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## I/IV B.Tech( Regular/Supplementary) DEGREE EXAMINATION

March,2023

First Semester

Time: Three Hours

Common to all branches  
Linear Algebra and ODE  
Maximum: 70 Marks

*Answer question 1 compulsory.**Answer one question from each unit.*

1. a) Define rank of a matrix CO1 L1 1M  
 b) Find the characteristic equation of  $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$  CO1 L2 1M  
 c) If  $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$  then find the eigen values of  $A^{-1}$  CO1 L1 1M  
 d) Define integrating factor CO2 L1 1M  
 e) Find the value of ' $m$ ', if  $ydx - mxdy = 0$  is an exact differential equation CO2 L3 1M  
 f) What do you mean by Linear differential equation. CO2 L1 1M  
 g) Find the general solution of  $\frac{d^3x}{dt^3} - x = 0$ . CO3 L1 1M  
 h) Find PI of  $(D^2 - 2D + 1)y = e^x$  CO3 L3 1M  
 i) Evaluate  $\frac{1}{D^2} \sin 2x$  where  $D$  is differentiable operator with respect to  $x$ . CO3 L1 1M  
 j) Find the Wronskian of  $e^{2x}$  and  $e^{-x}$  CO3 L3 1M  
 k) Find  $L\{2^t\}$  CO4 L1 1M  
 l) Find  $L^{-1}\left\{\frac{1}{s^2}\right\}$  CO4 L2 1M  
 m) Evaluate  $\int_0^\infty t^2 e^{-st} dt$  using Laplace transform CO4 L1 1M  
 n) State convolution theorem in Laplace transforms. CO4 L1 1M
- Unit -I**
2. a) Reduce the matrix  $\begin{bmatrix} -1 & 2 & 1 & 8 \\ 2 & 1 & -1 & 0 \\ 3 & 2 & 1 & 7 \end{bmatrix}$  into Echelon form and hence find its CO1 L3 7M  
 rank  
 b) Test for consistency and solve the system of equations CO1 L2 7M  
 $x+y+z=9, 2x+5y+7z=52, 2x+y-z=0$
- (OR)**
3. a) Find the Eigen values and the corresponding Eigen vectors of the matrix CO1 L3 7M  
 $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ .
- b) Find the inverse of a matrix  $A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$  using Gauss Jordon method. CO1 L2 7M

P.T.O

**Unit -II**

4. a) Solve  $2xydx + (x^2 + y^2)dy = 0$  CO2 L3 7M  
 b) If 30% of a radioactive substance disappears in 10 days, how long will it take CO2 L2 7M  
 for 90% of it to disappear?

**(OR)**

5. a) A body originally at  $80^0\text{C}$  cools down to  $60^0\text{C}$  in 20 minutes. The temperature CO2 L2 7M  
 of the air being  $40^0\text{C}$ . What will be the temperature of the body after 40  
 minutes from the original and when the temperature will be  $45^0\text{C}$ ?

- b) Solve  $\frac{dy}{dx} = \frac{1}{x+y}$  CO2 L1 7M

**Unit -III**

6. a) Solve  $(D^2 + 4)y = \sin 2x.$  CO3 L3 7M  
 b) Solve  $(D^2 - 4D + 4)y = 8e^{2x} \sin 2x$  CO3 L2 7M

**(OR)**

7. a) Solve  $(D^2 - 6D + 9)y = e^{2x}/x^2$  by the method of variation of parameters CO3 L3 7M  
 b) Solve  $(D^2 - 3D + 2)y = xe^{3x} + \sin 2x.$  CO3 L2 7M

**Unit -IV**

8. a) Find the Laplace transform of  $\frac{e^{at} - \cos bt}{t}$  CO4 L2 7M  
 b) Using convolution theorem, Evaluate  $L^{-1}\left[\frac{1}{(s^2 + 4)^2}\right].$  CO4 L3 7M

**(OR)**

9. Using Laplace transform technique find the solution of  
 $(D^3 - 3D^2 + 3D - 1)y = t^2 e^t;$   $y(0) = 1,$   $y'(0) = 0$  and  $y''(0) = -2.$  CO4 L4 14M



DEPARTMENT OF MATHEMATICS

I/IV B.Tech (Regular / Supplementary) DEGREE EXAMINATION  
 Linear Algebra and ODE Scheme of Evaluation March, 2023  
 (common to all branches) 20MA001/101

1(a). Rank of a matrix: A matrix is said to be of rank 'n' when (i) it has atleast one non-zero minor of order n, and (ii) every minor of order higher than n vanishes.

(b) Given  $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

The characteristic equation of A is  $\begin{vmatrix} -\lambda & 1 \\ -1 & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 + 1 = 0$

(c) Given  $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

The characteristic equation of A is  $\begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 4\lambda + 3 = 0 \Rightarrow \lambda = 1, 3$ .

The eigen values of A are 1, 3.

$\therefore$  The eigen values of  $A^{-1}$  are  $1, \frac{1}{3}$ .

(d) Integrating factor: Sometimes a differential equation which is not exact can be made so on multiplication by a suitable factor called an integrating factor.

(e)  $y dx - mx dy = 0$  is an exact DE.

$$\therefore \frac{\partial}{\partial y}(y) = \frac{\partial}{\partial x}(-mx) \Rightarrow 1 = -m \Rightarrow m = -1$$

(f) A DE is said to be linear if the dependent variable and its differential coefficients occur in the first degree and not multiplied together.

The standard form of a linear DE is  $\frac{dy}{dx} + P(x)y = Q(x)$ .

(2)

(g) Given DE is  $\frac{d^3x}{dt^3} - x = 0$ .

Its symbolic form is  $(D^3 - 1)x = 0$ , where  $D = \frac{d}{dt}$ .

Its auxiliary equation is  $D^3 - 1 = 0$

$$\Rightarrow (D-1)(D^2 + D + 1) = 0$$

$$\Rightarrow D=1, \frac{-1 \pm \sqrt{3}i}{2}$$

The general solution is  $x = c_1 e^t + e^{-t/2} \left[ c_2 \cos \frac{\sqrt{3}}{2}t + c_3 \sin \frac{\sqrt{3}}{2}t \right]$

(h) The PI of  $(D^2 - 2D + 1)y = e^x$  is  $\frac{1}{D^2 - 2D + 1} e^x = \frac{x}{2D-2} e^x = \frac{x}{2} e^x$ .

(i)  $\frac{1}{D^2} \sin 2x = \frac{1}{-2^2} \sin 2x = -\frac{\sin 2x}{4}$ .

(j) The wronskian of  $e^{2x}$  and  $\bar{e}^x$  is  $\begin{vmatrix} e^{2x} & \bar{e}^x \\ 2e^{2x} & -\bar{e}^x \end{vmatrix} = -3e^x$ .

(k)  $L[e^t] = L[e^{t \ln 2}] = \frac{1}{s - \ln 2}$

(l)  $L^{-1}\left[\frac{1}{s^2}\right] = t$

(m)  $\int_0^\infty t^r e^{-st} dt = L[t^r] = \frac{2}{s^3}$ .

(n) Convolution theorem: If  $L^{-1}\{\bar{f}(s)\} = f(t)$  and  $L^{-1}\{\bar{g}(s)\} = g(t)$ ,

then  $L^{-1}\{\bar{f}(s) \bar{g}(s)\} = f * g = \int_0^t f(u) g(t-u) du$ .

## UNIT-I

2(a) Let  $A = \begin{bmatrix} -1 & 2 & 1 & 8 \\ 2 & 1 & -1 & 0 \\ 3 & 2 & 1 & 7 \end{bmatrix}$

$$R_2 + 2R_1$$

$$\sim \begin{bmatrix} -1 & 2 & 1 & 8 \\ 0 & 5 & 1 & 16 \\ 0 & 8 & 4 & 31 \end{bmatrix}$$

$$5R_3 - 8R_2$$

$$\sim \begin{bmatrix} -1 & 2 & 1 & 8 \\ 0 & 5 & 1 & 16 \\ 0 & 0 & 12 & 27 \end{bmatrix}$$

The rank of  $A$ ,  $\text{r}(A) = 3$ .

2(b). Given equations are  $x+y+z=9$ ,  $2x+5y+7z=52$ ,  $2x+y-z=0$ .  
The given system of equations can be represented in the matrix equation as  $AX=B$ .

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$$

Consider  $[A|B] = \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2 & 5 & 7 & 52 \\ 2 & 1 & -1 & 0 \end{array} \right]$

$$R_2 - 2R_1 ; R_3 - 2R_1$$

$$\sim \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & -1 & -3 & -18 \end{array} \right]$$

$$3R_3 + R_2$$

$$\sim \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & 0 & -4 & -20 \end{array} \right] . \quad \text{r}(A) = \text{r}(A:B) = n = 3.$$

The equations are  $-4z = -20 \Rightarrow z = 5$

$$3y + 5z = 34 \Rightarrow y = 3$$

$$x + y + z = 9 \Rightarrow x = 1$$

Hence the equations are consistent and has a unique solution  
And its solution is  $(x, y, z) = (1, 3, 5)$ .

3(a) Given  $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

The characteristic equation of  $A$  is  $|A - \lambda I| = 0$ .

$$\Rightarrow \begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 + \lambda^2 - 21\lambda - 45 = 0$$

$$\Rightarrow \lambda = -3, -3, 5.$$

when  $\lambda = -3$ , the corresponding eigen vectors are  $x_1 = (-2, 1, 0)$   
 $x_2 = (3, 0, 1)$

when  $\lambda = 5$ , the corresponding eigen vector is  $x_3 = (1, 2, -1)$ .

3(b). Given  $A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

consider  $[A : I_3] = \left[ \begin{array}{ccc|ccc} -1 & 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array} \right]$

$$R_2 + R_1 ; R_3 + R_1 \quad \frac{R_2}{2}, \frac{R_3}{2}$$

$$\sim \left[ \begin{array}{ccc|ccc} -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 & 1 & 0 \\ 0 & 2 & 0 & 1 & 0 & 1 \end{array} \right] \sim \left[ \begin{array}{ccc|ccc} -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1/2 & 1/2 & 0 \\ 0 & 1 & 0 & 1/2 & 0 & 1/2 \end{array} \right]$$

$$R_{23} \sim \left[ \begin{array}{ccc|ccc} -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 & 1/2 & 0 \end{array} \right] \quad R_1 - R_2 \sim \left[ \begin{array}{ccc|ccc} -1 & 0 & 1 & 1/2 & 0 & -1/2 \\ 0 & 1 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 & 1/2 & 0 \end{array} \right]$$

$$R_1 - R_3 \sim \left[ \begin{array}{ccc|ccc} -1 & 0 & 0 & 0 & -1/2 & -1/2 \\ 0 & 1 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 & 1/2 & 0 \end{array} \right] \quad (-1)R_1 \sim \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 & 1/2 & 0 \end{array} \right]$$

$$\therefore A^{-1} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

4(a) Given DE is  $2xy \, dx + (x^2 + y^2) \, dy = 0$ .

Here  $M = 2xy$  and  $N = x^2 + y^2$ .

$$\Rightarrow \frac{\partial M}{\partial y} = 2x \quad \text{and} \quad \frac{\partial N}{\partial x} = 2x \quad \therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Thus the equation is exact and its solution is

$$\int M \, dx + \int (\text{Terms of } N \text{ not containing } x) \, dy = C.$$

(y constant)

$$\Rightarrow \int 2xy \, dx + \int y^2 \, dy = C$$

(y constant)

$$\Rightarrow x^2y + \frac{y^3}{3} = C$$

$$\Rightarrow \boxed{3x^2y + y^3 = C}$$

4(b). Let  $u$  be the amount of radioactive substance at any time  $t$ .

By decay law,  $\frac{du}{dt} \propto u \Rightarrow u = c e^{-kt}$  — ①

And let  $u_0$  be the initial amount of radioactive material present.  
i.e.,  $u = u_0$  when  $t = 0$ .

$$\text{from ①, } c = u_0.$$

$$\therefore u = u_0 e^{-kt} \quad \text{— ②}$$

Given 30% of radioactive material disappeared after 10 days.

i.e., when  $t = 10$  days,  $u = u_0 - 30\% \text{ of } u_0 = 0.7u_0$ .

$$\text{from ②, } k = \frac{1}{10} \ln\left(\frac{1}{0.7}\right)$$

$$\therefore u = u_0 e^{-\frac{t}{10} \ln\left(\frac{1}{0.7}\right)} \quad \text{— ③}$$

we have to find  $t$ , when  $u = u_0 - 90\% \text{ of } u_0 = 0.1u_0$

$$\text{from ③, } 0.1u_0 = u_0 e^{-\frac{t}{10} \ln\left(\frac{1}{0.7}\right)}$$

$$\Rightarrow t = \frac{10 \ln(0.1)}{\ln(0.7)} = 64.55 \text{ days.}$$

It will take 64.55 days to disappear 90% of radioactive substance from the original.

(OR)

5(a) Let ' $\theta$ ' be the temperature of the body at any time 't',  
and ' $\theta_0$ ' be the temperature of surrounding medium.  
By Newton's law of cooling,  $\theta = \theta_0 + c e^{-kt}$  —①

Given  $\theta_0 = 40$ .

And original temperature of the body is  $80^{\circ}\text{C}$ .  
i.e.,  $\theta = 80$  when  $t = 0$ .

$$\text{from } ①, 80 = 40 + c e^{-k(0)} \Rightarrow c = 40.$$

$$\therefore \theta = 40 + 40 e^{-kt} \quad ②$$

Also given after 20 minutes, temperature of the body is  $60^{\circ}\text{C}$ .  
i.e.,  $\theta = 60$  when  $t = 20$ .

$$\text{from } ②, 60 = 40 + 40 e^{-k(20)} \Rightarrow k = -\frac{1}{20} \ln\left(\frac{1}{2}\right)$$

$$\therefore \theta = 40 + 40 e^{\frac{t}{20} \ln\left(\frac{1}{2}\right)}$$

$$\text{when } t = 40 \text{ mins, } \theta = 40 + 40 e^{\frac{40}{20} \ln\left(\frac{1}{2}\right)} = 50^{\circ}\text{C}.$$

$$\text{when } \theta = 45^{\circ}\text{C}, t = \frac{20 \ln\left(\frac{1}{8}\right)}{\ln\left(\frac{1}{2}\right)} = 60 \text{ mins.}$$

5(b). Given DE is  $\frac{dy}{dx} = \frac{1}{x+y}$ .

let  $x+y = t$

$$\Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1.$$

$\therefore$  Given DE becomes,

$$\frac{dt}{dx} - 1 = \frac{1}{t}$$

$$\Rightarrow \frac{t}{t+1} dt = dx$$

$$\Rightarrow \int \left(1 - \frac{1}{t+1}\right) dt = \int dx$$

$$\Rightarrow t - \ln(t+1) = x + C$$

$$\Rightarrow x+y - \ln(x+y+1) = x+C$$

$$\Rightarrow y - \ln(x+y+1) = C.$$

## UNIT - III

6(a) Given DE is  $(D^2 + 4)y = \sin 2x$

Auxiliary equation is  $D^2 + 4 = 0 \Rightarrow D = \pm 2i$

$$CF = C_1 \cos 2x + C_2 \sin 2x$$

$$\begin{aligned} PI &= \frac{1}{D^2 + 4} \sin 2x \\ &= \frac{x}{2D} \sin 2x \\ &= \frac{x}{2} \int \sin 2x \, dx \\ &= \frac{x}{2} \left( -\frac{\cos 2x}{2} \right) = -\frac{x}{4} \cos 2x \end{aligned}$$

Hence the CS is  $y = CF + PI$

$$\Rightarrow y = C_1 \cos 2x + C_2 \sin 2x - \frac{x}{4} \cos 2x.$$

6(b). Given DE is  $(D^2 - 4D + 4)y = 8e^{2x} \sin 2x$

Auxiliary equation is  $D^2 - 4D + 4 = 0 \Rightarrow (D-2)^2 \Rightarrow D = 2, 2$ .

$$\therefore CF = (C_1 + C_2 x) e^{2x}$$

$$PI = \frac{1}{D^2 - 4D + 4} 8e^{2x} \sin 2x$$

$$= 8 \frac{1}{(D-2)^2} e^{2x} \sin 2x$$

$$= 8e^{2x} \frac{1}{D^2} \sin 2x$$

$$= 8e^{2x} \frac{1}{-2^2} \sin 2x$$

$$= -2e^{2x} \sin 2x$$

Hence the CS is  $y = CF + PI$

$$\Rightarrow y = (C_1 + C_2 x) e^{2x} - 2e^{2x} \sin 2x$$

(OR)

7(a) Given DE is  $(D^2 - 6D + 9)y = \frac{e^{2x}}{x^2}$

Auxiliary equation is  $D^2 - 6D + 9 = 0 \Rightarrow (D-3)^2 = 0$   
 $\Rightarrow D = 3, 3$

$$CF = (c_1 + c_2 x) e^{3x}$$

Here  $y_1 = e^{3x}$ ;  $y_2 = x e^{3x}$ ;  $x = \frac{e^{2x}}{x^2}$

$$\text{And } W = \begin{vmatrix} y_1 & y_2 \\ y'_1 & y'_2 \end{vmatrix} = \begin{vmatrix} e^{3x} & x e^{3x} \\ 3e^{3x} & 3x e^{3x} + e^{3x} \end{vmatrix} = e^{6x} \begin{vmatrix} 1 & x \\ 3 & 3x+1 \end{vmatrix} = e^{6x}$$

By the method of Variation of parameters,

$$\begin{aligned} PI &= -y_1 \int \frac{y_2 x}{W} dx + y_2 \int \frac{y_1 x}{W} dx \\ &= -e^{3x} \int \frac{x e^{3x} \cdot e^{2x}/x^2}{e^{6x}} dx + x e^{3x} \int \frac{e^{3x} \cdot e^{2x}/x^2}{e^{6x}} dx \\ &= -e^{3x} \int \frac{1}{x e^x} dx + x e^{3x} \int \frac{1}{x^2 e^x} dx. \end{aligned}$$

7(b) Given DE is  $(D^2 - 3D + 2)y = x e^{3x} + \sin 2x$ .

Auxiliary equation is  $D^2 - 3D + 2 = 0 \Rightarrow D = 1, 2$ .

$$CF = c_1 e^x + c_2 e^{2x}$$

$$PI = \frac{1}{D^2 - 3D + 2} (x e^{3x} + \sin 2x)$$

$$= \frac{1}{D^2 - 3D + 2} x e^{3x} + \frac{1}{D^2 - 3D + 2} \sin 2x$$

$$= e^{3x} \frac{1}{(D+3)^2 - 3(D+3) + 2} x + \frac{1}{-(3D+2)} \sin 2x$$

$$= e^{3x} \frac{1}{D^2 + 3D + 2} x + \frac{3D-2}{-(9D-4)} \sin 2x$$

$$= \frac{e^{3x}}{2} \left[ 1 + \frac{D+3D}{2} \right]^{-1} (x) - \frac{(6 \cos 2x - 2 \sin 2x)}{-36-4}$$

$$= \frac{e^{3x}}{2} \left[ 1 - \frac{D+3D}{2} \right] (x) + \frac{1}{20} (3 \cos 2x - \sin 2x) = \frac{e^{3x}}{4} (2x-3) + \frac{1}{20} (3 \cos 2x - \sin 2x)$$

Hence the CS is  $y = CF + PI \Rightarrow y = c_1 e^x + c_2 e^{2x} + \frac{e^{3x}}{4} (2x-3) + \frac{1}{20} (3 \cos 2x - \sin 2x)$

8(a) we have  $L\left[\frac{f(t)}{t}\right] = \int_s^{\infty} f(s) ds$

$$\begin{aligned} \Rightarrow L\left[\frac{e^{at}-\cos bt}{t}\right] &= \int_s^{\infty} \left(\frac{1}{s-a} - \frac{s}{s+b^2}\right) ds \\ &= \left[ \ln(s-a) - \frac{1}{2} \ln(s^2+b^2) \right]_s^{\infty} \\ &= \left[ \ln\left(\frac{s-a}{\sqrt{s^2+b^2}}\right) \right]_s^{\infty} \\ &= \left\{ \ln\left(\frac{1-a/s}{\sqrt{1+b^2/s^2}}\right) \right\}_s^{\infty} \\ &= \ln 1 - \ln\left(\frac{1-a/s}{\sqrt{1+b^2/s^2}}\right) = \ln\left(\frac{\sqrt{s^2+b^2}}{s-a}\right). \end{aligned}$$

$$\begin{aligned} 8(b). \quad L^{-1}\left[\frac{1}{(s^2+4)^2}\right] &= L^{-1}\left[\frac{1}{s^2+2^2} \cdot \frac{1}{s^2+2^2}\right] \\ &= \frac{1}{2} \sin 2t * \frac{1}{2} \sin 2t \\ &= \frac{1}{4} \int_0^t \sin 2u \sin(2t-2u) du \\ &= \frac{1}{8} \int_0^t [\cos(4u-2t) - \cos 2t] du \\ &= \frac{1}{8} \left[ \frac{\sin(4u-2t)}{4} - u \cos 2t \right]_0^t \\ &= \frac{1}{8} \left[ \frac{\sin 2t}{2} - t \cos 2t \right] = \frac{1}{16} [\sin 2t - 2t \cos 2t]. \end{aligned}$$

(OR)

9. Given DE is  $(D^3 - 3D^2 + 3D - 1)y = t^2 e^t$ ;  $y(0) = 1$ ,  $y'(0) = 0$  and  $y''(0) = -2$

Taking the Laplace transforms of both sides, we get

$$L\{y''' - 3y'' + 3y' - y\} = L\{t^2 e^t\}$$

$$\Rightarrow L(y''') - 3L(y'') + 3L(y') - L(y) = L(e^t t^2).$$

$$\rightarrow \left\{ s^3 \bar{y}(s) - s^2 y(0) - s y'(0) - y''(0) \right\} - 3 \left\{ s^2 \bar{y}(s) - s y(0) - y'(0) \right\} \\ + 3 \left\{ s \bar{y}(s) - y(0) \right\} - \bar{y}(s) = \frac{2}{(s-1)^3}$$

$$\Rightarrow \bar{y}(s) (s^3 - 3s^2 + 3s - 1) - s^2 + 3s - 1 = \frac{2}{(s-1)^3}$$

$$\Rightarrow \bar{y}(s) (s-1)^3 = \frac{2}{(s-1)^3} + s^2 - 3s + 1$$

$$\Rightarrow \bar{y}(s) = \frac{2}{(s-1)^6} + \frac{s^2 - 3s + 1}{(s-1)^3}$$

$$\Rightarrow \bar{y}(s) = \frac{2}{(s-1)^6} + \frac{(s-1)^2 - (s-1) - 1}{(s-1)^3}$$

$$\Rightarrow \bar{y}(s) = \frac{2}{(s-1)^6} + \frac{1}{s-1} - \frac{1}{(s-1)^2} - \frac{1}{(s-1)^3}$$

$$\Rightarrow y = L^{-1} \left\{ \frac{2}{(s-1)^6} + \frac{1}{s-1} - \frac{1}{(s-1)^2} - \frac{1}{(s-1)^3} \right\}$$

$$\Rightarrow y = 2e^t \frac{t^5}{5!} + e^t - e^t t - e^t \frac{t^2}{2!}$$

$$\Rightarrow y = e^t \left[ \frac{t^5}{60} - \frac{t^2}{2} - t + 1 \right]$$

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