20ME604D

Hall Ticket Number:

III/IV B.Tech (Regular) DEGREE EXAMINATION

July/August,2023 — **Mechanical Engineering**

Sixth Semester — **Operations Research**

Time: Three Hours — **Maximum:** 70 Marks

Answer question 1 compulsory. **(14X1 = 14Marks)**

Answer one question from each unit. **(4X14=56 Marks)**

1 a) Define Operations Research? CO1 L1

O.R. is the application of modern methods of mathematical science to complex problems involving management of large systems of men, machines, materials, and money in industry, business, government and defence

b) Explain the Mathematical formulation of LPP problem? CO1 L1

If $x_j\ (j = 1,2,3,\dots,n)$ are the n decision variables of the problem and if the system is subject to m constraints, the general mathematical model can be written in the form:

Optimize $Z = f(x_1, x_2, \dots\dots, x_n)$

Subject to $g_i(x_1, x_2, \dots x_n)\ \leq, =, \geq\ b_i, (i = 1,2,\dots\dots, m)$

(Called structural constraints)

And $x_1, x_2, \dots\dots\dots, x_n \geq 0$

(Called the non-negativity restrictions or constraints)

c) What are the characteristics of a good model? CO1 L1

1. It should be reasonably simple.
2. A good model should be capable of taking into account new changes in the situation affecting its frame significantly with ease i.e., updating the models should be as simple and easy as possible.
3. Assumptions made to simplify the model should be as small as possible.
4. Number of variables used should be as small in number as possible.
5. The model should be open to parametric treatment.

d) Define artificial variable CO2 L1

The procedure for starting "ill-behaved" LPs with (=) and (≥) constraints is to use *artificial variables* that play the role of slacks at the first iteration, and then dispose of them legitimately at a later iteration. They merely used to get the starting basic feasible solution.

e) Explain degeneracy in transportation problems? CO2 L1

In a transportation problem, whenever the number of non-negative independent allocations is less than m+n-1, the transportation problem is said to be a degenerate one. Degeneracy may occur either at the initial stage or at an intermediate stage at some subsequent iteration.

f) Distinguish transportation model and assignment model CO2 L1

The assignment problem is a particular case of transportation problem in which the number of jobs (or origins or sources) are equal to the number of facilities (or destinations or machines or persons and so on). The objective is to maximize total profit of allocation or to minimize the total cost. An assignment problem is a completely degenerate form of a transportation problem. The units available at each origin and the units demanded at each destination are all equal to one.

g) What is the distribution for service time? CO3 L1

Service time is the time required for completion of a service. i.e., it is the time interval between beginning of a service and its completion. The mean service rate 'μ' is the number of customers served per unit time (Assuming the service to be continuous throughout the entire time unit).The average service time 1/μ is the time required to serve one customer. The most common type of distribution used for service times is exponential distribution. It involves the probability of completion of a service. Mean service rate μ is assumed to be constant over time and independent of number of units already serviced, queue length or any other random property of the system.

h) Define Saddle point CO3 L1

If the maximin and minimax values are equal then mark the position of that pay-off in the matrix. This element represents the value of the game and it is called the *saddle point* of the game.

i) State Bellman's Principle of optimality. CO4 L1

It states, "An optimal policy (a sequence of decisions) has the property that whatever the initial state and decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision". This principle implies that a wrong decision taken at one stage does not prevent from taking of optimum decisions for the remaining stages.

j) Explain about pay off matrix — CO4 L1

It is the outcome of the game. Pay off (gain or game) matrix is the table showing the amounts received by the player named at the left-hand side after all possible plays of the game. The payment is made by player named at the top of the table.

k) What is pure strategy and mixed strategy — CO1 L1

1. *Pure strategy*: It is the decision rule to always select a particular course of action. It is usually represented by a number with which the course of action is associated.
2. *Mixed strategy*: It is decision, in advance of all plays, to choose a course of action for each play in accordance with some probability distribution. Thus, a mixed strategy is a selection among pure strategies with some fixed probabilities (proportions).

l) Explain about KENDALL'S notation. — CO1 L1

Kendall's notation is given by:

$$(a\,/b\,/c) : \ (d/\,e/f)$$

Where a = Arrivals distribution

b = Departures (service time) distribution

c = Number of parallel servers (=1, 2,...., ∞)

d = Queue discipline

e = Maximum number (finite or infinite) allowed in the system (in queue plus in-service)

f = Size of the calling source (finite or infinite)

m) What is simulation? — CO1 L2

Simulation is a numerical technique for conducting experiments that involve certain types of mathematical and logical relationships necessary to describe the behaviour and structure of a complex real world system over extended period of time.

n) Explain the advantages of simulation? — CO2 L2

It is an appropriate tool to use in solving a problem when experimenting on the real system (a) would be disruptive, (b) would be too expensive, (c) does not permit replication of events, and (d) does not permit control over key variables.

It is a desirable tool for solving a business problem when a mathematical model (a) is too complex to solve, (b) is beyond the capacity of available personnel, and (c) is not detailed enough to provide information on all important decision variables.

Unit-I

2 Solve the following LPP by Big M method — CO1 L3 14M

$$\text{Maximize } Z = 3X_1 - X_2$$

Subjected to Constraints

$$2X_1 + X_2 \geq 2$$
$$X_1 + 3X_2 \leq 3$$
$$X_2 \leq 4$$
$$X_1 ,\, X_2 \geq 0$$

	x1	x2		
Maximize	3.00	-1.00		
Subject to				
(1)	2.00	1.00	>=	2.00
(2)	1.00	3.00	<=	3.00
(3)	0.00	1.00	<=	4.00
Lower Bound	0.00	0.00		
Upper Bound	infinity	infinity		
Unrestr'd (y/n)?	n	n		

Iteration 1

Basic	x1	x2	Sx3	Rx4	sx5	sx6
z (max)	-203.00	-99.00	100.00	0.00	0.00	0.00
Rx4	2.00	1.00	-1.00	1.00	0.00	0.00
sx5	1.00	3.00	0.00	0.00	1.00	0.00
sx6	0.00	1.00	0.00	0.00	0.00	1.00
Lower Bound	0.00	0.00				
Upper Bound	infinity	infinity				
Unrestr'd (y/n)?	n	n				

Basic	Solution
z (max)	-200.00
Rx4	2.00
sx5	3.00
sx6	4.00

Iteration 2

Basic	x1	x2	Sx3	Rx4	sx5	sx6
z (max)	0.00	2.50	-1.50	101.50	0.00	0.00
x1	1.00	0.50	-0.50	0.50	0.00	0.00
sx5	0.00	2.50	0.50	-0.50	1.00	0.00
sx6	0.00	1.00	0.00	0.00	0.00	1.00
Lower Bound	0.00	0.00				
Upper Bound	infinity	infinity				
Unrestr'd (y/n)?	n	n				

Basic	Solution
z (max)	3.00
x1	1.00
sx5	2.00
sx6	4.00

Iteration 3

Basic	x1	x2	Sx3	Rx4	sx5	sx6
z (max)	0.00	10.00	0.00	100.00	3.00	0.00
x1	1.00	3.00	0.00	0.00	1.00	0.00
Sx3	0.00	5.00	1.00	-1.00	2.00	0.00
sx6	0.00	1.00	0.00	0.00	0.00	1.00
Lower Bound	0.00	0.00				
Upper Bound	infinity	infinity				
Unrestr'd (y/n)?	n	n				

Basic	Solution
z (max)	9.00
x1	3.00
Sx3	4.00
sx6	4.00

Ans:
Max Z= 9; x1=3, x2=0

(OR)

3 Solve the following Linear Programming Problem by graphical method. CO1 L3 14M

Maximize Z=3X1+ 5X2

Subject to the conditions

$X1 \leq 4$

$2X2 \leq 12$

$3X1 + 2X2 \leq 18$ and

$X1 , X2 \geq 0$

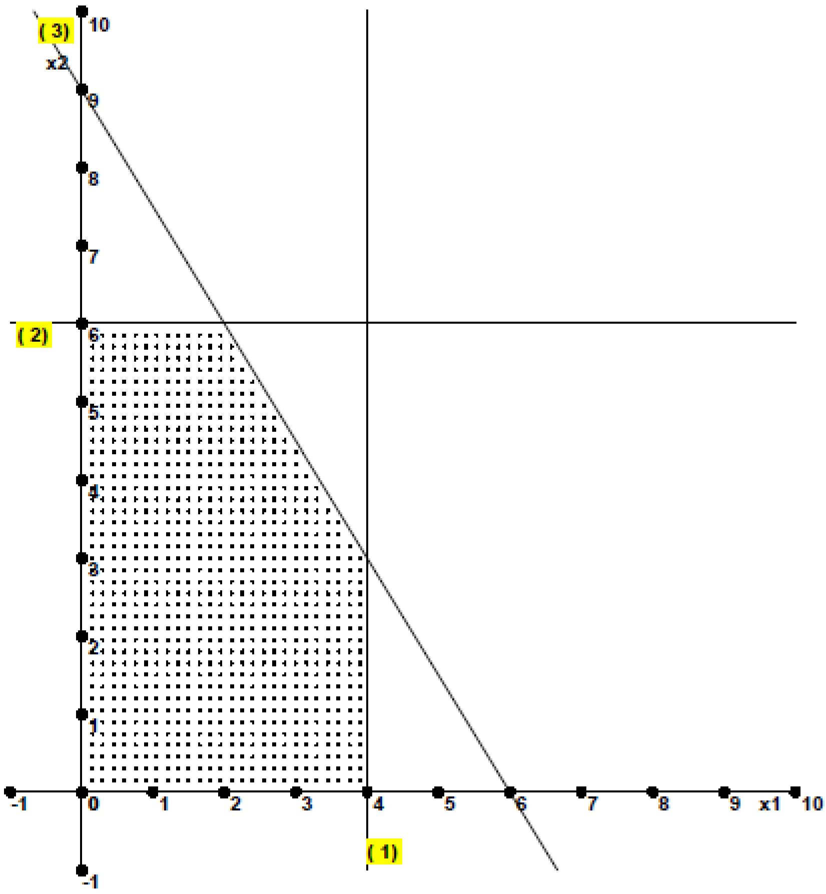
Answer:

Summary of Optimal Solution:

Objective Value = 36.00

x1 = 2.00

x2 = 6.00



Unit-II

4 Find the optimum transportation Cost CO2 L4 14M

	D_1	D_2	D_3	D_4	Supply
S_1	5	2	4	3	22
S_2	4	8	1	6	15
S_3	4	6	7	5	8
Demand	7	12	17	9	

Iteration 1: **ObjVal** **104.00**

	Name		D1 v1=2.00	D2 v2=2.00	D3 v3=4.00	D4 v4=3.00	**Supply**
S1		u1=0.00	5.00 -3.00	2.00 **12** 0.00	4.00 **2** 0.00	3.00 **8** 0.00	**22**
S2		u2=-3.00	4.00 -5.00	8.00 -9.00	1.00 **15** 0.00	6.00 -6.00	**15**
S3		u3=2.00	4.00 **7** 0.00	6.00 -2.00	7.00 -1.00	5.00 **1** 0.00	**8**
	Demand		**7**	**12**	**17**	**9**	

(OR)

5 A department of a company has five employees with five jobs to be performed. The time (in hours) that each man takes to perform each job is given in the effectiveness matrix.14 CO2 L4 14M

		Employees				
		I	*II*	*III*	*IV*	*V*
Jobs	*A*	10	5	13	15	16
	B	3	9	18	13	6
	C	10	7	2	2	2
	D	7	11	9	7	12
	E	7	9	10	4	12

Marks

How should the jobs be allocated, one per employee, so as to minimize the total man-hours?

$$\begin{array}{c|ccccc} & I & II & III & IV & V \\ \hline A & 10 & 5 & 13 & 15 & 16 \\ B & 3 & 9 & 18 & 13 & 6 \\ C & 10 & 7 & 2 & 2 & 2 \\ D & 7 & 11 & 9 & 7 & 12 \\ E & 7 & 9 & 10 & 4 & 12 \end{array} \Rightarrow \begin{bmatrix} 5 & \boxed{0} & 8 & 10 & 11 \\ \boxed{0} & 6 & 15 & 10 & 3 \\ 8 & 5 & \boxed{0} & \cancel{0} & \cancel{0} \\ \cancel{0} & 4 & 2 & \boxed{0} & 5 \\ 3 & 5 & 6 & \cancel{0} & 8 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 0 & 8 & 12 & 11 \\ 2 & 6 & 15 & 12 & 3 \\ 10 & 5 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 & 3 \\ 3 & 3 & 4 & 0 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 7 & \boxed{0} & 8 & 12 & 11 \\ \boxed{0} & 4 & 13 & 10 & 1 \\ 10 & 5 & \cancel{0} & 2 & \boxed{0} \\ \cancel{0} & 2 & \boxed{0} & \cancel{0} & 3 \\ 3 & 3 & 4 & \boxed{0} & 6 \end{bmatrix}$$

$A \rightarrow II$, $B \rightarrow I$, $C \rightarrow V$, $D \rightarrow III$, $E \rightarrow IV$

cost $\Rightarrow 5+3+2+9+4 = 23$ hours.

Unit-III

6 Solve the game whose payoff matrix is given below: CO3 L4 14M

	Player B			
Player A	B_1	B_2	B_3	B_4
A_1	3	2	4	0
A_2	3	4	2	4
A_3	4	2	4	0
A_4	0	4	0	8

The given pay-off matrix is:

	B_1	B_2	B_3	B_4	Row minimum
A_1	3	2	4	0	0
A_2	3	4	2	4	2
A_3	4	2	4	0	0
A_4	0	4	0	8	0
Column maximum	4	4	4	8	

There is no saddle point. Row 3 dominates over row 1 hence row 1 can be deleted.
Thus we get;

	B_1	B_2	B_3	B_4
A_2	3	4	2	4
A_3	4	2	4	0
A_4	0	4	0	8

Here we find that column 1 is dominated by column 3 and therefore we can delete column 1. The resulting matrix is given by;

	B_2	B_3	B_4
A_2	4	2	4
A_3	2	4	0
A_4	4	0	8

We find that column 1 (B_2) is inferior to a convex combination of columns 2 (B_3) and columns 3 (B_4).

$$4 \geq \frac{(2+4)}{2}, 2 \geq \frac{4+0}{2}, 4 \geq \frac{0+8}{2}, \qquad (\lambda_1 = \lambda_2 = \frac{1}{2})$$

$\therefore$ *column 1 can be deleted and the resulting matrix is*:

	B_3	B_4
A_2	2	4
A_3	4	0
A_4	0	8

Again in this matrix, row 1 is dominated by a convex combination of rows 2 and 3.

$2 \leq \frac{(4+0)}{2}, 4 \leq \frac{0+8}{2}, \ (\lambda_1 = \lambda_2 = \frac{1}{2})$ and hence row 1 can be deleted. Finally we get;

	B_3	B_4
A_3	4	0
A_4	0	8

$$p_3 = \frac{(a_{22} - a_{21})}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{8-0}{12} = \frac{2}{3}$$

$$p_4 = 1 - p_3 = 1 - \frac{2}{3} = \frac{1}{3}$$

$$q_3 = \frac{(a_{22} - a_{12})}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{(8-0)}{12} = \frac{2}{3}$$

$$q_4 = 1 - q_3 = 1 - \frac{2}{3} = \frac{1}{3}$$

Value of the game is

$$v = \frac{32}{12} = 8/3$$

The optimal strategies are;

$$S_A = \begin{pmatrix} A_1 & A_2 & A_3 & A_4 \\ 0 & 0 & 2/3 & 1/3 \end{pmatrix} \qquad and \qquad S_B = \begin{pmatrix} B_1 & B_2 & B_3 & B_4 \\ 0 & 0 & 2/3 & 1/3 \end{pmatrix}$$

(OR)

7 A television repairman finds that the time spent on his jobs has an exponential distribution with a mean of 30 minutes. If he repairs the sets in the order in which they came in, and if the arrival of sets follows a Poisson distribution with an approximate average rate of 10 per 8-hour day, what is the repairman's expected idle time each day? How many jobs are ahead of the average set just brought in? CO3 L3 14M

Arrival rate $\lambda = \frac{10}{8\times60} = \frac{1}{48}$ units /minute

Service rate $\mu = \frac{1}{30}$ units/minute

Number of jobs ahead of the set brought in = average number of jobs in the system

$$L_s = \frac{\lambda}{\mu-\lambda} = \frac{1/48}{1/30 - 1/48} = 1.67\ units$$

Number of hours for which the repairman remains busy in an 8-hour day

$$= 8\frac{\lambda}{\mu} = 8\times\frac{1/48}{1/30} = 8\times\frac{20}{32} = 5\ hours.$$

Therefore, Time for which repairman remains idle in an 8-hour day= 8-5 = 3 hours.

Unit-IV

8 Solve using dynamic programming CO4 L3 14M

$$Maximize\ Z = 3x_1 + 4x_2$$
$$subject\ to\ the\ constraints$$
$$2x_1 + 5x_2 \le 120$$
$$2x_1 + x_2 \le 40$$
$$\text{and } x_1, x_2 \ge 0$$

Here we have two stages for two variables.

Stage 1

$$f_1(B_{11}\ B_{21}) = Max\ (c_1 x_1)$$

$$= Max(3x_1) = 3\ Max(x_1)$$

From the constraints we find that

$$x_1 \le \frac{120-5x_2}{2} \quad and$$

$$x_1 \le \frac{40-x_2}{2}$$

In order to satisfy both the constraints

$$x_1 \le min\left\{\frac{120-5x_2}{2}, \quad (40-x_2)/2\right\}$$

$$\therefore f_1(B_{11}\ B_{21}) = 3min\left\{\frac{120-5x_2}{2}, \quad (40-x_2)/2\right\}$$

Stage 2

$$f_2(B_{12}\ B_{22}) = Max\ \{f_1(B_{11}\ B_{21}) + 4x_2\}$$

$$= Max\left[3\min\left\{\frac{120-5x_2}{2}, \frac{(40-x_2)}{2}\right\} + 4x_2\right]$$

$$= max\begin{cases}\frac{3(120-5x_2)}{2} + 4x_2\ if\ \frac{120-5x_2}{2} \le (40-x_2)/2 \\ \frac{3(40-x_2)}{2} + 4x_2\ if\ \frac{120-5x_2}{2} \ge (40-x_2)/2\end{cases}$$

Now $\frac{120-5x_2}{2} \le \frac{40-x_2}{2}$

$$\Rightarrow 120 - 5x_2 \le 40 - x_2$$

$$\Rightarrow 80 \le 4x_2 \Rightarrow x_2 \ge 20$$

$$\frac{120-5x_2}{2} \ge \frac{40-x_2}{2}$$

$$120 - 5x_2 \ge 40 - x_2$$

$$\Rightarrow x_2 \le 20$$

When $x_2 = 20$

$$\frac{3(120-5x_2)}{2} + 4x_2 = \frac{3(40-x_2)}{2} + 4x_2 = 110$$

$$\therefore f_2(120, 40) = 110$$

$x_2^* = 20\ and\ x_1^* = (40 - x_2^*)/2 = 10$ Solution is $x_1 = 10, x_2 = 20\ \ and\ Z^* = 110$

(OR)

9	Define simulation .Explain about the applications and Advantages and Disadvantages of simulation.	CO4	L2	14M

Simulation is a numerical technique for conducting experiments that involve certain types of mathematical and logical relationships necessary to describe the behaviour and structure of a complex real world system over extended period of time.

Simulation is the process of designing a model of a real system and conducting experiments with this model for the purpose of understanding the behaviour (within the limits imposed by a criterion or set of criteria) for the operation of the system.

Applications:

It is used in a variety of problems such as Investment analysis, queuing models, Inventory models, LPPs

Advantages:

It provides trial-and-error movement towards the optimal solution.

The decision maker selects an alternative, experiences the effect of the selection, and then improves the selection. In this way, the selection is adjusted until it approximates the optimal solution.

Disadvantages:

(i) It may be the only method available, because it is difficult to observe the actual reality.

(ii) Without appropriate assumption, it is impossible to develop a mathematical solution.

(iii)There may not be sufficient time to allow the system to operate for a very long time.