20EE404

Hall Ticket Number:

II/IV B.Tech (Regular\Supplementary) DEGREE EXAMINATION

July/August, 2023 **Electrical &Electronics Engineering**

Fourth Semester **Signals & Systems**

Time: Three Hours **Maximum:** 70 Marks

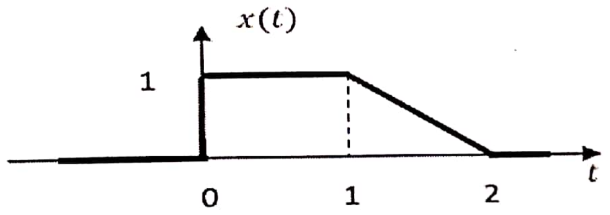
Answer question 1 compulsory. **(14X1 = 14Marks)**

Answer one question from each unit. **(4X14=56 Marks)**

			CO	BL	M
1	a)	What is the condition for orthonormality?	CO1	L2	1
	b)	What is the relation between Z- transform and Fourier transform of a signal?	CO3	L2	1
	c)	Find the FT of unit ramp function.	CO3	L2	1
	d)	What is anti- aliasing filter?	CO3	L2	1
	e)	What is aliasing effect?	CO4	L4	1
	f)	Define Nyquist interval.	CO4	L1	1
	g)	What is the relation between rise time and bandwidth of a linear system?	CO3	L2	1
	h)	Define Fourier complex spectrum	CO3	L3	1
	i)	What is the area under unit impulse function?	CO1	L1	1
	j)	Define Causal system	CO2	L1	1
	k)	What is the nature of ROC of Z-transform for an anti-causal sequence	CO1	L3	1
	l)	Write the expression for Fourier Transform of unit step sequence	CO1	L1	1
	m)	What is nyquist interval of signal $x(t)=19\cos^2 2000\pi t$	CO4	L3	1
	n)	What do you mean by even symmetry?	CO1	L1	1

Unit-I

2	a)	Find the even and odd parts of the signal shown in Figure.	CO1	L2	7M



	b)	Show that the unit impulse function is the derivative of unit step function	CO1	L2	7M

(OR)

3	a)	Define the following: i) Energy-type signals ii) Power-type signals	CO1	L1	7M
	b)	If x(t)= U(t)-U(t-1), sketch y(t)=x(5t+7)	CO1	L2	7M

Unit-II

4	a)	Explain the difference between the following systems i. Time invariant and Time variant systems ii. Causal and non-causal systems	CO2	L1	7M
	b)	Determine whether or not the following systems are Time Invariant and causal i. $y(t)=x(t-1)+t\,x(t)$ ii. $y(t)=\dot{x}(2t)$	CO2	L2	7M

P.T.O

		(OR)			
5	a)	Explain how input and output signals are related to impulse response of a LTI system.	CO2	L2	7M
	b)	Describe the following system classification i)Linear and Non Linear systems ii) Time variant and Time invariant iii)Stable and Unstable system	CO2	L2	7M
		Unit-III			
6	a)	Find the Fourier Transform and draw the spectrum of the following signals: i)unit step signal ii)signum function	CO3	L2	7M
	b)	State and prove the following Fourier Transform properties i)Time convolution, ii)Frequency Convolution iii) Time Integration	CO3	L2	7M
		(OR)			
7	a)	) State and prove z –transform time reversal and differentiation in z – domain properties.	CO3	L2	7M
	b)	Find the inverse z – transform of $X(z) = 1 - z^{-1} + \frac{z^{-2}}{(1-0.2z^{-1})(1-2z^{-1})(1-z^{-1})}$, With ROC of 0.2<IzI<1.	CO3	L2	7M
		Unit-IV			
8	a)	Distinguish between instantaneous sampling, natural sampling and flat top sampling.	CO4	L2	7M
	b)	What is zero order hold? Obtain the transfer function of zero order hold.	CO4	L2	7M
		(OR)			
9	a)	Determine the Nyquist sampling rate and Nyquist sampling interval for the signals. (a) sinc(100πt). (b) sinc2(100πt). (c) sinc(100πt) + sinc(50πt). (d) sinc(100πt) + 3 sinc2(60πt)	CO4	L2	7M
	b)	Discuss Natural sampling and effect of under sampling.	CO4	L2	7M

Scheme

II/IV B.Tech (Regular) Degree Examination

20EE404 : Signals & Systems

Fourth semester, July/August, 2023

1a. The set of vectors $\{x_1, x_2 \cdots x_n\}$ is called orthonormal vector space or orthogonal set if

$$x_m \cdot x_n = \begin{cases} 0 & m \neq n \\ 1 & m = n \end{cases}$$ 1M

1b: The relation between z-transform and discrete time fourier transform is: The z transform of $x(n)$ is same as the discrete time fourier transform of $x(n)r^{-n}$ i.e $Z\{x(n)\} = DTFT\{x(n)r^{-n}\}$ 1M

~~1b~~

$$u(t) \xleftrightarrow{FT} \pi\delta(\omega) + \frac{1}{j\omega}$$

$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 1 & \text{for } t < 0 \end{cases}$$

$$u(t) = \frac{1}{2} + \frac{1}{2}\,\mathrm{sgn}(t)$$

$$F[u(t)] = \pi\delta(\omega) + \frac{1}{j\omega}$$

1c. $r(t) = \int u(t)\,dt$

$F\{r(t)\} = F\{\int u(t)\,dt\}$ 1M

$$= \frac{1}{j\omega}\left[\pi\delta(\omega) + \frac{1}{j\omega}\right]$$

1d. The LPF used for band limiting a signal before sampling is generally referred to as anti-aliasing filter. It is primarily used for preventing aliasing 1M

1e. Aliasing is defined as the phenomenon ... frequency component in the frequency spectrum of ... takes identity of a lower frequency component in the spectrum of the sampled signal

(or)

The effect in which the individual terms in equation

$$X_s(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$$ 1M

overlap is called aliasing

1f. Nyquist interval is the time interval between any two adjacent samples when sampling rate is Nyquist rate. 1M

1g. Rise time $\alpha \frac{1}{\text{band width}}$. 1M

Bandwidth x Rise time = Constant

1h. $C_n = \frac{1}{T_0} \int_{t_0}^{t_0+T} x(t) e^{-jn\omega_0 t} dt$ 1M

The plot of magnitude & phase of exponential fourier series coefficient w.r.t frequency is called complex Fourier spectrum

1i. $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$ 1M

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1i. A system is said to be causal if the output of the system at any time depends only on present and past inputs, but does not depends on future inputs. 1M

1k. The ROC of a finite duration anticausal sequence is ~~exterior~~ entire z-plane except at $z = \infty$ 1M

1L. $x(n) = u[n] \Rightarrow x(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \Rightarrow \frac{1}{1-e^{-j\omega}}$ 1M

1F. $f[u(t)] = \pi\delta(\omega) + \frac{1}{j\omega}$

1E. ~~The Fourier transform of a unit ramp function also known as the ramp function is a well known result. The unit ramp function denoted as~~ 1M

~~$u(t)$ or $r(t)$~~

~~$u(t) = t$ for $t > 0$~~

~~$u(t) = 0$ for < 0~~

~~The Fourier transform of the unit ramp function $u(t)$ is given by,~~

~~$F[u(t)] = 2\pi * \delta(F) - j\pi x(\frac{t}{F})$~~

1m. $19\left[\frac{1+\cos 4000\pi t}{2}\right]$ 1M

$2\pi f_m = 4000$

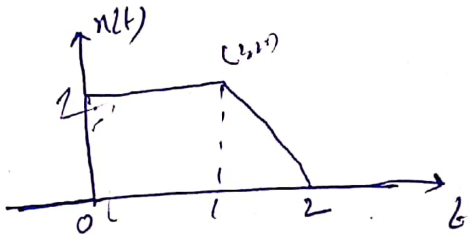
$f_m = 2000$

$F_s = 2f_m = 4000$

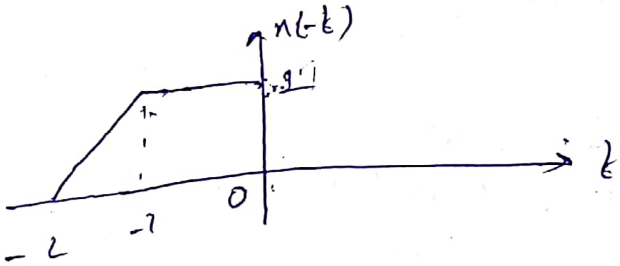
$T_s = \frac{1}{F_s} = 0.25\text{msec.}$

1.n. When a periodic function is symmetrical about vertical axis it is said to have a even symmetry or mirror symmetry $x(t) = x(-t)$ 1M

2a.

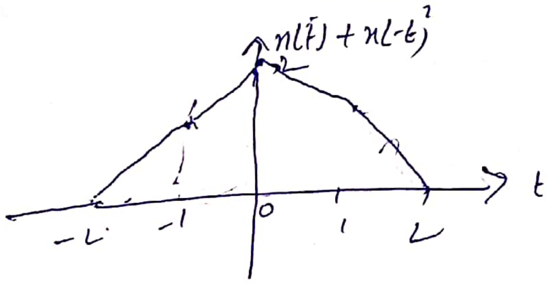


$x(t) = u(t) - u(t-1) - t + 2$



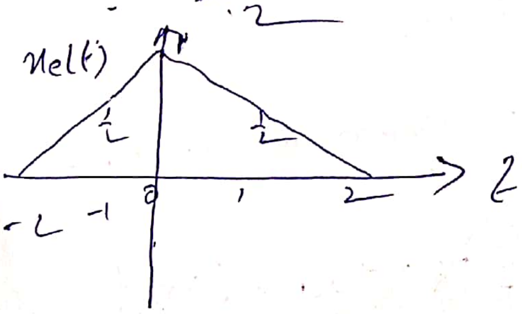
$x(-t) = u(-t) - u(-t+1) + t + 2$ 2M

$x_e(t) = \frac{x(t) + x(-t)}{2}$



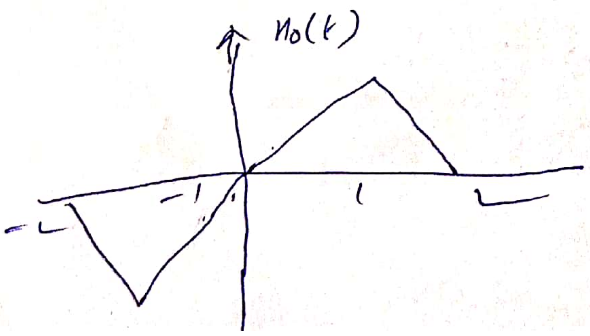
3M

$\frac{x(t)+x(-t)}{2}$



$x_e(t) = \frac{u(t) + u(-t) - u(t-1) - u(-t+1) + 4}{2}$ 4M

$x_o(t) = \frac{x(t) - x(-t)}{2}$



$x_o(t) = \frac{u(t) - u(-t) - u(t-1) + u(-t+1) - 2t}{2}$ 4M

2b

Unit Impulse function

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

$\delta(t) = 0 \quad$ for $t \neq 0$

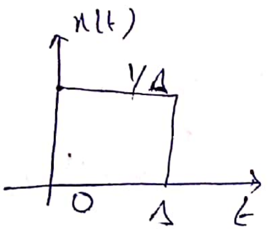
$$\delta(t) = \begin{cases} 1 & \text{for } t = 0 \\ 0 & \text{for } t \neq 0 \end{cases}$$ 2M

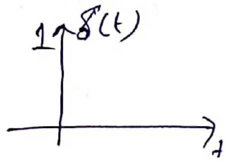
$\delta(t)$ can be represented as a limiting case of a rectangular pulse function

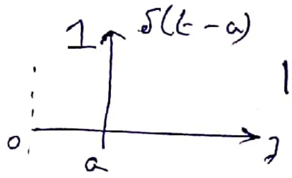
$$x(t) = \frac{1}{\Delta}\left[u(t) - u(t-\Delta)\right]$$

$$\delta(t) = \lim_{\Delta \to 0} x(t) = \lim_{\Delta \to 0} \frac{1}{\Delta}\left[u(t) - u(t-\Delta)\right]$$ 3M

$$\delta(t-a) = \begin{cases} 1 & \text{for } t = a \\ 0 & \text{for } t \neq a \end{cases}$$







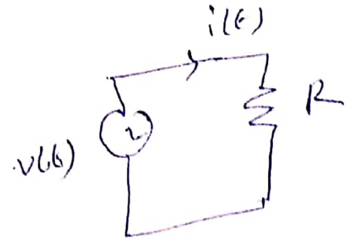
1M

The integral of unit impulse function is a unit step function and the derivate of unit step function is a unit impulse function

$$u(t) = \int_{-\infty}^{\infty} \delta(t)\, dt$$ 1M

$$\delta(t) = \frac{d}{dt} u(t)$$

3a. 1



$P(t) = v(t)\, i(t)$

$= v(t) \frac{v(t)}{R} = \frac{v^2(t)}{R}$ 1M

$= i(t) R\, i(t) = i^2(t) R$

when $R = 1\Omega$ the power dissipated is called normalized power

normalized power $p(t) = v^2(t)$ or $i^2(t)$

$P(t) = |x(t)|^2$

The total energy or normalized energy of a continuous time signal $x(t)$

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt \text{ Joules}$$

The average power or normalized average power

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \text{ watts}$$ 3M

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

- A signal is said to be an energy signal. if and only if its total Energy E is finite (i.e $0 < E < \infty$)

For an energy signal, average power $P = 0$. Non Periodic signals are examples of energy signals 3m

- A signal is said to be a power signal if its average power P is finite (i.e $0<P<\infty$) for a power signal, total energy $E=\infty$. periodic signals are examples of power signal. 3M

- The signal that do not satisfy the above properties are neither energy signal nor power signals.

Example's – Any relative power & Energy 1M.

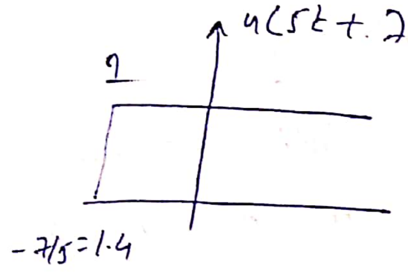
3b. $x(t) = u(t) - u(t-1)$

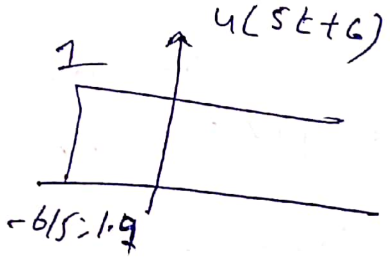
$y(t) = x(5t+7)$ 2M

$x(5t+7) = u(5t+7) - u(5t+7-1)$

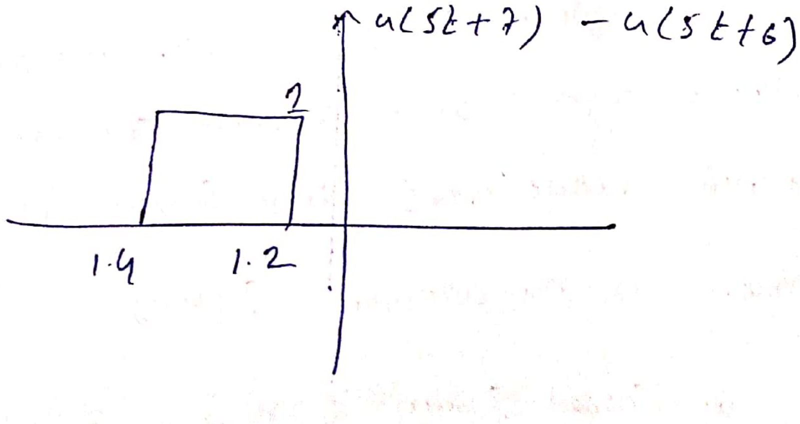
$x(5t+7) = u(5t+7) - u(5t+6)$

$y(t) = u(5t+7) - u(5t+6)$



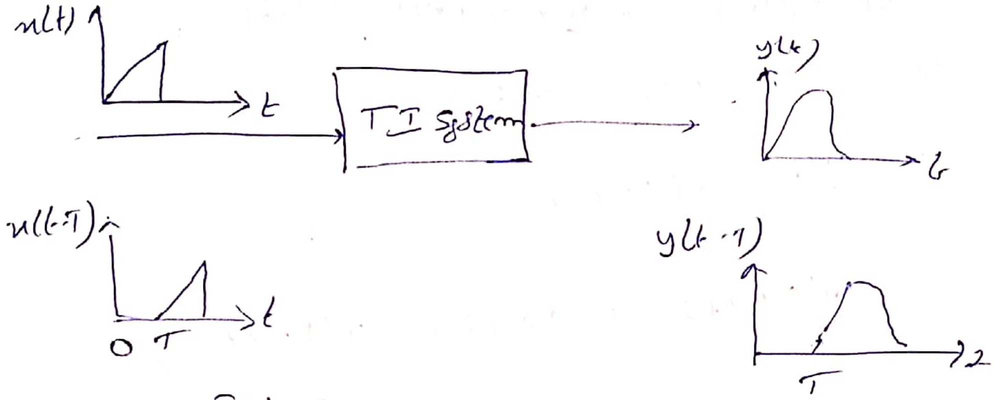


5M



4a.

i) A system is said to be time invariant if its input-output characteristics do not change with time. Suppose that we applied a signal $x(t)$ to a system and obtained an output $y(t)$. If we delay the input by T seconds, then for a time invariant system the output also will be delayed by T seconds.



3M

Fig: A Time invariant System

The output due to the delayed input $x(t-T)$ for a time invariant system is

$$y(t-T) = T[x(t-T)]$$

If the output due to input $x(t-T)$ is not equal to $y(t-T)$ then the system time variant

ii) Causal and non-Causal Systems:

A Causal system is one for which the output at any time t depends on the present and past inputs but not future inputs. These systems are also known as non-anticipative systems 3M

A non-Causal system is one whose output depends on future values

Examples for causal systems are

$y(t) = x(t) + x(t-1)$

$y(n) = n x(n) + x(n-3)$

Ex for non causal systems are 1M

$y(t) = x(t+3) + x^2(t)$

$y(n) = x(2n)$

4b. (i) $y(t) = x(t-1) + t\,x(t)$

$y(t,T) = x(t-1-T) + t\,x(t-T)$

$y(t-T) = x(t-T-1) + (t-T)\,x(t-T)$

$y(t,T) \neq y(t-T)$ Time variant system

$y(t) = x(t-1) + t\,x(t)$ 3.5M

for $t=-1$ $y(-1) = x(-2) - x(-1)$

$t=0$ $y(0) = x(-1) + 0$

$t=1$ $y(1) = x(0) + x(1)$

Causal system

(ii) $y(t) = x(2t)$

$y(t,T) = x(2t-T)$

$y(t-T) = x(2t-2T)$

$y(t,T) \neq y(t-T)$ Time variant system

3.5M

$y(t) = x(2t)$

for $t=-2$ $y(-2) = x(-4)$

$t=0$ $y(0) = x(0)$

$t=2$ $y(2) = x(4)$ non causal system

5b

5a. In a linear Time-invariant (LTI) System, the Output - Input relationship can be described using the impulse response of the system. understanding the relationship between the input signal, output signal and impulse response is fundamental in analyzing and designing such system. 2M

Impulse response: the impulse response of an LTI System is the system's output when the input is an impulse function. the impulse response is denoted as h(t) where t is the time variable.

Output(t) = h(t) * Input(t) 3M

Convolution:

$$\text{Output}(t) = h(t) * x(t) = \int [h(\tau) * x(t-\tau)] d$$ — (1)

the convolution operation is a way to compute the output of an LTI System for any given input signal. Given an input signal x(t) and an impulse response h(t) the convolution of x(t) and h(t). in equation (1).

2M

The impulse response of an LTI System represents the system's output when the input is an Impulse function. the Input-output relationship for an LTI System is given by the convolution of the Input signal with the impulse response.

5b. i) Linear and non linear Systems: A system which obeys the principle of superposition and principle of homogeneity is called a linear system, and a system which does not obey the principle of superposition and homogeneity is called a non-linear system.

$$x(t) \longmapsto y(t)$$

$$ax(t) \longmapsto ay(t)$$

3M

$$x_1(t) + x_2(t) \longleftrightarrow y_1(t) + y_2(t)$$

$$ax_1(t) + bx_2(t) \longleftrightarrow ay_1(t) + by_2(t)$$

$$T[ax_1(t) + bx_2(t)] = aT[x_1(t)] + bT[x_2(t)]$$

For Discrete time linear systems

$$T[ax_1(n) + bx_2(n)] = aT[x_1(n)] + bT[x_2(n)]$$

ii) Time variant and Time Invariant:

Time invariant is the property of a system which makes the behaviour of the system independent of time. 2M

A system is said to be time invariant if its input/output characteristics do not change with time.

$$x(t) \to y(t) \;;\quad x(t-T) \to y(t-T)$$

A system not satisfying the above requirements is called a time varying system.

Stable and unstable systems:-

A bounded signal is a signal whose magnitude is always a finite value. For example, a sine wave is a bounded signal. A system is said to be bounded input bounded-output stable, if and only if every bounded input produces a bounded output. 2M

The output of a stable system does not diverge or does not grow unreasonably large.

Let the input signal $x(t)$ be bounded (finite)

$$|x(t)| \leq M_x < \infty \text{ for all } t$$

M_x is a positive real number

$$|y(t)| \leq M_y < \infty$$

i.e If $y(t)$ is also bounded then the system is BIBO stable. otherwise the system is unstable. That is we say that a system is unstable even if one bounded input produces an unbounded output.

6a.

unit step function u(t)

The unit step function is defined by

$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Since the unit step function is not absolutely integrable, we cannot directly find its Fourier transform

$$u(t) = \frac{1}{2} + \frac{1}{2}\,\mathrm{sgn}(t)$$

$$x(t) = u(t) = \frac{1}{2}[1 + \mathrm{sgn}(t)]$$

$$X(\omega) = F[u(t)] = F\left\{\frac{1}{2}[1 + \mathrm{sgn}(t)]\right\}$$ 3-M

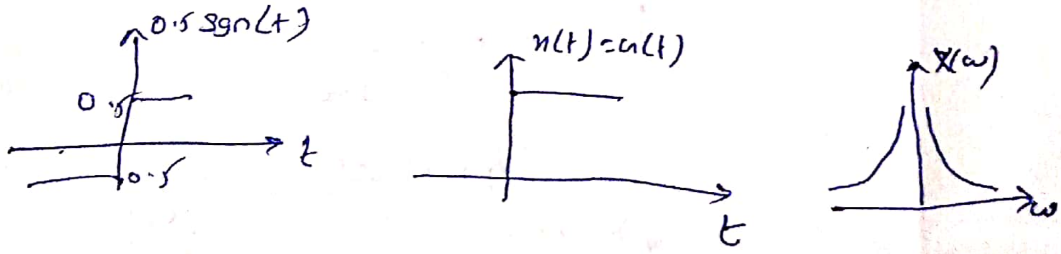
$$= \frac{1}{2}\left\{F[1] + F[\mathrm{sgn}(t)]\right\}$$

$$F[1] = 2\pi\delta(\omega) \text{ and } F[\mathrm{sgn}(t)] = \frac{2}{j\omega}$$

$$F[u(t)] = \frac{1}{2}\left[2\pi\delta(\omega) + \frac{2}{j\omega}\right] = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$F[u(t)] = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$u(t) \xleftrightarrow{F.T} \pi\delta(\omega) + \frac{1}{j\omega}$$



Signum Function $Sgn(t)$:-

The Signum Function is denoted by $Sgn(t)$ and is de

$$Sgn(t) = \begin{cases} 1 & \text{for } t>0 \\ -1 & \text{for } t<0 \end{cases}$$

Let us consider the function $e^{-a|t|} Sgn(t)$

3.M

$$x(t) = Sgn(t) = \lim_{a\to 0} e^{-a|t|} Sgn(t) = \lim_{a\to 0} \left[e^{-at}u(t) - e^{at}u(-t) \right]$$

$$X(\omega) = F[Sgn(t)] = \int_{-\infty}^{\infty} \left[\lim_{a\to 0} \left[e^{-at}u(t) - e^{at}u(-t) \right] \right] e^{-j\omega t}\, dt$$

$$= \lim_{a\to 0} \left[\int_{-\infty}^{\infty} e^{-at} e^{-j\omega t} u(t)\, dt - \int_{-\infty}^{\infty} e^{at} e^{-j\omega t} u(-t)\, dt \right]$$

$$= \lim_{a\to 0} \left[\frac{e^{-\infty} - e^{0}}{-(a+j\omega)} - \frac{e^{-\infty} - e^{0}}{-(a-j\omega)} \right]$$

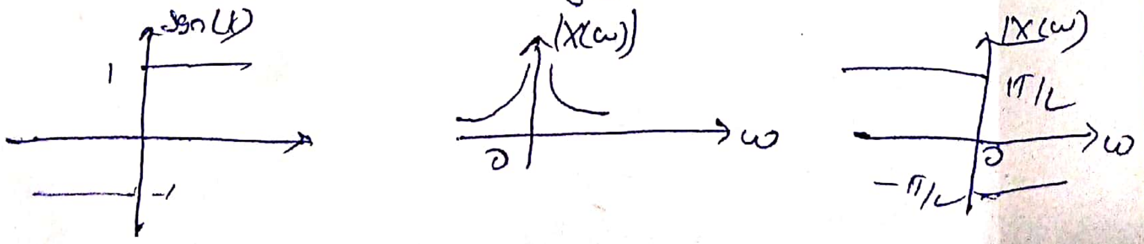
$$= \lim_{a\to 0} \left[\frac{1}{a+j\omega} - \frac{1}{a-j\omega} \right]$$

$$= \frac{2}{j\omega}$$

$$F[Sgn(t)] = \frac{2}{j\omega}$$

$$Sgn(t) \xleftrightarrow{FT} \frac{2}{j\omega}$$

1M



7a.

Time Reversal Property:

The time reversal property of z-Transform state that

If $x(n) \xleftrightarrow{ZT} X(z)$ with ROC = R

$x(-n) \xleftrightarrow{ZT} X\left[\frac{1}{z}\right]$ with ROC = $\frac{1}{R}$

Proof:

$$Z[x(n)] = X[z] = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$Z[x(-n)] = \sum_{n=-\infty}^{\infty} x(-n) z^{-n}$$

$p = -n$

$$Z[x(-n)] = \sum_{p=-\infty}^{\infty} x(p) z^{p}$$

$$Z[x(-n)] = \sum_{p=-\infty}^{\infty} x(p) (z^{-1})^{-p}$$

$$= X(z^{-1}) = X\left[\frac{1}{z}\right]$$

$$x(-n) \xleftrightarrow{ZT} X[z^{-1}]$$ 3M

Differentiation in z-domain property:

The multiplication by n or differentiation in z domain property of z-transform states that

$x(n) \xleftrightarrow{ZT} X[z]$ with ROC = R 4M

$n x[n] \xleftrightarrow{ZT} -z \frac{d}{dz} X[z]$ with ROC = R

Proof:

$$z\{x(n)\} = X[z] = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\frac{d}{dz} X(z) = \frac{d}{dz}\left[\sum_{n=-\infty}^{\infty} x(n) z^{-n}\right]$$

$$= \sum_{n=-\infty}^{\infty} x(n) \frac{d}{dz}(z^{-n})$$

$$= \sum_{n=-\infty}^{\infty} x(n)(-n) z^{-n-1}$$

$$= -z^{-1}\sum_{n=-\infty}^{\infty} [n x(n)] z^{-n}$$

$$= -z^{-1} z\{n x(n)\}$$

$$z[n x(n)] = -z \frac{d}{dz} X(z)$$

$$n x(n) \xleftrightarrow{ZT} -z \frac{d}{dz} X[z]$$

$$z[n^k x(n)] = (-1)^k z^k \frac{d^k X(z)}{dz^k}.$$

7b $x[z] = 1 - z^{-1} + \frac{z^{-2}}{(1-0.2z^{-1})(1-2z^{-1})(1-z^{-1})}$

ROC of $0.2 < |z| < 1$

Sol:

$$x(z) = 1 - z^{-1} + \frac{z^{-2}}{(1-0.2z^{-1})(1-2z^{-1})(1-z^{-1})}$$

$$\frac{z^{-2}}{(1-0.2z^{-1})(1-2z^{-1})(1-z^{-1})} = \frac{A}{(1-0.2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{C}{(1-2z^{-1})}$$

2M

$$z^{-2} = A\{1-2z^{-1}\}\{1-z^{-1}\} + B\{1-0.2z^{-1}\}\{1-z^{-1}\} + C\{1-0.2z^{-1}\}\{1-2z^{-1}\}$$

Compared coefficients

$$A + B + C = 0$$

$$2A + 0.2B + 0.4C = 1$$

$$-3A - 1.2B - 2.2C = 0$$

2M

$$A = -0.476, \quad B = 1.08 \quad C = -0.6122$$

$$X(z) = 1 - \frac{1}{z} - \frac{0.476}{1-0.2z^{-1}} + \frac{1.088}{1-z^{-1}} - \frac{0.6122}{1-2z^{-1}}$$

$$X(z) = 1 - \frac{1}{z} - \frac{0.476z}{z-0.2} + \frac{1.088z}{z-1} - \frac{0.612z}{z-2}$$

If ROC is $0.2 < |z| < 1$ 3 signals must be causal. 4 & 5 must be anticausal

3M

$x(n) = \delta(n) - u(n)$ ...

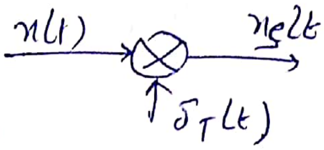
8a Sampling techniques

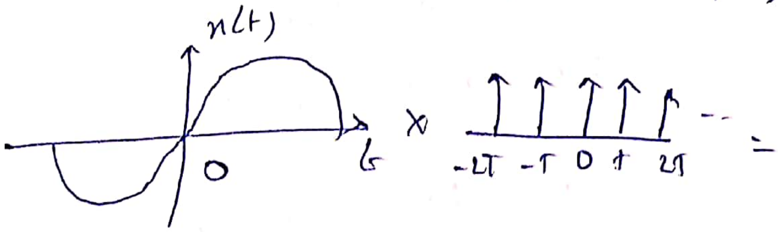
1) Instantaneous sampling (or) impulse sampling
2) natural sampling
3. flat top sampling

1M

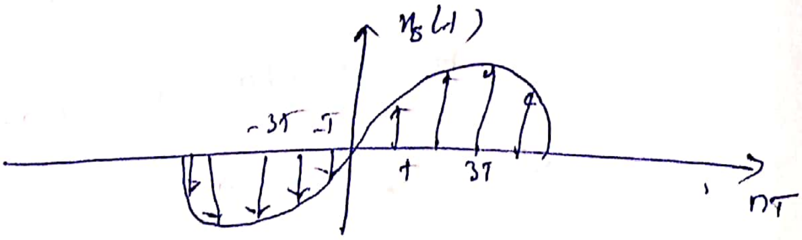
Impulse sampling:

Ideally sampling should be done Instantaneously so that the k^{th} element of the sequence obtained by sampling represents the value of $x(t)$ at $t = kT$.





2M



The Impulse train also called the sampling function.

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

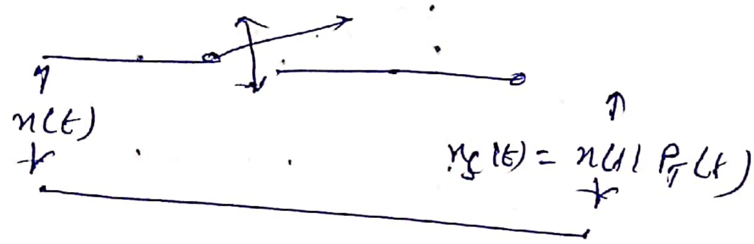
$$x_s(t) = x(t)\,\delta_T(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$= \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t - nT)$$

$$X_s(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$$

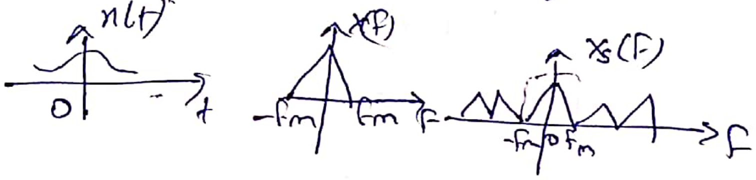
$$X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

Natural Sampling:



1M

Natural Sampler



$$x_s(t) = x(t)\, P_T(t)$$

$$P_T(t) = \sum_{n=-\infty}^{\infty} p(t - nT)$$

$$P_T(t) = \sum_{n=-\infty}^{\infty} p(t - nT) = \sum_{n=-\infty}^{\infty} C_n\, e^{+j2\pi n f_s t}$$

1M

$$C_n = \frac{1}{T} \int_{-\infty}^{\infty} p(t)\, e^{-j2\pi n f_s t}\, dt$$

$$C_n = f_s P(nf_s)$$

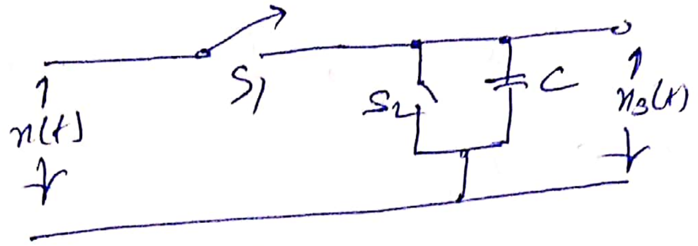
$$P_T(t) = f_s \sum_{n=-\infty}^{\infty} P(nf_s)\, e^{j2\pi n f_s t}$$

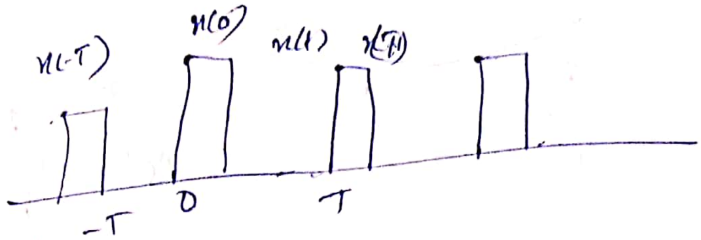
$$X_s(f) = F[x_s(t)]$$

$$= F\left\{ f_s \sum_{n=-\infty}^{\infty} P(nf_s)\, x(t)\, e^{j2\pi n f_s t} \right\}$$

$$X_s(f) = f_s \sum_{n=-\infty}^{\infty} P(nf_s)\, X(f - nf_s)$$

Flat Top Sampling:

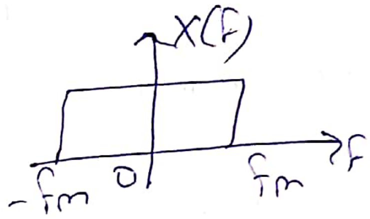




2M

$t = kT$

$x_s(f) = P(f) X_\delta(f)$



$f = \pm(1/\tau)$

$X_s(f) = P(f) X_\delta(f)$

$$H_e(f) = \frac{1}{P(f)} \quad ; \quad |f| \le f_m$$

8b. Zero order Hold:

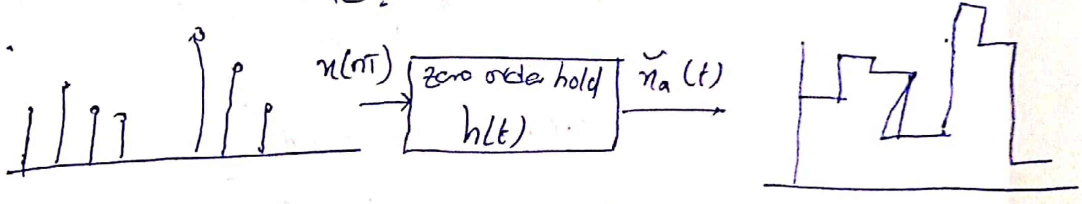


Fig: Zero order hold.

3M

$\tilde{x}_a(t) = x(n)$ for $nT \le n \le (n+1)T$

$\tilde{x}_a(t) = x(0)$ for $0 \le t \le T$

$= x(T)$ for $T \le t < 2T$

$= x(2T)$ for $2T \le T \le 3T$

;

The impulse response of a zero order hold

$$h(t) = 1 \quad 0 \le t \le T$$
$$= 0 \quad \text{otherwise}$$

Transfer function of a zero order Hold: 2M

$$\tilde{x}_a(t) = x(nT) * h(t) = \sum_{n=-\infty}^{\infty} x(nT)h(t-nT)$$

For zero order hold

$$h(t) = u(t) - u(t-T)$$
$$h(t-nT) = u(t-nT) - u[t-(n+1)T]$$
$$\tilde{x}_a(t) = \sum_{n=-\infty}^{\infty} x(nT)\{u(t-nT) - u[t-(n+1)T]\}$$

Taking Laplace transform on both sides

$$L[\tilde{x}_a(t)] = \sum_{n=-\infty}^{\infty} x(nT)\left[\frac{e^{-nTs}}{s} - \frac{e^{-(n+1)Ts}}{s}\right]$$
$$= \left[\frac{1-e^{-Ts}}{s}\right] X^*(s)$$ 2M

∴ Transfer function of zero order hold $= \dfrac{\tilde{X}_a(s)}{X^*(s)}$

$$= \frac{1-e^{-Ts}}{s}$$

9.a) $x(t) = \text{Sinc}(100\pi t)$

$$x(t) = \frac{\sin 100\pi t}{100\pi t} = \frac{\sin(\omega_m t)}{\omega_m t}$$

$\omega_m = 100\pi$ 2M

$$f_m = \frac{\omega_m}{2\pi} = \frac{100\pi}{2\pi} = 50\text{ Hz}$$

$f_s = 2f_m = 100\,Hz$

$\frac{1}{f_s} = \frac{1}{100} = 10ms.$

b) $Sinc2(100\pi t)$

$x(t) = Sinc2(100\pi t)$

$x(t) = \left[\frac{Sin\,100\pi t}{100\pi t}\right]^2$

$x(t) = Sinc\,200\pi t = \frac{Sin(200\pi t)}{200\pi t}$ 2M

$\omega_m = 200\pi$

$f_m = \frac{200\pi}{2\pi} = 100$

$f_s = 2f_m = 200$

$\frac{1}{f_s} = \frac{1}{200} = \cancel{0.5ms}\ 0.005\,sec$

c) $Sinc(100\pi t) + Sinc(50\pi t)$

$x(t) = \frac{Sin(100\pi t)}{100\pi t} + \frac{Sin(50\pi t)}{50\pi t}$

$\omega_m : 100\pi + 50\pi = 150\pi$ 1M

$f_m = \frac{150\pi}{2\pi} = 75\,Hz$

$f_s = 2f_m = 150\,Hz$

$\frac{1}{f_s} = \frac{1}{150} = 0.0066\,sec$

d) $Sinc(100\pi t) + 3Sinc^2(60\pi t)$

$f_m = \frac{100\pi}{2\pi} = 50$

P-23

d) $\sin c(100\pi t) + 3 \sin c\, 2(60\pi)$

$\omega m = 100\pi + 120\pi = 220\pi$

$fm = \frac{220\pi}{2\pi} = 110 Hz$ 2M.

$f_N = 2fm = 220 Hz$

$\frac{1}{f_N} = \frac{1}{220} = 0.0045 sec$

9b. natural sampling also known as uniform sampling is the process of taking samples of a continuous time signal at regular intervals in time. Each sample is taken at a specific time instant and the value of the signal at that instant is recorded. These samples can then be used to represent and process the continuous signal digitally. 2M

In natural sampling the signal is sampled uniformly in time which means that the time intervals between consecutive samples are constant.

The interval between samples is often denoted by 'T' which is the sampling period. The inverse of the sampling period $fs = 1/T$ is the sampling frequency, representing the number of samples taken per unit time. 1M 1M

Effect of undersampling:

undersampling also known as aliasing occurs when the sampling rate is insufficient to accurately capture the content of a continuous time signal. When a signal is undersampled high frequency components in the signal can fold back into lower frequency ranges, creating misleading or distorted representations of the original signal. 2M

The Nyquist -shannon sampling theorem establishes that inorder to avoid aliasing the sampling rate must be at least twice the highest frequency present in the signal. If the sampling rate is too low, aliasing can occur leading to artifacts and loss of information. The aliasing effect is particularly noticeable when a high frequency component in the original signal is folded back into the frequency range that overlaps with lower-frequency components. 2M.

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