18MA002

Hall Ticket Number:

I/IV B.Tech (Regular/Supplementary) DEGREE EXAMINATION

January, 2021 **Common to all branches**

Second Semester **Numerical Methods and Advanced Calculus**

Time: Three Hours **Maximum:** 50 Marks

Answer ALL Questions from PART-A. (1X10 = 10 Marks)
Answer ***ANY FOUR*** *questions from PART-B.* (4X10=40 Marks)

Part - A

1. Answer all questions (1X10=10 Marks)

a) Define an Algebraic equation. 1M

b) Decompose A= $\begin{bmatrix} 4 & 1 \\ 3 & -5 \end{bmatrix}$ as LU. Here L and U are lower and upper triangular matrices respectively. 1M

c) Write Lagrange's interpolation formula. 1M

d) State the Simpson's 1/3rd rule of integration. 1M

e) Write the general formula to find y_1 for the initial value problem $\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$ in Runge-Kutta method of 4th order. 1M

f) Evaluate the double integral $\int_0^1 \int_0^{1-x} dx\, dy$. 1M

g) Transform $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$ into polar form. 1M

h) Is the vector field $3x^4y^2I + 4x^3z^2J + 3x^2y^2K$ solenoidal? 1M

i) Find a vector normal to the surface f(x,y,z)=xyz. 1M

j) State Gauss divergence theorem. 1M

Part - B

2. a) Find a root of the equation $x^3 - 2x - 5 = 0$ using the Bisection method. 5M

b) Solve $10x + y + z = 12,\ 2x + 10y + z = 13,\ 2x + 2y + 10z = 14$ by using Factorization method. 5M

3. a) Find by Newton's method, the real root of the equation $3x = \cos x + 1$ 5M

b) Solve the system of equations $27x + 6y - z = 85,\ 6x + 15y + 2z = 72,\ x + y + 54z = 110$ using Gauss-Seidel iteration method. Do five iterations. 5M

4. a) Find the cubic polynomial which takes the following values by using Newton's forward interpolation formula

x:	0	1	2	3
f(x):	1	2	1	10

5M

b) Given the values

x:	5	7	11	13	17
y:	150	392	1452	2366	5202

Evaluate f(9), using Newton's divided difference formula. 5M

5. a) Find the value of y for x=0, using Picard's method for the initial value problem $\frac{dy}{dx} = y,\ y(0) = 1$. 5M

   b) Use the Trapezoidal rule to estimate the integral $\int_0^2 e^{x^2} dx$ taking 10 intervals. 5M

6. a) Evaluate $\iint_A xy\, dx\, dy$, where A is the domain bounded by x-axis, ordinate x=2a and the curve $x^2 = 4ay$. 5M

   b) Evaluate $\iint r^3\, dr\, d\theta$ over the area bounded between the circles $r = 2\sin\theta,\ r = 4\sin\theta$. 5M

7. a) Evaluate the triple integral $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz\, dy\, dx$ 5M

   b) Find the volume bounded by the xy-plane, the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 3$ 5M

8. a) Find the directional derivative of $f(x, y, z) = xy^2 + yz^3$ at the point (2,-1,1) in the direction of the vector I+2J+2K. in what direction the directional derivative is maximum? 5M

   b) Using Stoke's theorem evaluate $\int_C \left[(x+y)dx + (2x-z)dy + (y+z)dz \right]$ where C is the boundary of the triangle with vertices (2,0,0), (0,3,0),(0,0,6). 5M

9 a) Verify Green's theorem for $\int_C \left[(xy + y^2)dx + x^2 dy \right]$ where C is bounded by $y = x$ and $y = x^2$. 10M