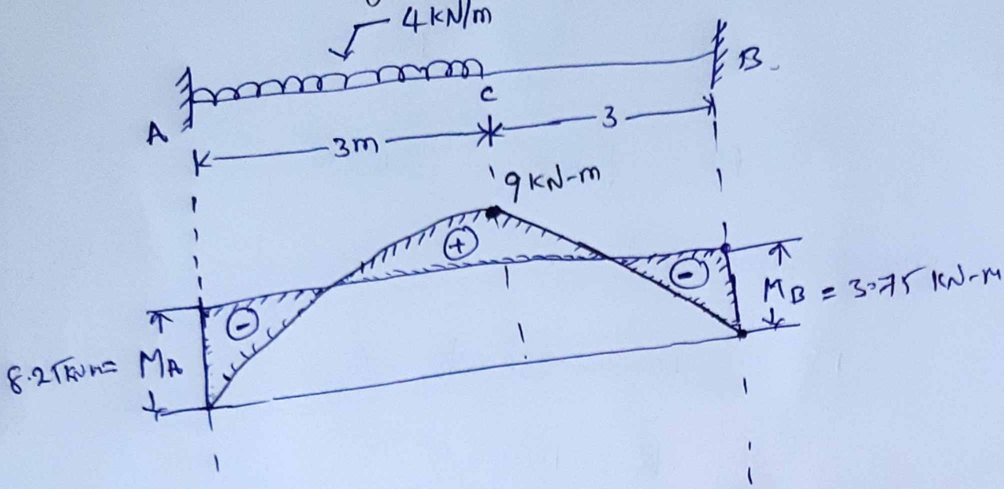
$$= 2(3.162+3.073+3.018+1.5) = 21.506\,m.$$

Maximum tension in the cable,

$$T_{max} = \sqrt{V^2+H^2} = \sqrt{120^2+360^2} = 379.47\,kN$$

$$T_{max} = \sigma \cdot A \quad \therefore A = \frac{T_{max}}{\sigma} = 505.96\,mm^2$$

4.) A Fixed beam AB of span 6 m is carrying a UDL of 4 kN/m over the left half of the span Find the fixing moments and support reactions. Also draw the bending moment diagram for the fixed beam AB.



Taking moments about A,

$$R_B \times 6 = 4 \times 3 \times 1.5$$

$$R_B = \frac{18}{6} = 3\,kN$$

$$R_A = 3 \times 4 - 3 = 9\,kN$$

We know that bending moment diagram will be parabolic from A to C and triangular from C to B as shown in the above Fig.

The bending moment at 'c' (treating the beam as simply supported),

$$M_C = R_B \times 3 = 3 \times 3 = 9\,kN\text{-}m$$

The bending moment at any section x in AC, at a distance 'x' from 'A' (treating the beam as simply supported)

$$M_x = 9x - 4x \cdot \frac{x}{2} = 9x - \frac{4x^2}{2}$$

$$= 9x - 2x^2$$

∴ Area of bending moment from A to B (treating beam as simply supported)

$$a = \int_0^3 (9x - 2x^2)\,dx + \frac{1}{2} \times 9.0 \times 3$$

$$= \left(\frac{9x^2}{2} - \frac{2x^3}{3}\right)\Big|_0^3 + \frac{1}{2} \times 9.0 \times 3$$

$$= 36$$

Area of fixed bending moment diagram.

$$a' = \left[\frac{M_A + M_B}{2}\right] \times 6 = 3(M_A + M_B)$$

We know that

$$a = -a'$$

$$\therefore \quad 3(M_A + M_B) = -36$$

$$M_A + M_B = -12 \longrightarrow (1)$$

Moment of simply supported bending moment diagram area about 'A'

$$a\bar{x} = \int_0^3 (9x^2 - 2x^3)\,dx + \frac{1}{2} \times 9 \times 3 \times 4$$

$$a\bar{x} = \left[\frac{9x^3}{3} - \frac{2x^4}{4}\right]_0^3 + 54$$

$$= 94.5$$

Moment of fixed bending moment diagram area about 'A'

$$a'\bar{x}' = \left(M_A \times \frac{6}{2} \times \frac{6}{3}\right) + M_B \times \frac{6}{2} \times \frac{2 \times 6}{3}$$

$$= 6M_A + 12M_B$$

$\therefore \quad M_A + 2M_B = -15.75 \quad \text{---} ②$

Solving equation ① & ②

$$M_A = -8.25 \text{ kN-m}$$

$$M_B = -3.75 \text{ kN-m}$$

Support reaction

Let $\quad R_A$ = Reaction at A, and

R_B = Reaction at B.

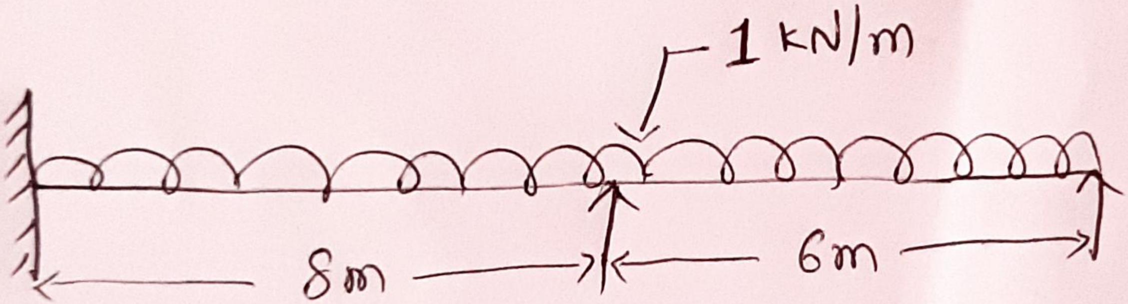
Take the moments about support 'A'

$$R_B \times 6 + 8.25 = \left(4 \times 3 \times \frac{3}{2}\right) + 3.75$$

$$R_B = 2.25 \text{ kN}$$

$$R_A = 9.75 \text{ kN}$$

5.) A continuous beam ABC of uniform section, with span AB as 8 m and BC as 6 m, is fixed at A and simply supported at B and C. The beam is carrying a UDL load of 1 kN/m throughout its length. Find the moments along the beam and the reactions at the supports. Also draw the shear force diagrams. Bending moment and



Moments along the Beam

Since the beam is fixed at A, therefore assume a zero span to the left of A.

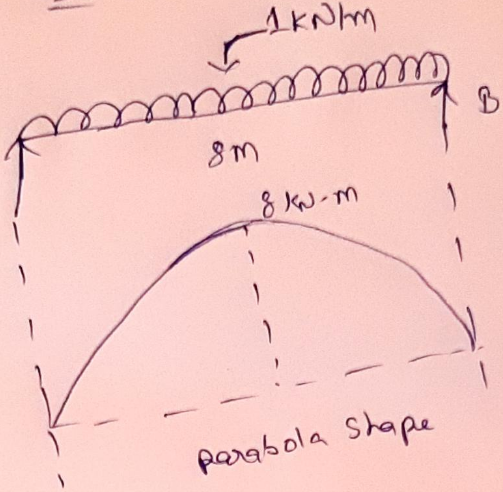
$M_0 = 0$ = Fixing moment at the left hand support of zero span.

M_A = Fixing moment at End A.

M_B = Fixing moment at End B.

* $M_C = 0$ = Fixing moment at End C.

Consider span AB & Simply Supported B.M diagram



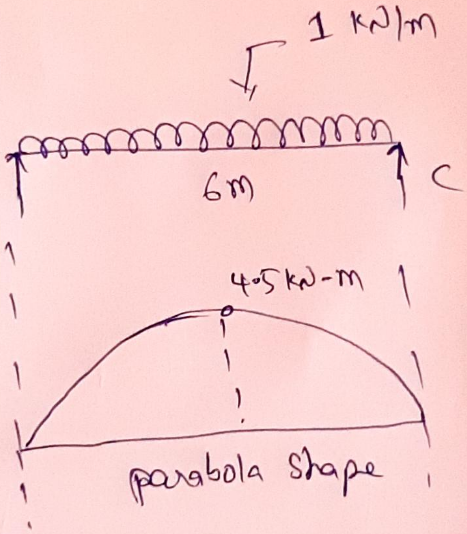
B.m. at mid span of AB

(1)

$$M_{AB} = \frac{wl^2}{8}$$

$$= \frac{1 \times 8^2}{8} = 8.0 \text{ kN-m}$$

For Span BC



B.m at midspan of BC

(2)

$$M_{BC} = \frac{wl^2}{8}$$

$$= \frac{1 \times 6^2}{8} = 4.5 \text{ kN-m}$$

For imaginary span AA'

(0)

$$M_{AA'} = 0$$

$$a_0 \bar{x}_0 = 0$$

Applying Three moment Theorem to Span AA' & AB

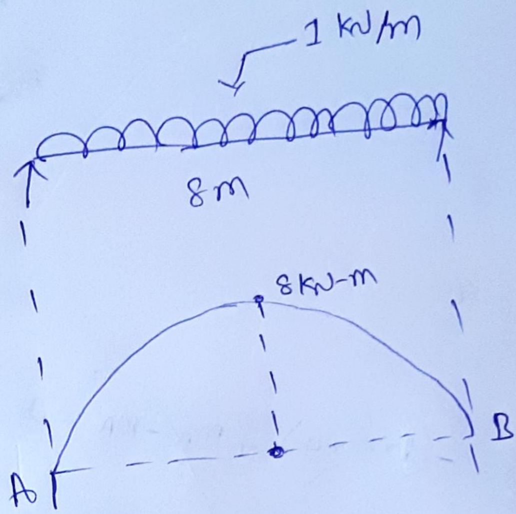
$$M_0 l_0 + 2M_A(l_0 + l_1) + M_B l_1 = \frac{6 a_0 \bar{x}_0}{l_0} + \frac{6 a_1 \bar{x}_1}{l_1} \rightarrow (1)$$

For span AA'

$l_0 = 0$

$\frac{a_0 \bar{x}_0}{l_0} = 0$

For Span AB



$a_1 \bar{x}_1 = \left(\frac{2}{3} \times 8 \times 8 \times 4\right) = 170.67$

Substituting in Eq (1)

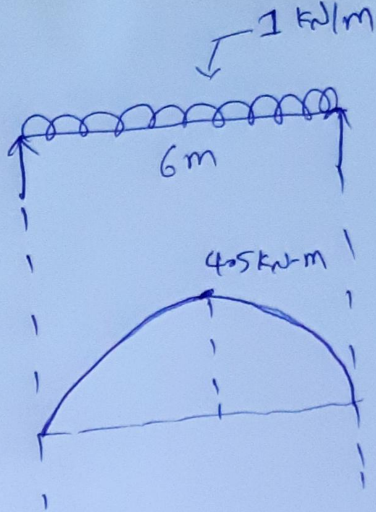
$$0 + 2M_A(0+8) + (M_B \times 8) = 0 + \frac{6 \times 170.67}{8}$$

$$16 M_A + 8 M_B = 128 \rightarrow (2)$$

Applying Three moment Theorem to Span AB & BC

$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 = \frac{6 a_1 \bar{x}_1}{l_1} + \frac{6 a_2 \bar{x}_2}{l_2} \rightarrow (3)$$

For span BC



$$a_2\bar{x}_2 = \frac{2}{3} \times 4.5 \times 6 \times 3$$

$$= 54$$

Substituting $a_1\bar{x}_1$ & $a_2\bar{x}_2$ values in Eq (3)

$$M_A \times 8 + 2M_B(8+6) + 0 = \frac{6 \times 170.67}{8} + \frac{6 \times 54}{6}$$

$$8M_A + 28M_B = 182 \longrightarrow (4)$$

Solving Eq (2) & (4)

$M_A = 5.75$ kN-m & $M_B = 4.5$ kN-m

Reactions at the supports

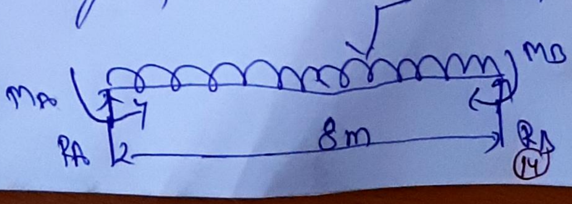
R_A = Reaction at A.

R_B = Reaction at B.

R_C = Reaction at C.

Taking moments about B, considering span AB.

$$R_A \times 8 - (1 \times 8 \times 4) + M_B - M_A = 0$$



$\therefore \quad R_A \times 8 - (4 \times 8) + 4.5 - 5.75 = 0$

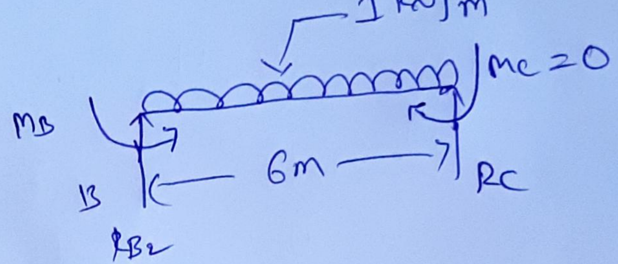
$R_A = 4.15\ kN$

$R_A + R_{B1} = \text{Total load}$

$4.15 + R_{B1} = 1 \times 8$

$R_{B1} = 8 - 4.15 = 3.85\ kN.$

Taking moments about 'B' considering span 'BC'



$R_C \times 6 - (1 \times 6 \times 3) + M_B = 0$

$R_C \times 6 - 18 + 4.5 = 0$

$R_C = 2.2\ kN$

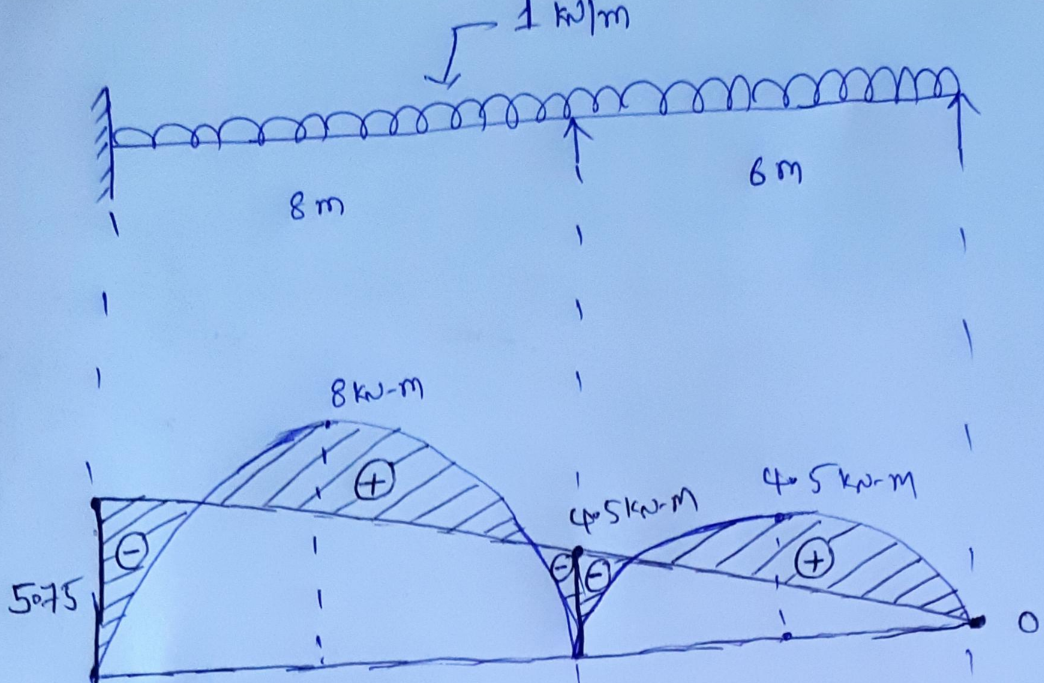
$R_{B2} + R_C = \text{Total load}$

$R_{B2} + 2.2 = 1 \times 6$

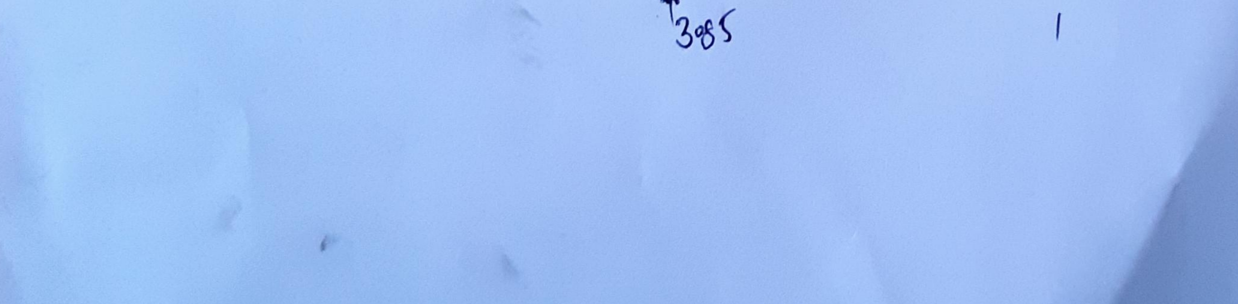
$R_{B2} = 3.8\ kN$

$\therefore \quad R_B = R_{B1} + R_{B2} = 3.85 + 3.8$

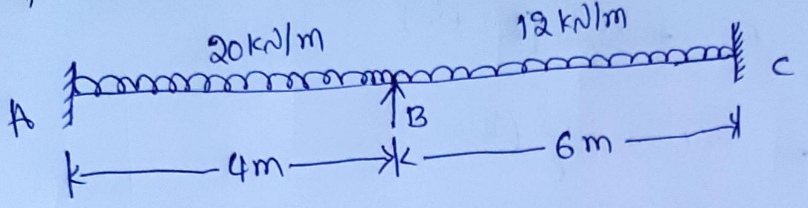
$R_B = 7.65\ kN.$



Bending moment diagram



6.) A continuous beam ABC fixed at 'A' and 'C' and simply supported at 'B' consists of spans AB and BC of lengths 4 m and 6 m respectively. The span AB carries a UDL of 20 kN/m while the span BC carries a UDL of 12 kN/m. Find the moments and reactions at the supports using slope-deflection method. Also draw B.M and S.F diagrams for the beam.



Fixed end Moments

$$\bar{M}ab = -\frac{wl^2}{12} = -\frac{20 \times 4^2}{12} = -26.67 \text{ kN-m}$$

$$\bar{M}ba = +\frac{20 \times 4^2}{12} = 26.67 \text{ kN-m}$$

$$\bar{M}bc = -\frac{12 \times 6^2}{12} = -36 \text{ kN-m}$$

$$\bar{M}cb = +36 \text{ kN-m}$$

Since A & C are fixed, we know $i_a = i_c = 0$

Applying Slope-deflection Equations to

SPAN AB:

$$Mab = \bar{M}ab + \frac{2EI}{l}(2i_a + i_b)$$

$$Mab = -26.67 + \frac{2EI}{4}(0 + i_b)$$

$$= -26.67 + \frac{1}{2}EI\, i_b$$

$$M_{ba} = \bar{M}_{ba} + \frac{2EI}{l}(2i_b + i_a)$$

$$= +26.67 + \frac{2EI}{4}(2i_b + 0)$$

$$= +26.67 + EI\,i_b$$

Span BC:

$$M_{bc} = \bar{M}_{bc} + \frac{2EI}{l}[2i_b + i_c]$$

$$= -36 + \frac{2EI}{6}(2i_b + 0)$$

$$= -36 + \frac{2}{3}EI\,i_b$$

$$M_{cb} = \bar{M}_{cb} + \frac{2EI}{l}(2i_c + i_b)$$

$$= +36 + \frac{1}{3}EI\,i_b$$

Equilibrium condition at B,

$$M_{ba} + M_{bc} = 0; \quad 26.67 + EI\,i_b - 36 + \frac{2}{3}EI\,i_b = 0$$

$$EI\,i_b = \frac{3}{5}(9.33) = 5.598$$

substituting for $EI\,i_b$

$$M_{ab} = -26.67 + \frac{1}{2}(5.598) = -23.87 \text{ KN-m}$$

$$M_{ba} = 26.67 + 5.598 = +32.27 \text{ KN-m}$$

$$M_{bc} = -36 + \frac{2}{3}(5.598) = -32.27 \text{ KN-m}$$

$$M_{cb} = +36 + \frac{1}{2}(5.598) = +37.87 \text{ KN-m}$$

Reactions:

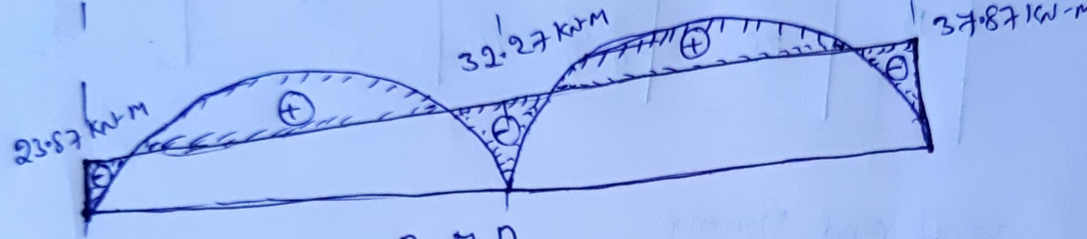
$$B.M \text{ at } B = V_a \times 4 - 23.87 - 20 \times 4 \times 2 = -32.27$$

$$\therefore V_a = 37.90 \text{ KN.}$$

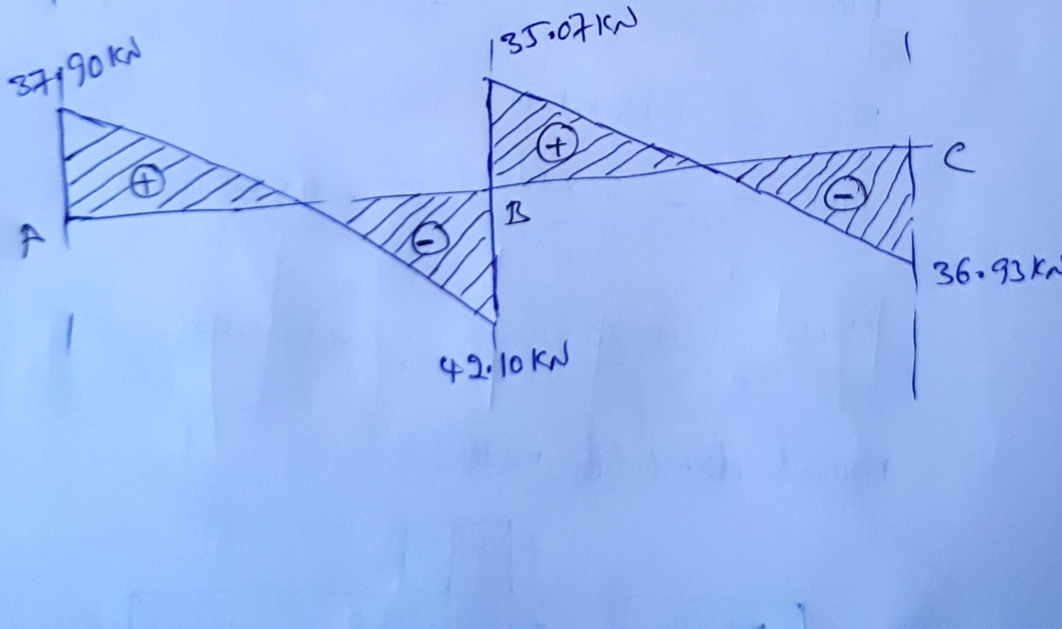
B.M at B = $V_c \times 6 - 37.87 - 12 \times 6 \times 3 = -32.27$

$\therefore \quad V_c = 36.93$ kN

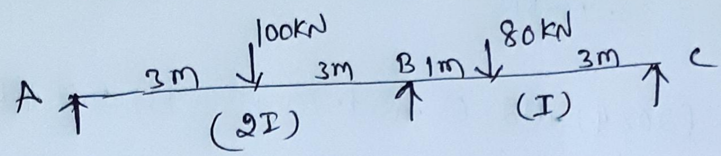
$\therefore \quad V_b = (20 \times 4) + (12 \times 6) - (37.90 + 36.93) = 77.17$ kN



B. M. D



7.) Find the Support moments and draw B.M and S.F diagrams for the continuous beam shown in fig.1



Distribution Factors:

Joint	Member	Relative Stiffness	Total Relative Stiffness	Distribution Factor
B	BA	$\frac{3}{4}\times\frac{2I}{6} = \frac{I}{4} = \frac{4I}{16}$	$\frac{7I}{16}$	$\frac{4}{7}$
	BC	$\frac{3}{4}\cdot\frac{I}{4} = \frac{3I}{16}$		$\frac{3}{7}$

Fixed End Moments

$$M_{ab} = -\frac{100\times 6}{8} = -75\ kN\text{-}m$$

$$M_{ba} = +75\ kN\text{-}m$$

$$M_{bc} = -\frac{80\times 1\times 3^2}{4^2} = -45\ kN\text{-}m$$

$$M_{cb} = +\frac{80\times 1^2\times 3}{4^2} = +15\ kN\text{-}m$$

Moment distribution Method.

		$\frac{4}{7}$	$\frac{3}{7}$		
	-75.0	+75.0	-45.0	+15.0	
	+75.0	+37.50	-7.50	-15.0	
	0	+112.50	-52.50	0	
		-34.29	-25.71		
	0	+78.21	-78.21	0	Final Moments

Reactions:

B.M at B = $V_a \times 6 - 100 \times 3 = -78.21$

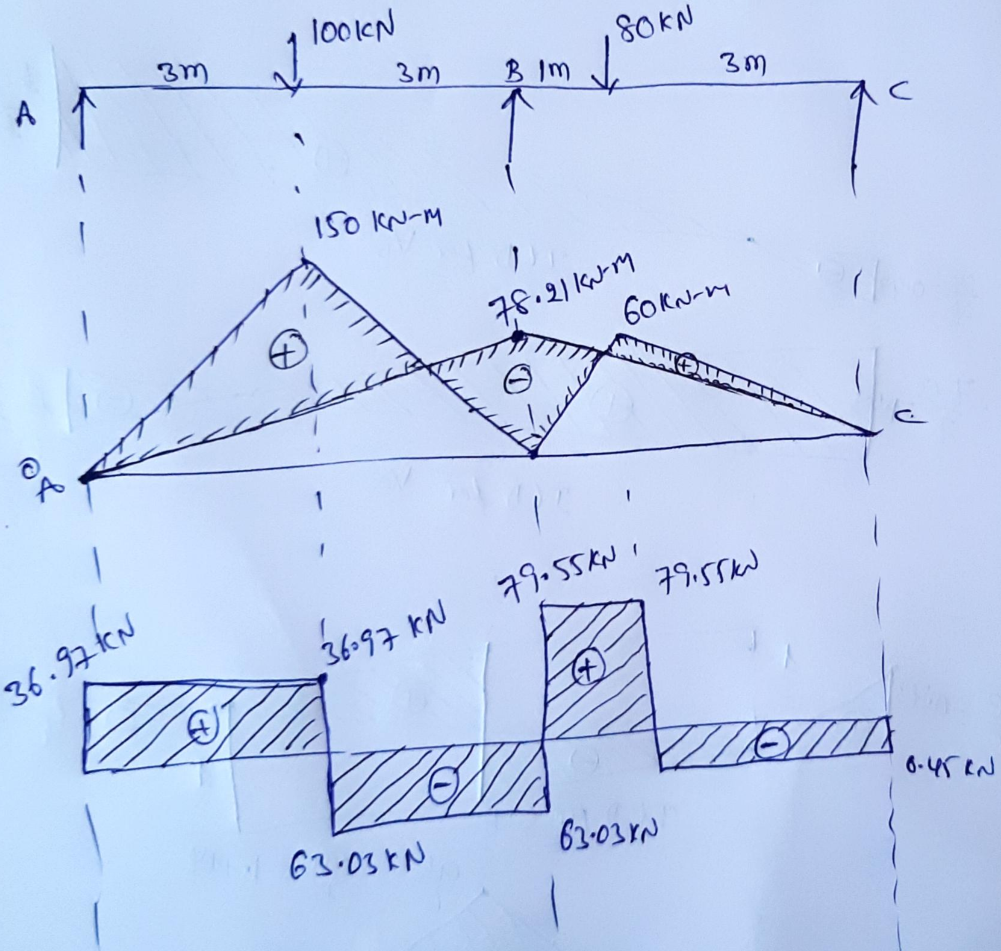
$\therefore$ $V_a = 36.97$ kN

B.M at B = $V_c \times 4 - 80 \times 1 = -78.21$

$\therefore$ $V_c = 0.45$ kN

$V_b = (100 + 80) - (36.97 + 0.45)$

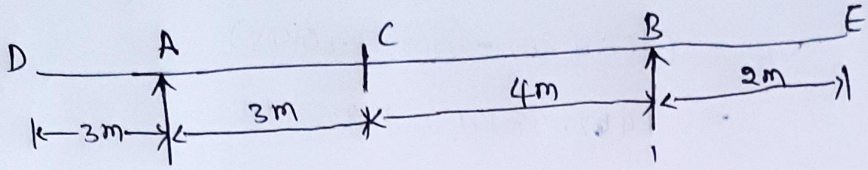
$= 142.58$ kN $= 142.58$ kN

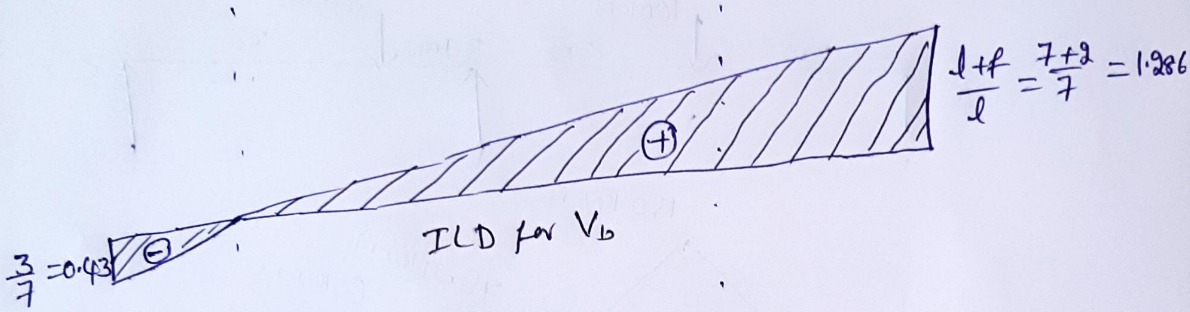


8.) Construct the ILD for overhanging beam shown in fig.2 for the following cases.

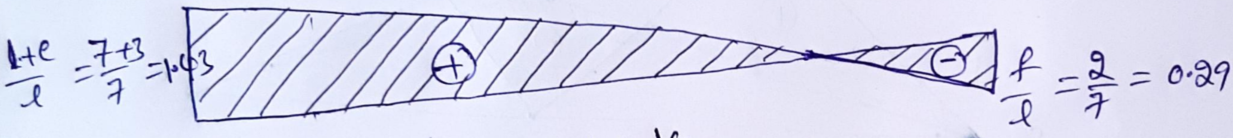
i) Reaction @ A (V_A) ii) Reaction @ B (V_B)

iii) Shear force @ C (V_C) iv) Bending moment @ C (M_C)

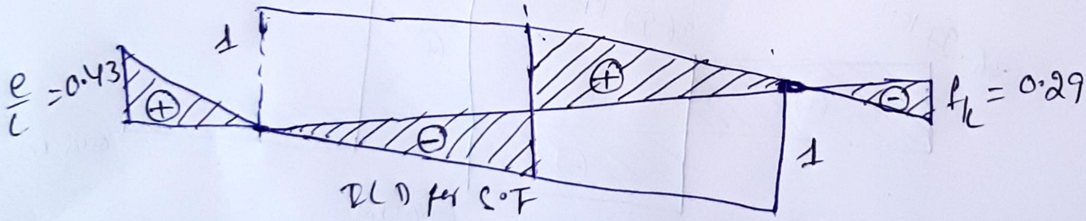




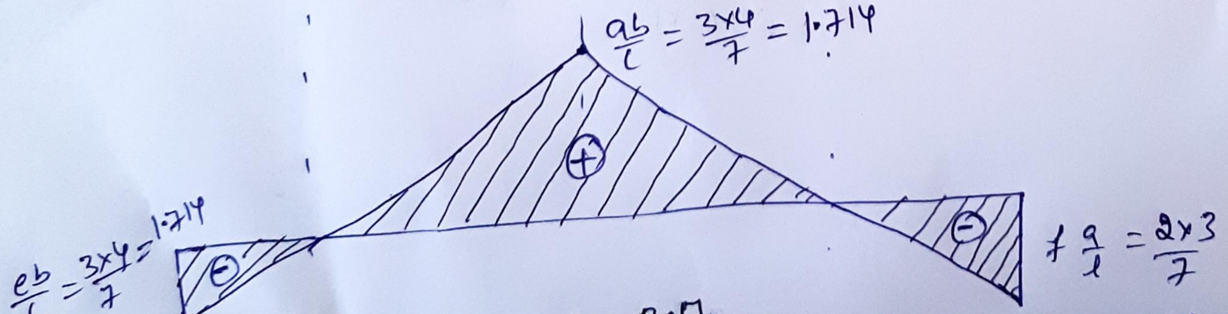
ILD for V_b



ILD for V_a

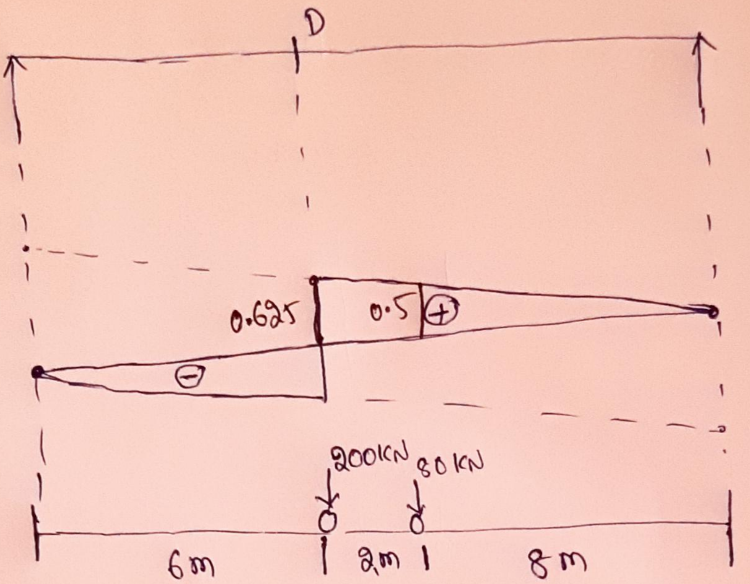


ILD for S.F



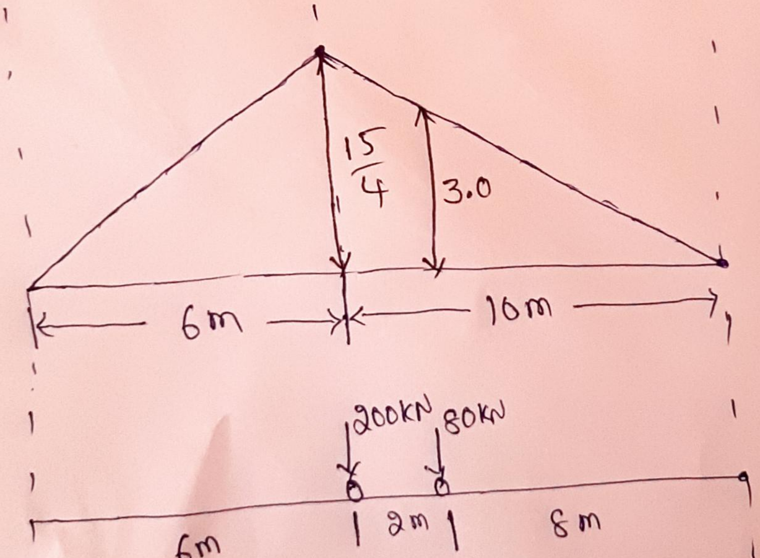
ILD for B.M

9.) Two wheel loads 200 kN and 80 kN spaced at 2 m apart move on the girder of span 16 m. Find the maximum positive shear force and bending moment that can occur at a section 6 m from the left end. Any wheel load can lead the other using ILD's.



ILD for S.F.D

position of loads for Max +ve S.F



ILD for B.M.D

position of loads for Max. B.M.

Maximum +ve Shear force $= 200 \times 0.625 + 80 \times 0.5$

$= 125 + 40$

$= 165$ kN.

Maximum B.M $= 200 \times \frac{15}{4} + 80 \times 3$

$= 990$ kN-m.