II/IV B.Tech (Regular) Degree Examination, AUG 2022

Civil Engineering

20CE404 | Hydraulics & Hydraulic Machines

Scheme of Evaluation

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AUGUST, 2022

20CE404/ Hydraulics & Hydraulic Machines

Scheme of Evaluation

1.

a) Energy Correction factor (α) $= \dfrac{\text{K.E/sec based on actual velocity}}{\text{K.E/sec based on average velocity}}$

b) Critical Depth (y_c): The depth at which specific energy is minimum. $y_c = \left(\dfrac{q^2}{g}\right)^{1/3}$.

c) $C = \dfrac{1}{N}(R)^{1/6} \Rightarrow N = \dfrac{1}{C}(R)^{1/6} = \dfrac{1}{50}(1)^{1/6} = 0.02.$

d) Most Economical Rectangular Channel conditions:

$b = 2d$; $R = \dfrac{d}{2}$.

e) $b = 2m$; $d = 1m$, $Q = 5\ m^3/sec$

$A = 2 \times 1 = 2m^2$; $V = Q/A = 2.5\ m/sec$

Froude Number (F_r) $= \dfrac{V}{\sqrt{gD}} = \dfrac{2.5}{\sqrt{9.81 \times 1}} = 0.798.$

f) $y = 2.6m$; $y_n = 1.5m$; $y_c = 2.5m$

$y_n < y_c \longrightarrow$ steep slope

$y_c > y > y_n \longrightarrow$ Zone-2

$\therefore$ The surface profile is 'S_2'

(g) $y_1 = 0.5m$; $y_2 = 4m$

$$y_1 y_2 (y_1 + y_2) = \frac{2q^2}{g} \Rightarrow q^2 = \frac{y_1 y_2 (y_1 + y_2) \times g}{2}$$

$$q^2 = \frac{0.5 \times 4 \times (0.5 + 4) \times 9.81}{2} = 44.145$$

$$\therefore q = 6.644 \ m^3/s/m.$$

(h) Froude number = 3.5 ; If it is between 2.5 to 4.5, Hence the hydraulic jump is "Oscillating jump."

(i) Impulse Momentum principle :

$$F_{Net} = \frac{Mass}{sec} [\text{Final velocity} - \text{Initial velocity}]$$

(j)

$$\text{Specific speed of turbine } (N_s) = \frac{N\sqrt{P}}{H^{5/4}}$$

(k) Draft tube : It is a pipe of gradually increasing area which connects the outlet of the runner to the tail race. This pipe of gradually increasing area is called a Draft tube.

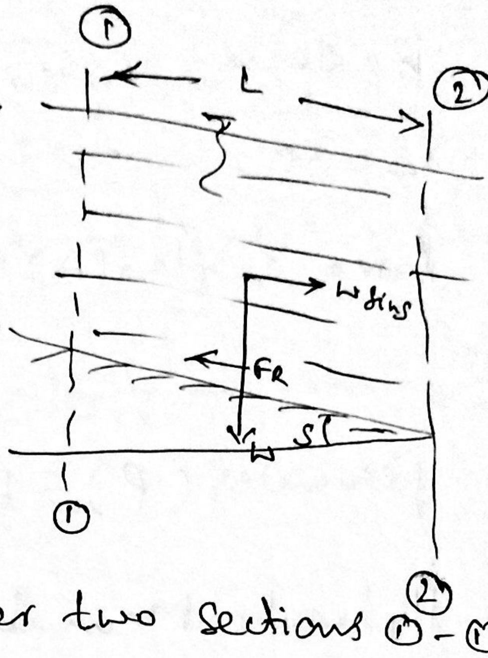
(L) When pressure at the discharge end of the pump is too high, causing high velocity fluid to recirculate between pump impeller and housing, causing Cavitation.

(m) The dimensions of each term on both sides of an equation are the same, the equation is known as dimensionally homogeneous equation.

(n) $$\text{Reynold's number } (Re) = \frac{\rho v d}{\mu}.$$

UNIT - I

2(a)



Consider uniform flow of water in a channel. As the flow is uniform, it means the velocity, depth of flow and area of flow will be constant for a given length of the channel. Consider two sections 1-1 & 2-2 — (2M)

The forces acting on the water between sections 1-1 & 2-2 are:

i) Pressure force acting section 1-1, P_1

ii) Pressure force acting section 2-2, P_2

iii) Component of weight of water along the direction of flow

iv) Friction resistance against flow of water, F_R

∴ Resolving all forces in the direction of flow; — (2M)

$$P_1 - P_2 + W \sin S - F_R = 0 \quad \text{—— (1M)}$$

$$\gamma A L \sin S - f' P L V^2 = 0$$

$$V^2 = \frac{\gamma A \cancel{L} \sin S}{f' P \cancel{L}} = \frac{\gamma}{f'} \times \frac{A}{P} \times \sin S$$

$$\therefore V^2 = C R \sin S$$

$$\boxed{\therefore V = C\sqrt{RS}}$$

$$\therefore Q = AC\sqrt{RS} \quad \text{—— (2M)}$$

2(b): $Q = 200\,lit/sec = \frac{200}{1000} = 0.2\ m^3/sec$ —— (1M)

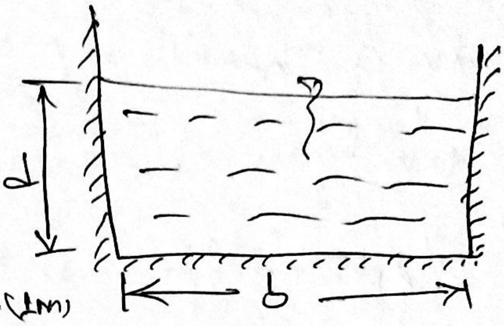
$b = 50\ cm = 0.5\ m$, $d = 20\ cm = 0.2\ m$

$C = 50$

Area of flow $(A) = bd$

$= 0.5 \times 0.2$

$= 0.1\ m^2$ —(1M)



Perimeter $(P) = b + 2d = 0.5 + 2(0.2) = 0.9\ m$ —(1M)

Hydraulic Mean depth $(R) = \frac{A}{P} = \frac{0.1}{0.9} = 0.11\ m$ —(1M)

According to Chezy's $Q = AC\sqrt{RS}$ —(1M)

$0.2 = 0.1 \times 50 \times \sqrt{0.11 \times S}$

$\therefore S = 0.0145 = \frac{1}{68.75}$ —(2M)

(3)

(a) Critical flow: The flow at which Specific Energy is minimum is called Critical flow. (or) $F_r = 1$.

Critical flow conditions for Rectangular channel: —(1M)

i) Critical depth (y_c): The depth at which specific energy is minimum. —(1M)

$$\frac{d(E)}{d(y)} = 0 \Rightarrow \frac{d}{dy}\left[y + \frac{v^2}{2g}\right] = 0$$

$$\frac{d}{dy}\left[y + \frac{q^2}{2gy^2}\right] = 0$$

$$\Rightarrow 1+\frac{q^2}{2g}\left[\frac{-2}{y^3}\right]=0$$

$$1-\frac{q^2}{gy^3}=0 \Rightarrow 1=\frac{q^2}{gy^3} \Rightarrow y^3=\frac{q^2}{g}$$

$$\therefore y_c=\left(\frac{q^2}{g}\right)^{1/3}$$ —— (1M)

ii) Critical velocity (V_c): The velocity at which specific energy is minimum. —— (1M)

$$q=\frac{Q}{b}=\frac{\cancel{b}\times y\times V}{\cancel{b}}=Vy$$

$$y_c^3=\frac{q^2}{g} \Rightarrow gy_c^3=q^2=V_c^2y_c^2$$

$$\therefore V_c=\sqrt{gy_c}$$ —— (1M)

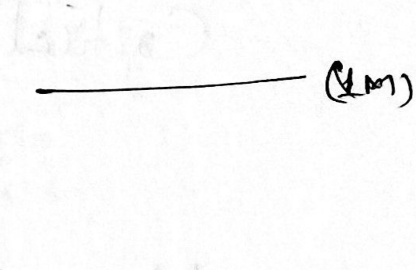
iii) Minimum specific energy (E_{min}):

$$E_{min}=y_c+\frac{V_c^2}{2g}=y_c+\frac{\cancel{g}y_c}{2\cancel{g}}=\frac{3}{2}y_c$$

$$\therefore E_{min}=\frac{3}{2}y_c$$ —— (2M)

(3)
(b) Width (b) = 5m, bed slope (S) = 0.0001

normal depth (y) = 2m, C = 50



Area of flow (A) = b × y = 5 × 2
= 10 m²

Perimeter (P) = b + 2d = 5 + 2(2) = 9m

Hydraulic mean depth (R) = $\frac{A}{P}=\frac{10}{9}$ = 1.11 m —— (1M)

According to chezy's $Q = AC\sqrt{RS}$

$$Q = 10 \times 50 \times \sqrt{1.11 \times 0.0001}$$

$$\therefore Q = 5.268 \ m^3/sec \quad \text{—— (1M)}$$

$$q = \frac{Q}{b} = \frac{5.268}{5} = 1.053 \ m^3/s/m \quad \text{—— (1M)}$$

$$V = Q/A = \frac{5.268}{10} = 0.527 \ m/sec \quad \text{—— (1M)}$$

$$\text{Specific Energy } (E) = y + \frac{V^2}{2g} = 2 + \frac{(0.527)^2}{2 \times 9.81}$$

$$= 2.028 \ Nm/N \quad \text{—— (1M)}$$

$$\text{Critical depth } (y_c) = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{1.053^2}{9.81}\right)^{1/3}$$

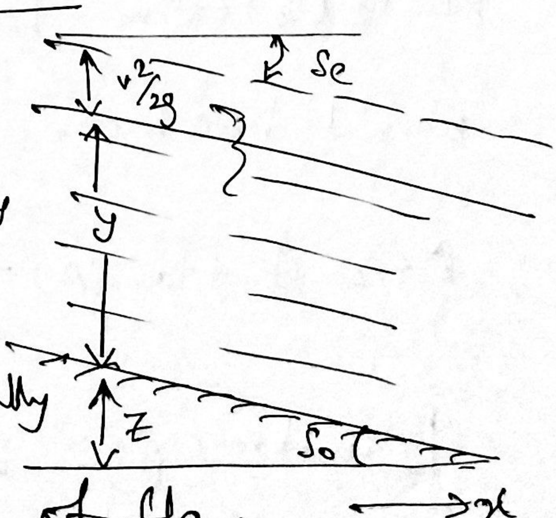
$$= 0.483 \ m. \quad \text{—— (1M)}$$

$$\text{Minimum Specific Energy } (E_{min}) = 3/2 \ y_c$$

$$= 3/2\,(0.483) = 0.724 \ Nm/N \quad \text{—— (1M)}$$

UNIT-II

(4)
(a)



Consider a Rectangular having gradually Varied flow.
The depth of flow is gradually decreasing in the direction of flow.

—— (1M)

The energy equation at any section is given by B.E

$$E = y + \frac{V^2}{2g} + z$$ ——— (1M)

differentiating this equation with respect to x-direction

$$\frac{dE}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right)$$ ——— (1M)

$$-S_e = -S_o + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{Q^2}{2gA^2}\right)$$

$$S_o - S_e = \frac{dy}{dx} + \frac{d}{dx}\left(\frac{Q^2}{2gb^2y^2}\right)$$

$$= \frac{dy}{dx} + \frac{d}{dy}\left[\frac{1}{y^2}\right]\frac{dy}{dx} \cdot \frac{Q^2}{2gb^2}$$

$$= \frac{dy}{dx} + \frac{Q^2}{2gb^2}\left[\frac{-2}{y^3}\right]\frac{dy}{dx}$$

$$= \frac{dy}{dx} + \frac{-\cancel{2}Q^2}{\cancel{2}gy^3b^2}\frac{dy}{dx}$$

$$= \frac{dy}{dx} + \left[-\frac{V^2}{gy}\right]\frac{dy}{dx}$$

$$S_o - S_e = \frac{dy}{dx}\left[1 - \frac{V^2}{gy}\right]$$

$$\therefore \frac{dy}{dx} = \frac{S_o - S_e}{\left[1 - V^2/gy\right]}$$ ——— (3M)

Assumptions: 1) The channel is prismatic

2) Energy correction factor (α) is unity

3) bed slope is small ——— (1M)

4) The flow is steady and hence discharge is constant

4(b) Bed width $(b) = 5m$, Discharge $(Q) = 15\ m^3/sec$

depth $(y) = 1.5m$, bed slope $(S_0) = 0.0001$

Manning's constant $(N) = 0.012$

Soln: Area of flow $(A) = by = 5 \times 1.5 = 7.5\ m^2$

perimeter $(P) = b + 2y = 5 + 2(1.5) = 8m$

Hydraulic Mean depth $(R) = A/P = \frac{7.5}{8} = 0.937\ m$

Velocity $(V) = Q/A = 15/7.5 = 2\ m/sec$ — (1M)

According to Manning's $Q = \frac{A}{N} R^{2/3} S_e^{1/2}$ — (1M)

$$15 = \frac{7.5}{0.012} [0.937]^{2/3} [S_e]^{1/2}$$

$$S_e = (0.025)^2 = 6.282 \times 10^{-4}$$ — (1M)

$\therefore$ Slope of water surface $\left(\frac{dy}{dx}\right) = \frac{S_0 - S_e}{\left(1 - V^2/gy\right)}$ — (1M)

$$= \frac{0.001 - 6.282 \times 10^{-4}}{\left[1 - \frac{(2)^2}{9.81 \times 1.5}\right]}$$

$$= \frac{-5.282 \times 10^{-4}}{0.728}$$

$$= -7.255 \times 10^{-4}$$

— (3M)

$\therefore$ Hence, Depth of flow is decreasing,

(5)
(a) Bed width (b) = 2 m, Pre-Jump (y_1) = 0.5 m

Discharge (Q) = 15 m³/sec

Discharge per unit width (q) = Q/b

$$= \frac{15}{2} = 7.5 \text{ m}^3/\text{s}/\text{m}$$
—— (1M)

The depth of flow after the Jump (or Post Jump

$$y_2 = -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2q^2}{g y_1}}$$ —— (2M)

$$= -\frac{0.5}{2} + \sqrt{\frac{0.5^2}{4} + \frac{2(7.5)^2}{9.81 \times 0.5}}$$

$$= -0.25 + \sqrt{0.0625 + 22.9357}$$

$$= -0.25 + 4.796$$

$\therefore$ Post Jump (y_2) = 4.54 m —— (2M)

$$\therefore \text{Head loss } (h_L) = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$$ —— (1M)

$$= \frac{[4.54 - 0.5]^3}{4 \times 0.5 \times 4.54}$$

$$\therefore h_L = 7.262 \text{ m}$$ —— (1M)

(5)
(b) Energy loss (or Head loss (h_L) = $E_1 - E_2$ —— (1M)

$$h_L = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right) \quad \text{—— (1M)}$$

$$= \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - (y_2 - y_1)$$

$$= \left(\frac{q^2}{2gy_1^2} - \frac{q^2}{2gy_2^2}\right) - (y_2 - y_1)$$

$$= \frac{q^2}{2g}\left[\frac{1}{y_1^2} - \frac{1}{y_2^2}\right] - (y_2 - y_1)$$

$$= \frac{q^2}{2g}\left[\frac{y_2^2 - y_1^2}{y_1^2 y_2^2}\right] - (y_2 - y_1) \quad \text{—— (2M)}$$

$$= gy_1y_2\frac{(y_2+y_1)}{2} \times \frac{y_2^2 - y_1^2}{2g\,y_1^2 y_2^2} - (y_2 - y_1)$$

$$= \frac{(y_2+y_1)(y_2^2 - y_1^2)}{4y_1y_2} - (y_2 - y_1)$$

$$= \frac{(y_2+y_1)(y_2+y_1)(y_2-y_1)}{4y_1y_2} - (y_2 - y_1)$$

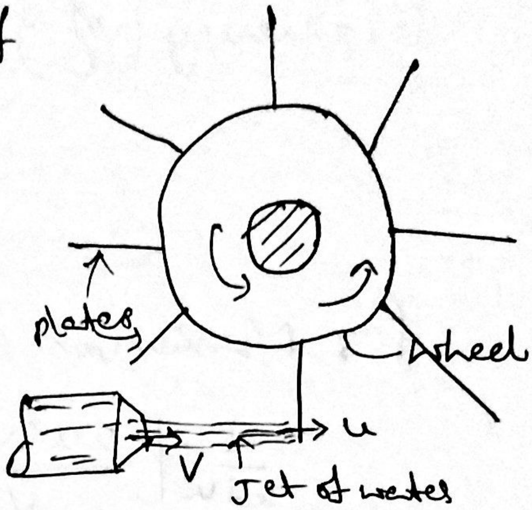
$$= (y_2 - y_1)\left[\frac{(y_2+y_1)^2}{4y_1y_2} - 1\right]$$

$$= (y_2 - y_1)\left[\frac{y_2^2 + y_1^2 + 2y_1y_2 - 4y_1y_2}{4y_1y_2}\right]$$

$$\therefore h_L = \frac{(y_2 - y_1)(y_2 - y_1)^2}{4y_1y_2} = \frac{(y_2 - y_1)^3}{4y_1y_2} \quad \text{—— (3M)}$$

(6)

(a) The force exerted by a Jet of water on a single moving plate is not practically feasible.



This case is only a theoretical one. In actual practice, a large number of plates are connected on the circumference of a wheel at a fixed distance. The jet strikes a plate and due to the force exerted by the jet on the plate, the wheel starts moving and the 2nd plate mounted on the wheel appears before the jet, which again exerts the force on the 2nd plate. Thus each plate appears successively before the jet and the jet exerts force on each plate. The wheel starts moving at a constant speed. ——— (2M)

∴ Force exerted on a series of plates (F_x)

Apply impulse momentum Equation:

$$F_x = \frac{\text{Mass}}{\text{Sec}} [\text{Initial velocity} - \text{final velocity}]$$

$$\therefore F_x = \rho a V (V - u)$$ ——— (1M)

$$\text{Work done/sec} = F_x \times u = \rho a V(V-u)u \quad \text{—— (1M)}$$

$$\text{Efficiency } (\eta) = \frac{\text{Work done/sec}}{\text{K.E/sec}} = \frac{\rho a V(V-u)u}{\frac{1}{2}\rho a V^3}$$

$$= \frac{2u[V-u]}{V^2} \quad \text{—— (1M)}$$

for Maximum efficiency $\frac{d(\eta)}{d(u)} = 0 \Rightarrow$

$$\frac{d}{du}\left[\frac{2u(V-u)}{V^2}\right] = 0 \Rightarrow 2V - 4u = 0$$

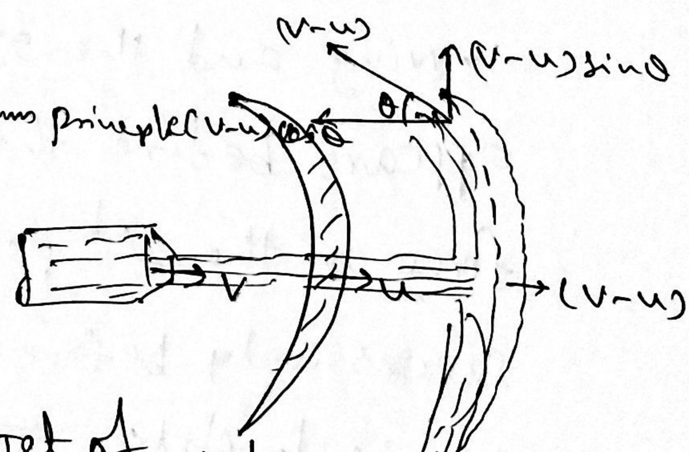
$$V = 2u \; ; \; u = V/2 \quad \text{—(1M)}$$

$$\therefore \text{Maximum efficiency } (\eta_{max}) = \frac{2u[2u-u]}{(2u)^2} \times 100$$

$$= \frac{1}{2} \times 100 = 50\% \quad \text{—(1M)}$$

(6)
(b)

Applying Impulse Momentum principle



$$F_{Net} = \frac{\text{Mass}}{\text{sec}}\left[\text{I.V} - \text{F.V}\right]$$

(2M)

$\therefore$ force exerted by the jet of water on the curved plate in the direction of the jet (F_x)

$$F_x = \rho a(V-u)\left[(V-u) - (-(V-u)\cos\theta)\right]$$

$$= \rho a(V-u)\left[(V-u) + (V-u)\cos\theta\right]$$

$$= \rho a(V-u)^2\left[1 + \cos\theta\right] \quad \text{—— (4M)}$$

$$\therefore \text{Work done/sec} = F_x \times u = \rho a(V-u)^2[1+\cos\theta] \times u$$

(1M)

7(a)

Velocity of plate $(u) = S\sqrt{2gH}$

$$u \propto \sqrt{H} \quad \text{--- (1)}$$

Velocity of flow $(V_f) = F\sqrt{2gH}$

$$V_f \propto \sqrt{H} \quad \text{--- (2)}$$

Tangential velocity $(u) = \frac{\pi D N}{60}$

$$u \propto DN \quad \text{--- (3)}$$

from Eq (1) & (3) $\Rightarrow \sqrt{H} \propto DN$

$$D \propto \frac{\sqrt{H}}{N} \quad \text{--- (4)}$$

(1M)

Discharge (Q) = Area x velocity of flow

$$Q \propto D^2 \times V_f$$

$$Q \propto \left(\frac{\sqrt{H}}{N}\right)^2 \times \sqrt{H}$$

$$Q \propto \frac{H^{3/2}}{N^2} \quad \text{--- (5)}$$

(1M)

Overall efficiency $(\eta_o) = \frac{S.P}{W.P} = \frac{P}{\gamma Q H}$

$$P = \eta_o \times \gamma Q H$$

$$P \propto Q H$$

$$P \propto \frac{H^{3/2}}{N^2} \times H$$

$$P \propto \frac{H^{5/2}}{N^2}$$

$$P = K \frac{H^{5/2}}{N^2} \quad \text{--- (6)}$$

(1M)

If $P = 1$ kw, $H = 1$ m, the Speed $(N) = N_s$

$$1 = K \frac{(1)^{5/2}}{N_s^2} \Rightarrow K = N_s^2$$

$$P = N_s^2 \frac{H^{5/2}}{N^2} \Rightarrow N_s^2 = \frac{N^2 P}{H^{5/2}}$$

$$\therefore N_s = \frac{N\sqrt{P}}{H^{5/4}}$$ ——— (1M)

7(b)

Net Head $(H) = 40$ m, Speed $(N) = 500$ rpm

Shaft power $(P) = 400$ kw, $\eta_o = 85\%$, $\eta_h = 90\%$,

flow ratio $(F) = 0.2$, breadth ratio $(n) = 0.1 = \frac{B_1}{D_1}$

$D_1 = 2D_2$, Thickness of vanes $= 5\%$ of circumferential area

$V_{f_1} = V_{f_2}$, $V_{w_2} = 0$

Velocity of flow $(V_{f_1}) = F\sqrt{2gH}$

$$= 0.2\sqrt{2 \times 9.81 \times 40}$$
$$= 5.60 \text{ m/sec}$$ ——— (1M)

$V_{f_1} = V_{f_2} = 5.60$ m/sec

$$\eta_o = \frac{\text{Shaft power}}{\text{Water power}} \Rightarrow 0.85 = \frac{400 \times 10^3}{\frac{\gamma \times Q \times H}{1000}}$$

$$0.85 = \frac{400 \times 10^3}{\left(\frac{9810 \times Q \times 40}{1000}\right)}$$

$Q = 1.199$ m³/sec ——— (1M)

Q = Actual area of flow × velocity of flow

$= 0.95 \times \pi \times D_1 \times B_1 \times V_{f_1}$

$1.199 = 0.95 \times \pi \times D_1 \times 0.1 D_1 \times 5.60$

$D_1 = 0.846 \text{ m}$ ———— (1M)

$\frac{B_1}{D_1} = 0.1 \Rightarrow B_1 = 0.1 \times D_1 = 0.1 \times 0.846$

$= 0.0846 \text{ m} = 84 \text{ mm}$ —— (1M)

Tangential velocity $(u_1) = \frac{\pi D_1 N}{60}$

$= \frac{\pi \times 0.846 \times 500}{60}$

$= 22.15 \text{ m/sec}$ ——— (1M)

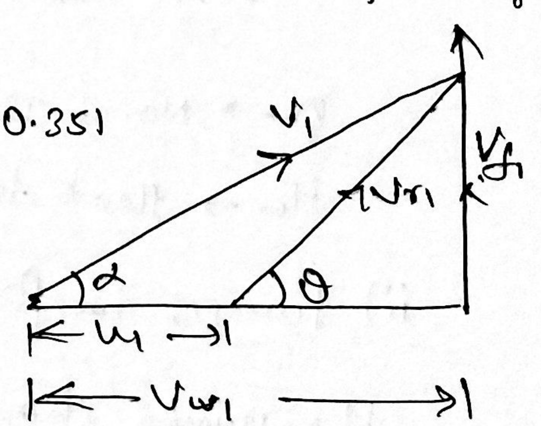
Hydraulic efficiency $(\eta_h) = 0.9 = \frac{V_{w_1} u_1}{gH}$

$0.9 = \frac{V_{w_1} \times 22.15}{9.81 \times 40}$

$\therefore V_{w_1} = 15.94 \text{ m/sec}$ ——— (1M)

Guide blade angle (α); Using Inlet velocity Triangle

$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{5.60}{15.94} = 0.351$



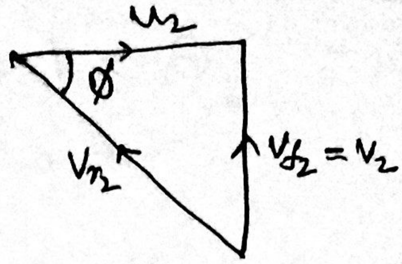
$\alpha = \tan^{-1}(0.351)$

$\therefore \alpha = 19^\circ 21'$

$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{5.60}{(15.94 - 12.15)}$

$\theta = 42^\circ 2'$ ——— (2M)

Using outlet velocity Triangle



$$\tan\phi = \frac{V_{f_2}}{u_2} \Rightarrow$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi\left(\frac{D_1}{2}\right)N}{60} = \frac{\pi \times \left(\frac{0.846}{2}\right)\times 500}{60}$$

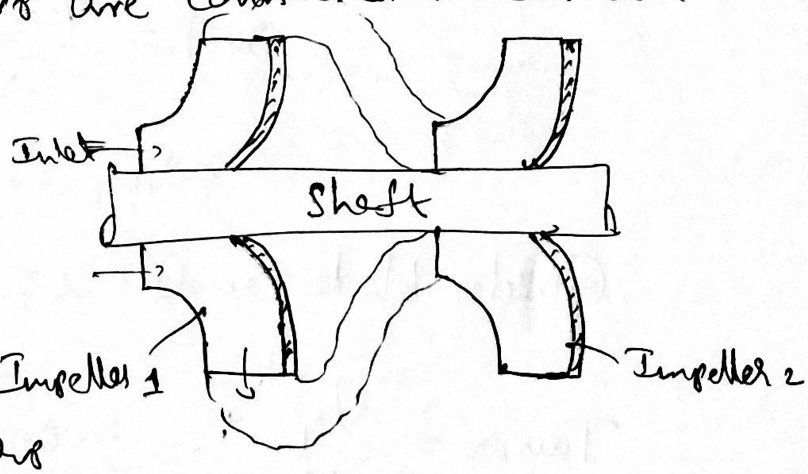
$u_2 = 11.074$ m/sec ——— (1M)

$$\tan\phi = \frac{5.60}{11.074} = 0.505$$

$\therefore \phi = 26.824 = 26^\circ 49'$ ——— (1M)

UNIT - IV

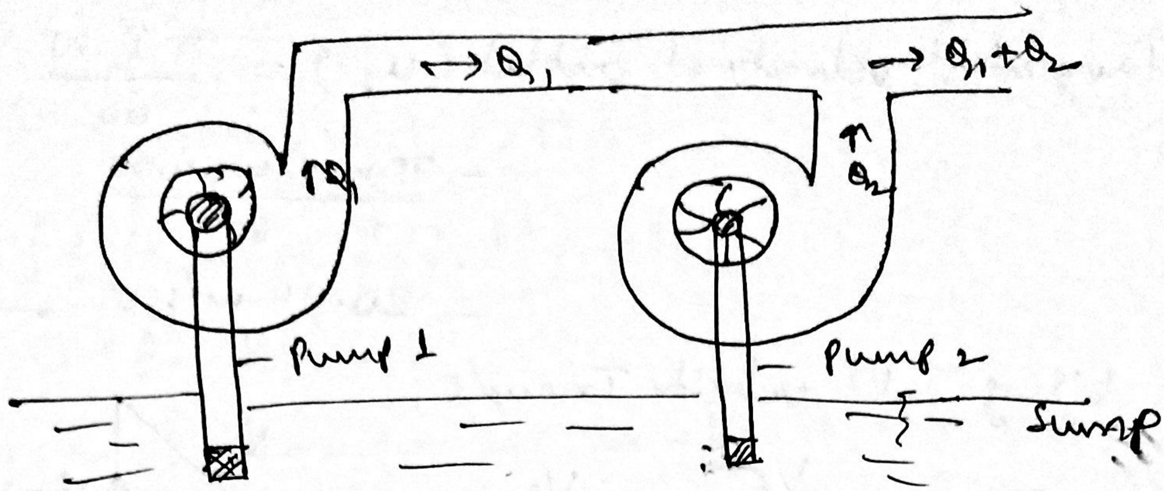
(8)
(a) i) Pumps in Series: For developing a high head, a number of impellers are connected in series (or) on the same shaft.



$\therefore$ Total Head developed
$= n \times H_m$

$n \rightarrow$ No. of Impellers
$H_m \rightarrow$ Head developed by each Impeller. ——— (3M)

ii) Pumps in Parallel: For obtaining high discharge, the pumps should be connected in parallel. Each of pump lifts water from a common pump and discharges water to a common pipe.



Total discharge $(Q_T) = n \times Q$

$n \rightarrow$ No. of pumps

$Q \rightarrow$ Discharge from one pump. (2M)

(iii) Efficiencies of pump:

a) Manometric Efficiency $(\eta_{mano}) = \dfrac{\text{Manometric Head}}{\text{Head at Impeller}}$

b) Mechanical Efficiency $(\eta_m) = \dfrac{\text{Power at Impeller}}{\text{Power at Shaft}}$

c) Overall efficiency $(\eta_o) = \dfrac{\text{Output power}}{\text{Input power}}$ (2M)

(8)
(b)

$D_1 = 200\text{ mm} = 0.2\text{ m}$; $D_2 = 400\text{ mm} = 0.4\text{ m}$

$N = 1000\text{ rpm}$; $\theta = 20°$; $\phi = 30°$

Water enters radially; $\alpha = 90°$ & $V_{w_1} = 0$

$V_{f_1} = V_{f_2}$

Tangential velocity at Inlet $(u_1) = \dfrac{\pi D_1 N}{60}$

$$= \frac{\pi \times 0.2 \times 1000}{60} = 10.47\text{ m/sec}$$

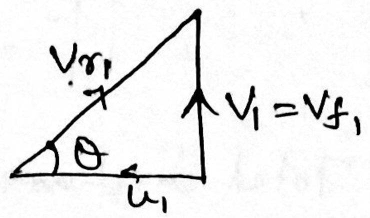
(1M)

Tangential velocity at outlet (u_2) $= \frac{\pi D_2 N}{60}$

$= \frac{\pi \times 0.4 \times 1000}{60}$

$= 20.94$ m/sec ——— (1M)

Using Inlet velocity Triangle;

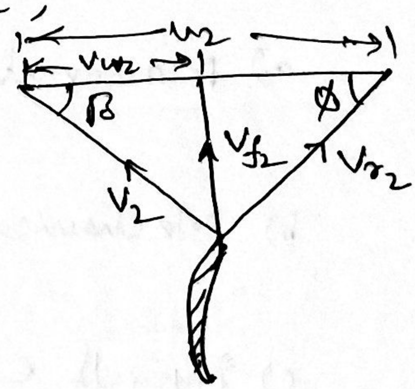


$\tan\theta = \frac{V_{f_1}}{u_1} = \frac{V_{f_1}}{10.47}$

$V_{f_1} = 10.47 \times \tan 20^\circ = 3.81$ m/sec

$V_{f_1} = V_{f_2} = 3.81$ m/sec ——— (2M)

Using Outlet velocity Triangle;



$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$

$\tan 30^\circ = \frac{3.81}{20.94 - V_{w_2}}$

$20.94 - V_{w_2} = 6.599$

$V_{w_2} = 14.34$ m/sec ——— (2M)

∴ Work done by impeller per kg of water per second

$= \frac{1}{g} V_{w_2} u_2$

$= \frac{1}{9.81} [14.34 \times 20.94]$

$= 30.61$ Nm/N. ——— (1M)

(9)
(a)

$$F = f(L, V, g, \rho, \mu)$$

$$f(F, L, V, g, \rho, \mu) = 0$$

$n = 6, \quad m = 3$

$\therefore$ No. of dimensionless variables = π - terms = $(n-m)$

$= 6 - 3 = 3.$

$$f(\pi_1, \pi_2, \pi_3) = 0$$

$$\pi_1 = L^{a_1} V^{b_1} \rho^{c_1} \cdot F$$

$$\pi_2 = L^{a_2} V^{b_2} \rho^{c_2} g$$

$$\pi_3 = L^{a_3} V^{b_3} \rho^{c_3} \mu$$ (1M)

$$\pi_1 = L^{a_1} V^{b_1} \rho^{c_1} . F$$

$$M^0 L^0 T^0 = L^{a_1} [LT^{-1}]^{b_1} [ML^{-3}]^{c_1} MLT^{-2}$$

power of M: $0 = c_1 + 1 \Rightarrow c_1 = -1$

power of L: $0 = a_1 + b_1 - 3c_1 + 1$

$a_1 = -b_1 + 3c_1 - 1 \Rightarrow a_1 = -2$

power of T: $0 = -b_1 - 2 \Rightarrow b_1 = -2$

$$\pi_1 = L^{-2} V^{-2} \rho^{-1} F$$

$$\therefore \pi_1 = \frac{F}{L^2 V^2 \rho} \quad \text{---} \ (1)$$ (2M)

$$\pi_2 = L^{a_2} V^{b_2} \rho^{c_2} \cdot g$$

$$M^0 L^0 T^0 = L^{a_2} [LT^{-1}]^{b_2} [ML^{-3}]^{c_2} [LT^{-2}]$$

power of M: $0 = c_2 \Rightarrow c_2 = 0$

power of L: $0 = a_2 + b_2 - 3c_2 + 1 \Rightarrow 0 = a_2 - 2 - 0 + 1$

$0 = a_2 - 1$

$a_2 = 1$

power of T: $0 = -b_2 - 2 \Rightarrow b_2 = -2$

$$\pi_2 = L^1 V^{-2} \rho^0 \cdot g$$

$$\pi_2 = \frac{gL}{V^2} \quad \text{——— ②} \qquad \text{———— (2M)}$$

$$\pi_3 = L^{a_3} V^{b_3} \rho^{c_3} \cdot \mu$$

$$M^0 L^0 T^0 = L^{a_3} (LT^{-1})^{b_3} (ML^{-3})^{c_3} \cdot ML^{-1} T^{-1}$$

power M: $0 = c_2 + 1 \Rightarrow c_2 = -1$

power of L: $0 = a_2 + b_2 - 3c_2 - 1$

$a_2 = -b_2 + 3c_2 + 1 \Rightarrow a_2 = -1$

power of T: $0 = -b_2 - 1 \Rightarrow b_2 = -1$

$$\pi_3 = L^{-1} V^{-1} \rho^{-1} \cdot \mu \qquad \text{———— (1M)}$$

$$\pi_3 = \frac{\mu}{LV\rho} \quad \text{——— ③}$$

$$f\left(\frac{F}{\rho L^2 V^2}, \frac{gL}{V^2}, \frac{\mu}{\rho LV}\right) = 0$$

$$F = \rho L^2 V^2 \phi \left[\frac{gL}{V^2}, \frac{\mu}{\rho LV}\right] \qquad \text{———— (1M)}$$

(9)(b)

Three types of similarities must exist between the model and prototype.

i) Geometric Similarity : The ratios of the length, breadth, diameter, Area, volume of the model and Prototype are equal.

$$\frac{L_p}{L_m} = \frac{b_p}{b_m} = \frac{D_p}{D_m} = L_r = \text{Scale ratio}$$

$$\frac{A_p}{A_m} = \frac{L_p \times b_p}{L_m \times b_m} = L_r \times L_r = L_r^2 = \text{Area ratio}$$

(2M)

ii) Kinematic Similarity : The ratio of the velocity and acceleration at the corresponding points in the model and prototype are equal.

$$\frac{V_p}{V_m} = V_r = \text{Velocity ratio}$$

$$\frac{V_{p_1}}{V_{m_1}} = \frac{V_{p_2}}{V_{m_2}}$$

$$\frac{a_p}{a_m} = a_r = \text{acceleration ratio}$$

$$\frac{a_{p_1}}{a_{m_1}} = \frac{a_{p_2}}{a_{m_2}}$$

(2M)

iii) Dynamic Similarity : The ratio of the corresponding forces acting at the corresponding points are equal.

$$\frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_v)_m} = \frac{(F_g)_p}{(F_g)_m} \dots\dots = F_r \quad \text{— (2M)}$$

$$= \text{force ratio}$$

Also the directions of corresponding forces at the corresponding points in the model and prototype should be same. — (1M)

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$$\frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_v)_m} = \frac{(F_g)_p}{(F_g)_m} \cdots\cdots = F_r \quad \text{— (2M)}$$

$= \text{force ratio}$

Also the directions of corresponding forces at the corresponding points in the model and prototype should be same. — (1M)

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