18MA004

Hall Ticket Number:

II/IV B.Tech (Regular/Supplementary – Repeat Exam) DEGREE EXAMINATION

January, 2021 **ECE & EIE**

Fourth Semester **COMPLEX VARIABLES & SPECIAL FUNCTIONS**

Time: Three Hours **Maximum:** 50 Marks

Answer ALL Questions from PART-A. (1X10 = 10 Marks)

*Answer **ANY FOUR** questions from PART-B.* (4X10=40 Marks)

PART-A

1.	a)	Find the argument of $1+i$.	**CO1**	**1M**
	b)	Check whether the function u = x^2 - y^2 is harmonic or not?	**CO1**	**1M**
	c)	Find the value of $\oint_c \frac{dz}{z-a}$, where c is $\lvert z-a\rvert = r$	**CO1**	**1M**
	d)	State Cauchy Residue Theorem.	**CO2**	**1M**
	e)	Find the nature of singularity of the function $f(z) = \frac{z - Sin\,Z}{z^2}$	**CO2**	**1M**
	f)	What is the Fourier complex Integral formula?	**CO3**	**1M**
	g)	State Linear property.	**CO3**	**1M**
	h)	Write the value of $F\left(\frac{\partial^2 u}{\partial x^2}\right)$	**CO3**	**1M**
	i)	State Bessel's Differential equation.	**CO4**	**1M**
	j)	Determine $J_1(0)$	**CO4**	**1M**

PART-B

2.	a)	Find all the roots of $(1+i)^{\frac{1}{4}}$	**CO1**	**5M**
	b)	Determine the analytic function whose real part is $\cos x \cosh y$ by using Milne Thomson's Method.	**CO1**	**5M**
3.		If F(ζ)=$\int_c \frac{4z^2+z+5}{(z-\zeta)}dz$ where c is the ellipse $\left(\frac{x}{2}\right)^2+\left(\frac{y}{3}\right)^2=1$, Find the values of (i). F(3.5) , (ii). F(i), (iii). F^1 (-1) and (iv). F^{11}(-i)	**CO1**	**10M**
4.	a)	Expand f (z) = $\frac{z}{(z-1)(z-3)}$ for $\lvert z-1\rvert < 2$ in Laurent series.	**CO2**	**5M**
	b)	Expand $Sin\,z$ in a Taylor's series about $z=0$ and determine the region of Convergence.	**CO2**	**5M**

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5. a) Evaluate $\oint_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$,where c is the circle $|z| = 3$ **CO2** **5M**

b) Find $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$ **CO2** **5M**

6. a) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$ **CO3** **5M**

b) Express f(x)= $\begin{cases} 1 & for\ 0 \le x \le \pi \\ 0 & for\ x > \pi \end{cases}$ as a Fourier sine integral and hence evaluate $\int_0^{\infty} \frac{1-\cos \pi\lambda}{\lambda} \sin x\lambda \ d\lambda$ **CO3** **5M**

7. Find the Fourier transform of f(x)= $\begin{cases} 1-x^2, & |x| \le 1 \\ 0 & , |x| > 1 \end{cases}$ Hence evaluate $\int_0^{\infty} \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2} dx$ **10M** **CO3**

8. Solve in series the equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ **CO4** **10M**

9. a) Prove that $J_n(x) = \frac{x}{2n}\left(J_{n-1}(x) + J_{n+1}(x)\right)$ **CO4** **5M**

b) Determine $J_{\frac{1}{2}}(x)$ **CO4** **5M**