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III/IV B.Tech (Regular/Supplementary) DEGREE EXAMINATION

February , 2021

Electrical and Electronics Engineering

Fifth Semester

Control Systems

Time: Three Hours

Maximum: 50 Marks

Answer ALL Questions from PART-A.

(1X10 = 10 Marks)

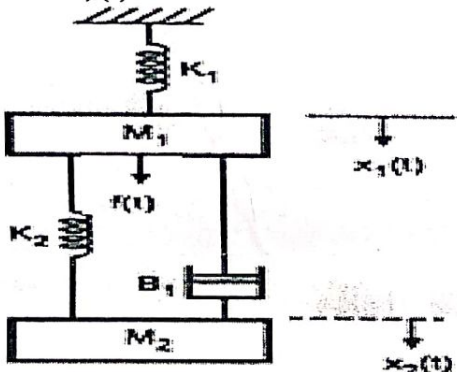
Answer ANY FOUR questions from PART-B.

(4X10=40 Marks)

PART-A

1.	a)	What are the advantages of Closed Loop System?	CO1	1M
	b)	Write any two applications for feedback control system?	CO1	1M
	c)	Write Monson's Gain Formulae?	CO1	1M
	d)	What are the standard test signals used in the analysis of control systems?	CO2	1M
	e)	What is difference between type number and order of the system?	CO2	1M
	f)	What is the effect of adding zero to a system?	CO2	1M
	g)	Define BIBO stability of a system.	CO3	1M
	h)	What are the limitations of RH stability criterion?	CO3	1M
	i)	Draw the polar plot for type 1 and order 2 System.	CO4	1M
	j)	What is meant by Observability of the system?	CO4	1M

PART-B

2.	a)	Write any four differences between open loop and closed loop systems?	CO1	4M
	b)	Determine the transfer function $\frac{x_1(t)}{f(t)}$ of the mechanical system shown in below Fig.	CO1	6M
				
3.	a)	Derive the transfer function of armature controlled DC servo Motor.	CO1	6M
	b)	Write any four rules, which are used in Block diagram reduction technique.	CO1	4M
4.	a)	Derive the expression for rise time and peak over shoot of second order system.	CO2	5M
	b)	Determine the step, ramp & parabolic error constants and their corresponding steady state errors for the following system with unity feedback.	CO2	5M
		$G(s) = \frac{1}{s^2(s+1)(s+2)}$		
5.	a)	Explain the effect of adding zeroes on the performance of over shoot, rise time and bandwidth.	CO2	5M
	b)	What is the effect of PI Controller on response of the system?	CO2	5M

6.	a)	Check the stability and comment on location of the poles of the system described by the characteristic equation. $S^7 + 5S^6 + 9S^5 + 9S^4 + 4S^3 + 20S^2 + 24S + 24 = 0$	CO3	
	b)	Describe the frequency domain specifications.	CO3	5M
7.		Sketch the bode plot for the transfer function $G(S) = \frac{20}{S(1+3S)(1+4S)}$. From The bode plot, Obtain Gain Crossover Frequency.	CO3	10M
8.		Define root locus and Clearly Explain about the construction rules of root locus.	CO4	10M
9.	a)	Obtain the state model for the following differential equation $\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y = -u(t)$	CO4	5M
	b)	Check the controllability for the above system	CO4	5M

* Logarithmic Graph need to be supplied

18EE502 Scheme of Evaluation

① a) Advantages of closed Loop Systems:-

- Accurate
- Accurate even in the presence of non-linearities.
- Less sensitive to disturbances.
- Less affected by noise.
- more stable.

Award one mark for any two.

b) Two Applications of feedback control systems:-

- Temperature control system.
- Traffic control system.
- Position control system using servomotor etc.
- Automatic Electric Iron
- Air conditioner
- Water level controller etc.

Award one mark for any two.

c) Mason's Gain Formula:-

$$\text{Overall Gain } T = \frac{1}{\Delta} \sum_k P_k \Delta_k$$

1m.

d) Standard test signals:-

- step signal
- Ramp signal
- Parabolic signal
- Impulse signal.

1m

e) Difference b/w type number and order of the system:

- Total no. of poles at origin represents type number of the system.
- Total no. of poles / highest power of s in denominator polynomial is called order of the system.

1m

f) Effect of adding zero to a system:-

If zero is added on LHP it makes the step response faster (decreases rise time & the peak time) and increases overshoot. Adding a RHP zero makes step response slower, and can make the response undershoot.

1m

g) BIBO Stability:-

If a system gives Bounded output for a Bounded input then that system is called stable system.

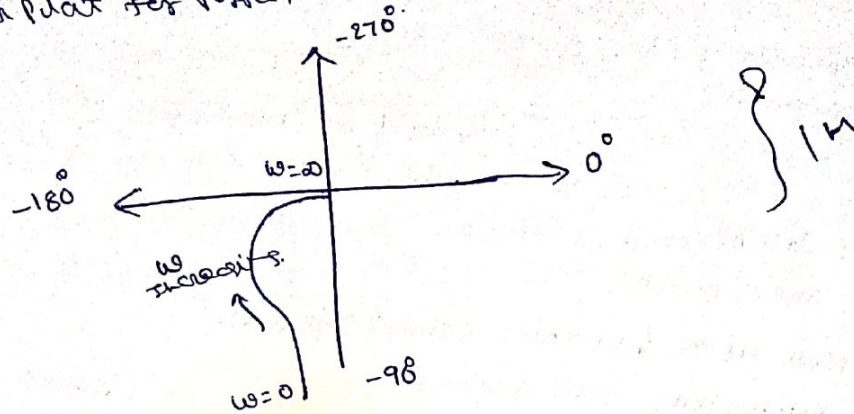
1m

h) Limitations of Routh criterion:-

- It is applicable only for a linear system.
- It does not provide the exact location of poles on the right and left half of the s plane.
- In case of the characteristic equation, it is valid only for real coefficients.

1m

i) Polar plot for type 1 and order 2 system.



i) observability of the system:-

A system is said to be completely observable if every state $x(t)$ can be completely identified by measurements of the output $y(t)$ over a finite time interval.

2 a) Differences b/w open loop and closed loop systems:

open loop system

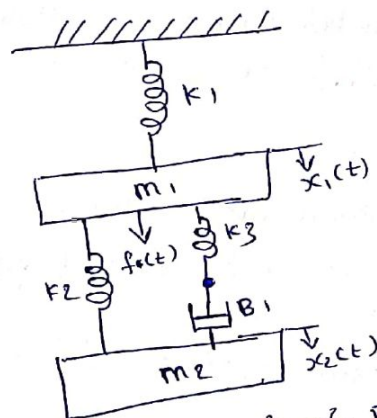
- i. Feedback element is absent.
- ii. more stable
- iii. construction Easy.
- iv. Economical
- v. In Accurate
- vi. unreliable
- vii. less maintenance

closed loop system

- i. Feedback element is always present.
- ii. may become unstable.
- iii. complicated construction.
- iv. costly.
- v. Accurate.
- vi. reliable.
- vii. more maintenance.

Answer
1M
For any
Form.

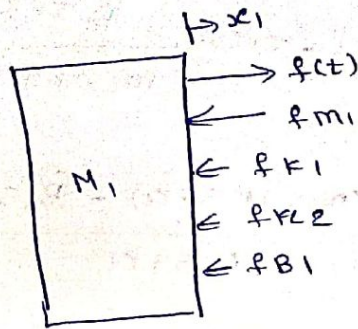
2 b) Transfer Function. $\frac{x_1(t)}{f(t)}$



Let Laplace Transform of $f(t) = L\{f(t)\} = F(s)$.
Laplace transform of $x_1 = L\{x_1\} = X_1(s)$.
Laplace transform of $x_2 = L\{x_2\} = X_2(s)$.

Required
 $T.F = \frac{X_1(s)}{F(s)} = ?$

Free body diagram of first mass element m_1 is,



$$f m_1 = M_1 \frac{d^2 x_1}{dt^2} \quad f k_1 = K_1 x_1 \quad f k_2 = K_2 (x_1 - x_2)$$

$$f B_1 = B_1 \frac{d}{dt} (x_1 - x_2)$$

By Newton's second Law,

$$f m_1 + f b_1 + f k_1 + f k_2 = f(t)$$

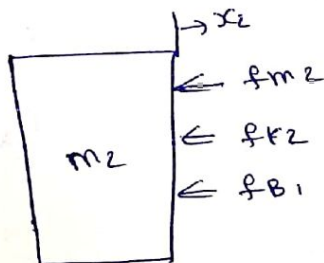
$$\therefore m_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{d}{dt} (x_1 - x_2) + K_1 x_1 + K_2 (x_1 - x_2) = f(t)$$

on taking Laplace transform of above equation with zero initial conditions we get,

$$m_1 s^2 x_1(s) + B_1 s x_1(s) - B_1 s x_2(s) + K_1 x_1(s) + K_2 x_1(s) - K_2 x_2(s) = F(s)$$

$$x_1(s) [m_1 s^2 + B_1 s + K_1 + K_2] - x_2(s) [B_1 s + K_2] = F(s) \quad \text{--- (1)}$$

Free body diagram of second mass element m_2 is,



$$f m_2 = m_2 \frac{d^2 x_2}{dt^2} \quad f k_2 = K_2 (x_2 - x_1) \quad f B_1 = B_1 \frac{d}{dt} (x_2 - x_1)$$

By Newton's second law,

$$f m_2 + f k_2 + f B_1 = 0$$

$$m_2 \frac{d^2 x_2}{dt^2} + K_2 (x_2 - x_1) + B_1 \frac{d}{dt} (x_2 - x_1) = 0$$

on taking Laplace transform we get,

$$m_2 s^2 x_2(s) + K_2 x_2(s) - K_2 x_1(s) + B_1 s x_2(s) - B_1 s x_1(s) = 0$$

$$x_2(s) [m_2 s^2 + K_2 + B_1 s] - x_1(s) [B_1 s + K_2] = 0$$

$$x_2(s) = x_1(s) \frac{B_1 s + K_2}{m_2 s^2 + B_1 s + K_2} \quad \text{--- (2)}$$

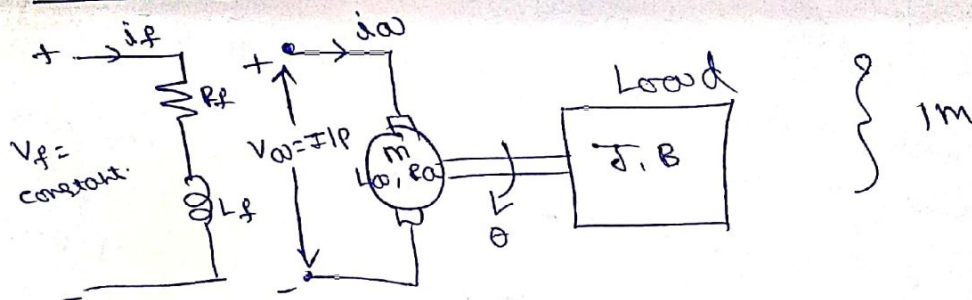
on substituting Eq (2) in Eq (1) we get,

$$X_1(s) [m_1 s^2 + B_1 s + K_1 + K_2] - X_2(s) \frac{[B_1 s + K_2]^2}{m_2 s^2 + B_1 s + K_2} = F(s)$$

$$X(s) \left[\frac{(m_1 s^2 + B_1 s + K_1 + K_2)(m_2 s^2 + B_1 s + K_2) - [B_1 s + K_2]^2}{m_2 s^2 + B_1 s + K_2} \right] = F(s)$$

$$\therefore \frac{X_1(s)}{F(s)} = \frac{m_2 s^2 + B_1 s + K_2}{(m_1 s^2 + B_1 s + K_1 + K_2)(m_2 s^2 + B_1 s + K_2) - (B_1 s + K_2)^2}$$

3 a) T.F. of armature controlled DC motor:-



diff. Equations are.

$$I_a s R_a + L_a \frac{dI_a}{dt} + E_b = V_w \quad (1)$$

$$T = K_t I_a \quad (2)$$

$$J \frac{d^2 \theta}{dt^2} + B \frac{d\theta}{dt} = T \quad (3)$$

$$E_b = K_b \frac{d\theta}{dt} \quad (4)$$

on taking L.f. we get,

$$I_a(s) R_a + L_a s I_a(s) + E_b(s) = V_w(s) \quad (5)$$

$$T(s) = K_t I_a(s) \quad (6)$$

$$J s^2 \theta(s) + B s \theta(s) = T(s) \quad (7)$$

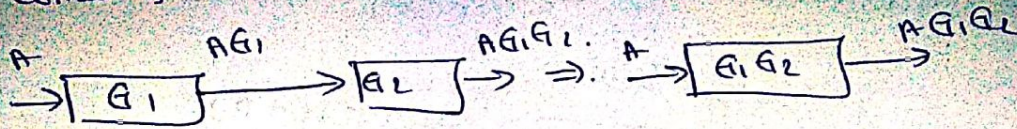
$$E_b(s) = K_b s \theta(s) \quad (8)$$

on solving above equations we get.

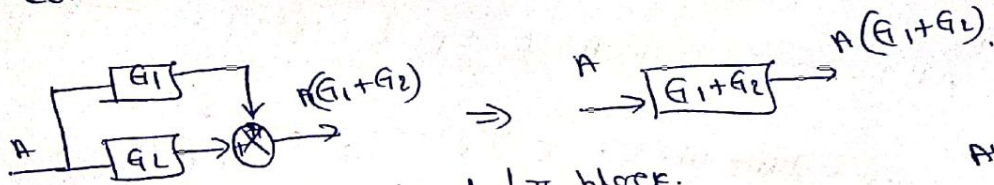
$$\frac{\theta(s)}{V_w(s)} = \frac{K_t}{(R_a + s L_a) (J s^2 + B s) (K_b K_t s)}$$

b) Block diagram reduction rules:

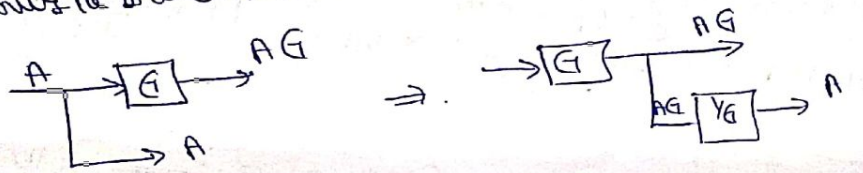
i, combining the blocks in cascade.



ii, combining parallel blocks.



iii, moving the branch point ahead of the block.



Answer 4 marks

for any four rules.

iv, moving the branch point before the block.



etc...

H(s)

Expression for rise time and peak overshoot of the second order system:-

i, Rise time (t_r)

$$C(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

$$At t = t_r, C(t) = C(t_r) = 1$$

$$C(t_r) = 1 - \frac{e^{-\xi \omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 1$$

$$\therefore - \frac{e^{-\xi \omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 0$$

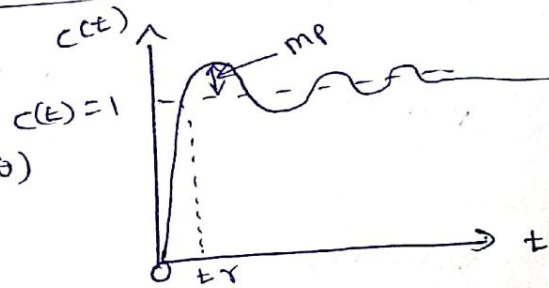
Since $-e^{-\xi \omega_n t_r} \neq 0$ then the term $\sin(\omega_d t_r + \theta) = 0$.

$\sin \phi = 0$ when $\phi = 0, \pi, 2\pi, 3\pi, \dots$

$$\omega_d t_r + \theta = \pi$$

$$\omega_d t_r = \pi - \theta$$

$$\therefore \text{Rise time } (t_r) = \frac{\pi - \theta}{\omega_d} = \frac{\pi - \theta}{\omega_n \sqrt{1-\xi^2}}$$



3M

ii, Peak overshoot (M.P).

$$\therefore \text{Peak overshoot} = \frac{C(t_p) - C(\infty)}{C(\infty)} \times 100$$

$$C(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

$$\text{At } t = \infty, C(t) = 1 \quad \text{At } t = t_p, C(t_p) = 1 - \frac{e^{-\xi \omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \theta)$$

$$\text{on solving we get } C(t_p) = 1 + e^{-\frac{\xi \pi}{\sqrt{1-\xi^2}}}$$

$$\therefore \text{M.P} = \frac{C(t_p) - C(\infty)}{C(\infty)} \times 100 = \left[1 + e^{-\frac{\xi \pi}{\sqrt{1-\xi^2}}} \right] \times 100$$

$$\therefore \text{Peak overshoot (M.P)} = e^{-\frac{\xi \pi}{\sqrt{1-\xi^2}}} \times 100$$

4b)

$$G(s) = \frac{1}{s^2(s+1)(s+2)}$$

For unity feedback system $H(s) = 1$

$$\text{Position Error constant } K_P = \lim_{s \rightarrow 0} G(s)H(s) = \lim_{s \rightarrow 0} \frac{1}{s^2(s+1)(s+2)} = \frac{1}{0} = \infty$$

Velocity Error constant

$$K_V = \lim_{s \rightarrow 0} s G(s)H(s)$$

$$= \lim_{s \rightarrow 0} \frac{1}{s(s+1)(s+2)} = \frac{1}{0} = \infty$$

Acceleration Error constant

$$K_A = \lim_{s \rightarrow 0} s^2 G(s)H(s)$$

$$= \lim_{s \rightarrow 0} \frac{1}{(s+1)(s+2)} = \frac{1}{1 \times 2} = 0.5$$

$\therefore$ Error constants are

$$K_P = \infty$$

$$K_V = \infty$$

$$K_A = 0.5$$

Steady state error

$$e_{ss} = \frac{1}{K_0} = \frac{1}{0.5} = 2 \quad \left. \vphantom{\frac{1}{K_0}} \right\} 1M$$

— X —

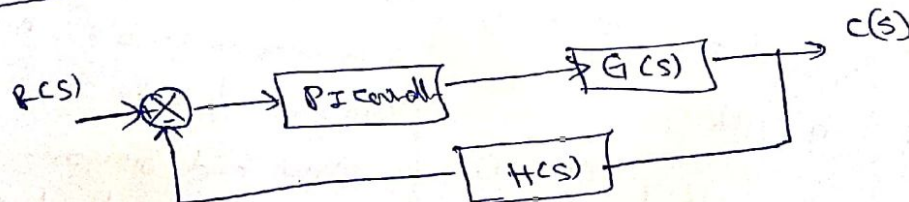
5 a) Effect of adding zeros on the performance of overshoot, rise time and bandwidth:-

Adding a zero to the closed-loop transfer function decreases rise time and increases the maximum overshoot of the step response. } 2M

When the added zero is very far away from the imaginary axis, the overshoot is large and the damping is very poor. The overshoot is reduced and damping improves when the zero moves to the right. Again when the zero moves closer to the origin, the overshoot increases but damping improves. } 3M

The conclusion is that although the characteristic equation roots are generally used to study the relative damping and relative stability of linear control systems, the zeros of the transfer function should not be overlooked in their effects on the transient performance of the system.

5 b) Effect of PI controller on response of the system:-



The PI-controller is a device that produces an output signal consisting of two terms - one proportional to input signal and the other proportional to the integral of input signal. } 2M

The transfer function of PI controller is,

$$G_c(s) = K_P + \frac{K_I}{s} = \frac{K_P (s + \frac{K_I}{K_P})}{s}$$

where K_I & K_P gains of Integral & Proportional res.

The C.L. transfer function of the system with our controller

$$\text{i.e. } \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

The C.L. transfer function of the system with PI controller

$$\text{i.e. } \frac{C(s)}{R(s)} = \frac{G_c(s)G(s)}{1 + G_c(s)H(s)}$$

where $G_c(s)$ = T.F of PI controller.

$G(s)$ = open loop system T.F.

$H(s)$ = Feedback gain.

The introduction of PI-controller into system reduces the steady state error and increases the order and type no. of the system by one.

6a) Routh's Equation: $s^7 + 5s^6 + 9s^5 + 4s^4 + 20s^3 + 24s^2 + 24s + 24 = 0$

s^7	1	9	4	24
s^6	5	20	24	
s^5	7.2	0	0	
s^4	9	16.5	0	
s^3	-13.2	-12.7	0	
s^2	-925.1	24	0	
s^1	+17157	0	0	
s^0	24			

$$\frac{5 \times 9 - 1 \times 9}{5} = 7.2$$

11.4.

There are two sign changes, Hence system is unstable. And two no. of poles are in right half of s-plane and remain 5 no's of poles are in left half of s-plane

— x —

6 b) Frequency domain specifications:-

i. Resonant peak (M_r):- the resonant peak M_r is the max value of M , the magnitude of the closed loop frequency response. A large resonant peak corresponds to a large overshoot in time response.

ii. Resonant Frequency (ω_r):- the resonant frequency ω_r is the frequency at which the resonant peak M_r occurs. This is related to the frequency of oscillations in the step response and thus is indicative of the speed of transient response.

iii. Bandwidth (ω_b):- the bandwidth is the range of frequency for which the system gain is more than -3 dB.

iv. Cut-off rate:- the cut-off rate is the slope of the log magnitude curve near the cut-off frequency.

v. Gain margin and phase margin:- Gain margin and phase margin are measures of relative stability of a system and are related to the closeness of the closed loop poles to the axes.

— X —

7) Bode plot of $G(s)$
$$= \frac{20}{s(1+3s)(1+4s)}$$

The sinusoidal T.F of $G(j\omega)$ is obtained by replacing s by $j\omega$ in the s-plane T.F.

$$G(j\omega) = \frac{20}{j\omega(1+j3\omega)(1+j4\omega)}$$

magnitude plot:-

The corner frequencies are $\omega_{c1} = \frac{1}{4} = 0.25 \text{ rad/sec}$

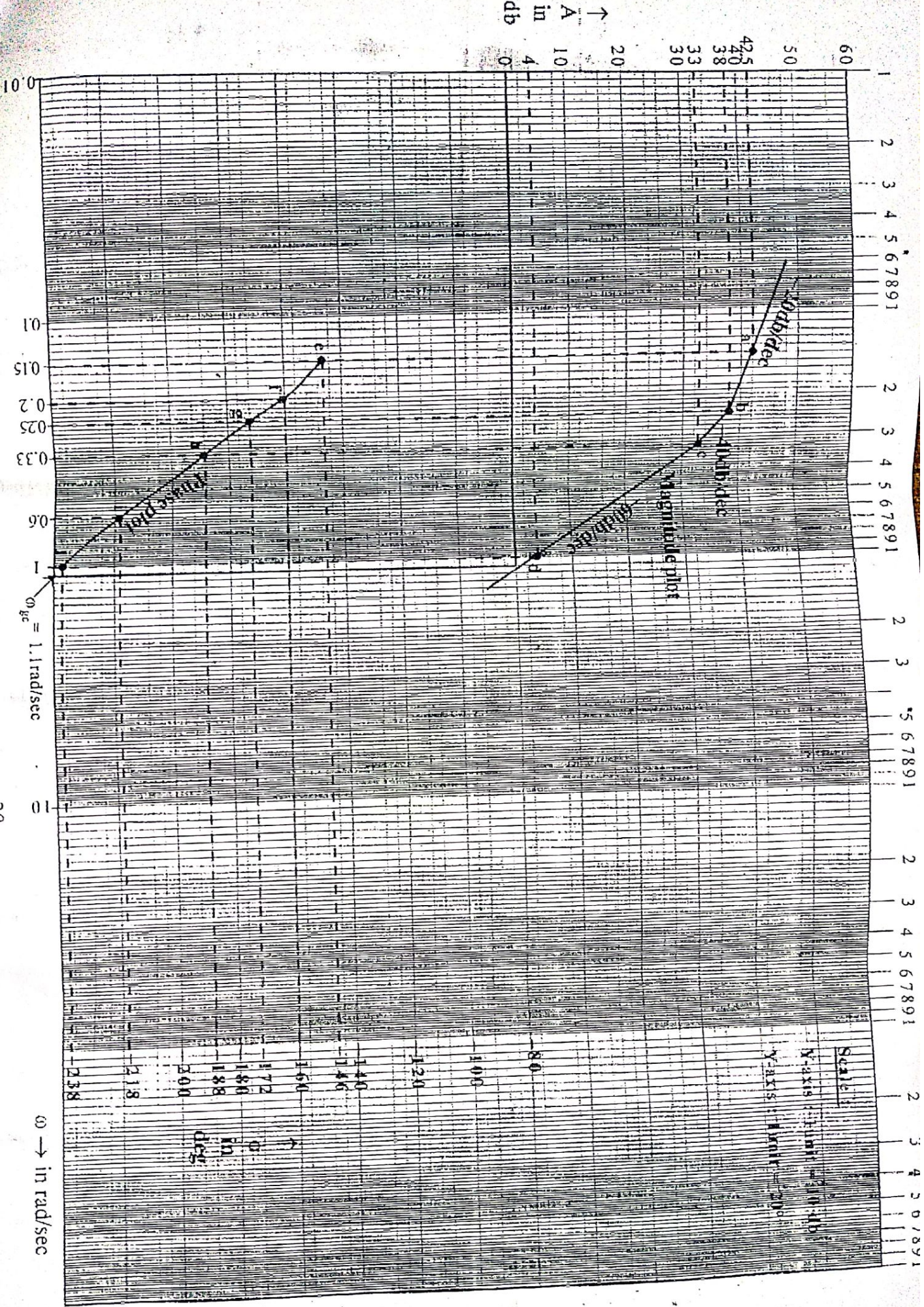
$$\omega_{c2} = \frac{1}{3} = 0.333 \text{ rad/sec}$$

Various forms of $G(s)$ listed in Table-1.

Table-1

Term	Carter frequency rad/sec	Slope db/dec.	Change in slope db/dec.		
$\frac{20}{s}$	---	-20		2m	
$\frac{1}{1+s4s}$	$\omega_{c1} = \frac{1}{4} = 0.25$	-20	-20-20 = -40		
$\frac{1}{1+s3s}$	$\omega_{c2} = \frac{1}{3} = 0.33$	-20	-40-20 = -60	2m	
Choose ω between ω_{c1} and ω_{c2} such that $\omega_{c1} < \omega < \omega_{c2}$ & ω is such that $\omega > \omega_{c1}$. Let $\omega = 0.15$ rad/sec and $\omega_{c1} = 1$ rad/sec. Let us calculate A at ω_{c1}, ω_{c2} & ω.					
At $\omega = \omega_{c1}$		$A = G(s) = 20 \text{ db} + \left \frac{20}{0.15} \right = 112.5 \text{ db}$		2m	
At $\omega = \omega_{c2}$		$A = G(s) = 20 \text{ db} + \left \frac{20}{0.25} \right = 38 \text{ db}$			
At $\omega = \omega_{c1}$		$A = \left[\text{Slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \frac{\omega_{c2}}{\omega_{c1}} \right] + A_{at \omega = \omega_{c1}}$			2m
		$= -40 \times \log \frac{0.33}{0.25} + 38 = 33 \text{ db}$			
At $\omega = \omega$		$A = \left[\text{Slope from } \omega_{c2} \text{ to } \omega \times \log \frac{\omega}{\omega_{c2}} \right] + A_{at \omega = \omega_{c2}}$			2m
		$= -60 \times \log \frac{1}{0.33} + 33 = 4 \text{ db}$			
Plot ω, b, c, d and join by straight line. <u>Phase plot:</u> $\angle G(s) \Rightarrow \phi = -90^\circ - \tan^{-1} 3\omega - \tan^{-1} 4\omega$					
ω rad/sec	$\tan^{-1} 3\omega$ deg	$\tan^{-1} 4\omega$ deg	$\phi = \angle G(s)$ deg		
0.15	24.92	30.96	-146	e	
0.2	30.96	38.66	-160	f	
0.25	36.86	45.0	-172	g	
0.33	44.7	52.8	-188	h	
0.6	60.14	67.38	-218	i	
1	71.56	75.96	-238	j	
Phase: 2m					

Fig. : Bode plot for transfer function, $G(j\omega) = \frac{20}{j\omega(1+j3\omega)(1+j4\omega)}$



8) Root Locus:-

The path taken by the roots of the characteristic equation when one of the system parameters (usually the open-loop gain K) is varied from 0 to ∞ is known as Root Locus.

Rules for construction of Root Locus:-

Rule 1: The root locus is symmetrical about the real axis.

Rule 2: Each Branch of the root locus originates from the open-loop poles corresponding to $K=0$ and terminates at either a finite open-loop zero corresponding to $K=\infty$.

Rule 3: - Segments of the real axis having an odd number of real axis open-loop poles plus zeros to their right are part of the root locus.

Rule 4:- Angle of asymptotes. $\phi_A = \frac{180(2q+1)}{n-m}$
 $q = 0, 1, 2, \dots, n-m$

Rule 5:- The point of intersection of the asymptotes with the real axis is at $S = \sigma_A$ where $\sigma_A = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m}$

Rule 6:- The breakaway and break-in points of the root locus are determined from the roots of the equation $dK/ds = 0$.

Rule 7:- The angle of departure from a complex open-loop zero is given by

$$\phi_p = \pm 180(2q+1) + \phi \quad q = 0, 1, 2, \dots$$

The angle of arrival at a complex open-loop zero is given by

$$\phi_z = \pm 180(2q+1) + \phi \quad q = 0, 1, 2, \dots$$

Rule 8: The points of intersection of the root locus branches with the imaginary axis can be determined by use of Routh Criteria.

Rule 9:- The open-loop gain K at any point $S = S_a$ on the root locus is given by

$$K = \frac{\text{Product of Vectors from open-loop poles to the point } S_a}{\text{Product of Vectors from open-loop zeros to the point } S_a}$$

7 a) state model of the differential equation

$$\frac{d^3 y}{dt^3} + 6 \frac{d^2 y}{dt^2} + 11 \frac{dy}{dt} + 6y = -u(t)$$

The system is governed by third order differential equation
so the no. of state variables are three.

The state variables x_1, x_2 and x_3 are related to phase variables as follows.

$$x_1 = y$$

$$x_2 = \frac{dy}{dt} = \dot{x}_1$$

$$x_3 = \frac{d^2 y}{dt^2} = \dot{x}_2$$

$$\text{Put } y = x_1, \quad \frac{dy}{dt} = x_2 \quad \& \quad \frac{d^2 y}{dt^2} = x_3 \quad \& \quad \frac{d^3 y}{dt^3} = \dot{x}_3$$

$$\therefore \dot{x}_3 + 6x_3 + 11x_2 + 6x_1 + 4 = 0$$

$$\text{or } \dot{x}_3 = -6x_1 - 11x_2 - 6x_3 - 4$$

The state equation are,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -6x_1 - 11x_2 - 6x_3 - 4$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} u$$

$$y = x_1 \quad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

b) controllability matrix

$$Q_c = [B \quad AB \quad A^2 B]$$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 6 \end{bmatrix}$$

$$A \cdot A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -6 & -11 & -6 \\ 36 & 60 & 25 \end{bmatrix}$$

$$\begin{aligned} -6 + 66 \\ -11 + 36 \end{aligned}$$

$$A^T B = \begin{bmatrix} 0 & 0 & 1 \\ -6 & -11 & -6 \\ 36 & 60 & 25 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \\ -25 \end{bmatrix}$$

$$\omega_c = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 6 \\ -1 & 6 & -25 \end{bmatrix}$$

$$|\omega_c| = 1 \neq 0$$

Hence the system is completely stable system. $\} 1m$

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