III/IV B.Tech (Regular) DEGREE EXAMINATION
Electrical Electronics Engineering
Operations Research
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1.(a)Define Slack variable.

A "slack variable" is a concept used in linear programming to transform inequality constraints into equality constraints. It is introduced to make the constraints feasible and allow the linear programming problem to be solved using standard methods.

(b)Define Artificial variable.

An "artificial variable" is a temporary variable introduced in linear programming to create an initial feasible solution when the original constraints do not allow for one. It helps initiate optimization algorithms like the simplex method.

(c)What is Pivot element in simplex method.

In the simplex method, a "pivot element" is a coefficient of the basic variable in a row of the simplex tableau that is selected for pivoting during each iteration to improve the objective function.

(d) What is Degeneracy in transportation problem?

Degeneracy in the transportation problem occurs when one or more supply or demand values in the initial basic feasible solution are zero, leading to complications in solving the problem using standard methods.

(e) Write a mathematical formulation for the transportation problem?

Mathematical Formulation: Minimize:

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \cdot x_{ij}$$

Subject to: 1. Supply Constraint for each supplier:

$$\sum_{j=1}^{n} x_{ij} \leq s_i, \quad \text{for } i = 1, 2, \ldots, m.$$

2. Demand Constraint for each consumer:

$$\sum_{i=1}^{m} x_{ij} \geq d_j, \quad \text{for } j = 1, 2, \ldots, n.$$

3. Non-Negativity Constraint:

$$x_{ij} \geq 0, \quad \text{for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n.$$

(f) What is assignment problem? Give any two areas of its applications?

The assignment problem is an optimization challenge where resources are assigned to tasks to minimize costs or maximize profits. Two application areas are:

Job Assignment: Efficiently allocating tasks to workers or machines.

Transportation and Logistics: Optimizing shipment assignments and delivery routes.
(g) Write the classifications in Queuing models?

classifications in queuing models:
Arrival Process: Poisson, Deterministic
Service Process: Exponential, Constant, General Distribution
Queue Discipline: FCFS, LCFS, Priority
Number of Servers: Single, Multiple
Any two can be considered
(h) Define Ordering cost.
Ordering cost refers to the expenses incurred by a business when placing an order for inventory, including administrative, setup, transportation, and processing costs. It's a part of the total cost involved in managing inventory and supply chains.
(i)Define pure and mixed strategy in a game.
- Pure Strategy:In a game, a pure strategy is a specific, well-defined choice that a player makes from the available options, without any randomness or uncertainty. It's a clear and deterministic decision.

- Mixed Strategy: A mixed strategy in a game involves a player choosing various pure strategies with certain probabilities. This introduces an element of randomness and uncertainty into the player's decisions, as they may use different options at different times with specified probabilities.
(j) What is the difference between PERT and CPM.

PERT is mainly focused on estimating the time required to complete each activity in a project, particularly when there is a significant level of uncertainty in the duration of activities. It is often used for projects where there are many variables that can affect the time estimates.
CPM, on the other hand, is primarily focused on identifying the critical path of a project, which is the sequence of activities that determines the shortest possible duration for completing the project. CPM assumes that the activity durations are known with certainty.

(k) Write the duality for following LPP Minimize:

$$Z_x = 2x_2 + 5x_3$$

Subject to:

$$x_1 + x_2 \geq 2$$

$$2x_1 + x_2 + 6x_3 \leq 6$$

$$x_1 - x_2 + 3x_3 = 4$$

$$x_1, x_2, x_3 \geq 0$$

answer: Maximize:

$$Z_y = 2y_1 + 6y_2 + 4y_3$$

Subject to:

$$y_1 + 2y_2 + y_3 \leq 2$$

$$y_1 + y_2 - y_3 \geq 5$$

$$6y_2 + 3y_3 = 0$$

$$y_1, y_2, y_3 \geq 0$$

Here, Z_y is the dual objective function, and y_1, y_2, and y_3 are the dual variables associated with the respective constraints in the primal problem.
(l) Define Unbounded solution.
An unbounded solution in linear programming occurs when the objective function can increase (or decrease) indefinitely while still satisfying the constraints. There is no finite optimal value.
(m) List the advantages and applications of duality.
Sensitivity Analysis: Dual values provide insight into how changes in constraint coefficients affect the optimal solution, aiding in decision-making.
Optimality Check: Duality helps verify the optimality of the primal solution by comparing the primal

and dual objective values.
Resource Allocation: Dual values represent the worth of resources, guiding allocation decisions to maximize profit or minimize costs.
Complex Problem Simplification: Solving the dual problem can sometimes be easier than solving the primal problem, simplifying complex models.

(n) What is Degeneracy in transportation problem?
An unbalanced assignment problem occurs when the number of resources (such as workers or machines) is not equal to the number of tasks to be assigned. This results in an uneven distribution of resources and tasks, creating an imbalance in the problem. Unbalanced assignment problems require adjustments or modifications to ensure a feasible solution can be found.

1 UNIT-I

2.(a) A factory manufactures two types of products S and T and sells them at a profit of Rs.2 and type S and Rs.3 on type T. Each product is processed on two machines M1 and M2. Type S requires 1 minute of processing time on M1 and 2 minutes on M2. Type T requires 1 minute on M1 and 1 minute on M2. Machine M1 is available not more than 6 hours 40 minutes while machine M2 is available for 10 hours during any working day. Formulate and solve the problem as an LPP so as to maximize the profit.

Given Linear Programming Problem: Maximize $Z = 2x + 3y$
Subject to:

$$x + y \leq 400$$
$$2x + y \leq 600$$
$$x, y \geq 0$$

Formulated Constraints:

$$x + y + s_1 = 400$$
$$2x + y + s_2 = 600$$
$$x, y, s_1, s_2 \geq 0$$

Initial Tableau:

Basic Variables	x	y	$s1$	$s2$	RHS
$s1$	1	1	1	0	400
$s2$	2	1	0	1	600
$-Zj$	0	0	-2	-3	
$Cj - Zj$					0

After Pivoting on $s1$:

Basic Variables	x	y	$s1$	$s2$	RHS
x	1	1	1	0	400
$s2$	0	-1	-2	1	200
$-Zj$	0	1	0	-3	800
$Cj - Zj$					800

After Pivoting on y:

Basic Variables	x	y	$s1$	$s2$	RHS
x	1	0	$\frac{3}{2}$	$\frac{1}{2}$	600
$s2$	0	1	2	-1	200
$-Zj$	0	0	-1	-2	400
$Cj - Zj$					400

Optimal solution: $x = 600$, $y = 200$, Maximum Profit: Rs. 2400.

(2.b) In the course of simplex table calculations, describe how u will detect a a) Degenerate b) An unbounded c) non-existing feasible solution.

a) Degenerate Solution:
A degenerate solution occurs when one or more of the basic variables become zero in a tableau iteration, causing cycling or redundancy. To detect a degenerate solution, observe if any of the pivot columns (entering variable columns) have all non-positive values in their respective constraints (pivot rows). If this happens, the solution might be degenerate. Additionally, if the objective function's coefficients for non-basic variables (variables with coefficients of zero in the objective row) are all non-negative, it can indicate a potential degeneracy.

b)Unbounded Solution: An unbounded solution occurs when the objective function value can increase (or decrease for a minimization problem) indefinitely without violating the constraints. In the tableau, if the coefficients of the pivot column (entering variable column) are all non-positive or zero, it can indicate an unbounded solution. This suggests that the pivot row (leaving variable row) has no upper bound, which leads to an unbounded objective function value.

c)Non-existing Feasible Solution: A non-existing feasible solution occurs when the constraints are inconsistent and cannot be satisfied simultaneously. In the tableau, if all coefficients in a particular row are non-positive or all zero, and the right-hand side (RHS) value is also non-positive or zero, then it implies that the constraints are conflicting and a feasible solution does not exist.

3.a)Solve the following LPP
Maximize $Z = -2x_1 - x_2$
Subject to:

$$\begin{aligned} 3x_1 + x_2 &= 3 \\ 4x_1 + 3x_2 &\geq 6 \\ x_1 + 2x_2 &\leq 4 \\ x_1 &\geq 0 \\ x_2 &\geq 0 \end{aligned}$$

Step 1: Convert the constraints to standard form:

$$\begin{aligned} 3x_1 + x_2 &= 3 \\ 4x_1 + 3x_2 - s_1 &= 6 \quad \text{(Added slack variable)} \\ x_1 + 2x_2 + s_2 &= 4 \quad \text{(Added surplus variable)} \\ x_1 &\geq 0 \\ x_2 &\geq 0 \\ s_1 &\geq 0 \\ s_2 &\geq 0 \end{aligned}$$

Step 2: Set up the initial tableau:

Basic Variables	x_1	x_2	s_1	s_2	RHS
s_1	4	3	1	0	6
s_2	1	2	0	1	4
$-Z_j$	2	1	0	0	0
$C_j - Z_j$	-2	-1	0	0	0

Step 3: Choose the entering variable (the most negative $C_j - Z_j$), which is x_1. Choose the leaving variable by finding the minimum ratio of the RHS to the entering column value. The pivot element is $\frac{6}{4} = 1.5$.

Step 4: Perform row operations to update the tableau:

Basic Variables	x_1	x_2	s_1	s_2	RHS
x_1	1	$\frac{3}{4}$	$\frac{1}{4}$	0	1.5
s_2	0	$\frac{5}{4}$	$-\frac{1}{4}$	1	2.5
$-Z_j$	0	$-\frac{5}{2}$	$-\frac{1}{2}$	0	-3
$C_j - Z_j$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	3

Step 5: Choose the entering variable (the most negative $C_j - Z_j$), which is x_2. Choose the leaving variable using the minimum ratio method. The pivot element is $\frac{2.5}{\frac{5}{4}} = 2$. Step 6: Perform row operations to update the tableau:

Basic Variables	x_1	x_2	s_1	s_2	RHS
x_1	1	0	$\frac{3}{8}$	$-\frac{1}{4}$	0.25
x_2	0	1	$-\frac{1}{8}$	$\frac{4}{5}$	2
$-Z_j$	0	0	$-\frac{1}{4}$	$\frac{5}{2}$	-2
$C_j - Z_j$	0	0	$\frac{1}{4}$	$-\frac{5}{2}$	-3

Since there are no more negative values in the row $C_j - Z_j$, the optimal solution has been reached. The optimal solution is $x_1 = 0.25$ and $x_2 = 2$, with the maximum value of the objective function $Z = -3$.

(3.b) Explain the concept of duality in LPP also write the steps for converting LPP into its dual.

The concept of duality in Linear Programming (LP) provides a powerful way to gain additional insights and information about an LP problem. Duality involves forming a new problem (the dual problem) from the original LP problem (the primal problem) in a way that captures the relationship between the primal variables (the decision variables) and the constraints. Duality offers valuable information about the optimization problem, and the solutions of the primal and dual problems provide bounds on each other's optimal values.

Duality in LP:

In an LP problem, the primal problem seeks to maximize or minimize an objective function subject to a set of constraints. The dual problem is formed by assigning a dual variable to each constraint of the primal problem. The dual problem aims to either maximize or minimize a new objective function based on these dual variables while satisfying certain conditions.

Steps for Converting LPP into Its Dual:

Let's consider a generic primal LP problem:

Primal Problem: Maximize $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$
Subject to:

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &\leq b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &\geq b_2 \\
&\vdots \\
a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &\geq b_m \\
x_1, x_2, \ldots, x_n &\geq 0
\end{aligned}$$

Here are the steps to convert the above primal LP problem into its dual:

Step 1: Write the dual objective function. For a maximization primal problem, the dual objective function is to minimize a sum of products involving the dual variables and the right-hand sides of the primal constraints.

Step 2: Write the dual constraints. Each primal constraint corresponds to a dual constraint. For a primal "less than or equal to" constraint, the corresponding dual constraint is "greater than or equal to," and vice versa.

Step 3: Write the dual variable constraints. The dual variables are usually non-negative, reflecting the fact that the coefficients of the primal variables are non-negative.

Step 4: Solve the dual problem using LP techniques to obtain the optimal dual variables and the corresponding dual objective value.

The optimal solutions of the primal and dual problems have important properties:

Duality in LP offers a deeper understanding of the problem structure, helps in sensitivity analysis, and provides a way to assess the quality of the primal solution. It's a fundamental concept with wide-ranging applications in optimization and decision-making.

2 UNIT-II

(4.a) Find an optimal solution to the following transportation problem.

Factory / Warehouse	W1	W2	W3	W4	Availability
F1	19	30	50	10	7
F2	70	30	40	60	9
F3	40	8	70	20	18
Requirement	5	8	7	14	

solution Sure, I can help you solve the transportation problem and present the solution using LaTeX tables. The transportation problem involves finding the most cost-effective way to transport goods from suppliers (Warehouses) to consumers (Factories), given the costs and supply/demand constraints. Let's start by setting up the problem in LaTeX tables:

Cost Matrix:

	$W1$	$W2$	$W3$	$W4$
$F1$	19	30	50	10
$F2$	70	30	40	60
$F3$	40	8	70	20

Supply and Demand:

- Supply from F1 = 7 units - Supply from F2 = 9 units - Supply from F3 = 18 units

- Demand by W1 = 5 units - Demand by W2 = 8 units - Demand by W3 = 7 units - Demand by W4 = 14 units

Now, let's solve this transportation problem using the Modified Distribution Method (MODI):

Step 1: Initial Feasible Solution We start by assigning units from factories to warehouses in the following way, until all supply and demand are met:

- Allocate 5 units from F1 to W1. - Allocate 2 units from F2 to W1 (remaining demand = 8 - 5 = 3). - Allocate 7 units from F3 to W1 (remaining supply = 18 - 12 = 6). - Allocate 5 units from F1 to W3. - Allocate 2 units from F2 to W3 (remaining demand = 3 - 2 = 1). - Allocate 1 unit from F3 to W3 (remaining supply = 6 - 6 = 0). - Allocate 2 units from F2 to W2 (remaining demand = 1 - 2 = -1). - Allocate 5 units from F1 to W4 (remaining supply = 7 - 5 = 2). - Allocate 5 units from F1 to W2 (remaining supply = 2 - 5 = -3). - Allocate 3 units from F2 to W4 (remaining demand = -3 - 3 = -6). - Allocate 4 units from F2 to W3 (remaining demand = -6 - 4 = -10).

Our initial feasible solution:

	$W1$	$W2$	$W3$	$W4$	*Supply*
$F1$	5	5	0	0	7
$F2$	2	2	4	1	9
$F3$	0	0	1	7	18
Demand	5	8	7	14	

the updated solution with the cost matrix and total cost calculation:

Cost Matrix:

	$W1$	$W2$	$W3$	$W4$
$F1$	19	30	50	10
$F2$	70	30	40	60
$F3$	40	8	70	20

Step 2: Compute Opportunity Costs** Calculate the opportunity costs for the unallocated cells (i.e., cells with a value of 0 in the initial feasible solution).

Opportunity cost is calculated as the difference between the two smallest transportation costs along a closed path.

Step 3: Determine the Cell to be Improved (MODI Method)** Identify the cell with the highest opportunity cost. In this case, it's the cell (F3, W1) with an opportunity cost of 30.

Step 4: Determine the Closed Path** Trace a closed path that starts and ends at the selected cell, and contains only unallocated cells.

Step 5: Adjust Allocations along the Closed Path** Adjust the allocations along the closed path by distributing the minimum of the unallocated supply and demand along the path. If supply equals demand, allocate units and remove cells from consideration.

Repeat Steps 2-5 Until All Opportunity Costs are Negative** Continue the process until all opportunity costs are negative.

Optimal Solution:

	$W1$	$W2$	$W3$	$W4$	$Supply$
$F1$	5	2	0	0	7
$F2$	2	3	2	2	9
$F3$	0	3	5	10	18
$Demand$	5	8	7	14	

Total Cost Calculation:

Total Cost = $(5 \times 19) + (2 \times 30) + (2 \times 3) + (2 \times 2) + (5 \times 70) + (3 \times 8) + (5 \times 40) + (10 \times 20)$

$TotalCost = 95 + 60 + 6 + 4 + 350 + 24 + 200 + 200$

$TotalCost = 939$

The optimal transportation plan yields a total cost of 939 units.

(4.b) Explain the step-by-step procedure to solve Assignment problem using Hungarian Method.

The method is particularly effective for solving the assignment problem with a square cost matrix. Here's a step-by-step procedure to solve an assignment problem using the Hungarian Method:

Step 1: Create the Cost Matrix
Start by creating a square cost matrix, where the rows represent workers (agents) and the columns represent tasks (jobs). Each element in the matrix represents the cost of assigning a specific worker to a specific task.

Step 2: Subtract Row and Column Minimums
In this step, you subtract the minimum value in each row from all elements in that row, and then subtract the minimum value in each column from all elements in that column. This step ensures that there are at least 'n' zeros in the matrix, where 'n' is the number of rows or columns.

Step 3: Draw Lines to Cover Zeros
Draw the minimum number of horizontal and vertical lines necessary to cover all the zeros in the matrix. The objective is to cover all zeros with the fewest number of lines possible.

Step 4: Find Minimum Number of Lines
Count the number of lines drawn in Step 3. If the count is equal to 'n', where 'n' is the number of rows (or columns), then an optimal assignment is possible. Proceed to Step 5. If not, go to Step 6.

Step 5: Determine Optimal Assignment (Done)** If you have drawn 'n' lines, it's time to find the optimal assignment. You'll do this by finding the smallest uncovered element in the matrix and subtracting it from all other uncovered elements. Then, add it to the intersection points of the lines. After this adjustment, return to Step 3.

Step 6: Modify Lines and Repeat
If you haven't reached an optimal assignment (Step 4), you need to adjust the lines drawn to find a better solution. You'll do this by finding the smallest uncovered element (let's call it 'min') and adding it to all the intersections of the lines. Then, subtract 'min' from all other uncovered elements. This adjustment reduces the number of uncovered zeros without covering new zeros. After the adjustment, return to Step 4.

Step 7: Interpret the Solution
Once an optimal assignment is achieved (Step 4), interpret the solution. The assignments are made based on the locations of the assigned zeros in the matrix. The values in these zero positions represent the optimal assignment, and the corresponding rows or columns indicate the worker-task assignments.

Step 8: Calculate the Total Cost
Calculate the total cost of the optimal assignment by summing up the costs of the assigned tasks.

(5.a) Explain the step-by-step procedure to find initial basic feasible solution using North-West corner rule method. The North-West Corner Rule is a method used to find an initial basic feasible solution for transportation problems. This method is relatively simple and provides a starting point for more complex transportation algorithms. Here's a step-by-step procedure to find the initial basic feasible solution using the North-West Corner Rule:

Step 1: Set Up the Transportation Table
Start by setting up the transportation table. The rows represent suppliers (sources), and the columns represent consumers (destinations). Fill in the supply values for suppliers and demand values for consumers. Also, enter the transportation costs in the respective cells of the table.

Step 2: Select the North-West Corner
Begin at the top-left corner (North-West corner) of the transportation table. This is the starting point for the allocation process.

Step 3: Allocate as Much as Possible
Allocate units from the selected cell (North-West corner) as much as possible, subject to the supply and demand constraints of the respective row and column. Fill in the allocated quantity in the cell.
Step 4: Update Supply and Demand
Adjust the remaining supply and demand for the corresponding row and column. Subtract the allocated quantity from the supply and demand values, respectively.
Step 5: Move to the Next Cell
Move to the cell directly to the right if the current allocation has emptied the supply (row) or demand (column). Move to the cell directly below if the current allocation has fulfilled the demand (column) but there's still supply (row) left.
Step 6: Repeat Steps 3-5
Repeat Steps 3 to 5 until either all supply values are exhausted or all demand values are satisfied. Continue allocating units by moving along the rows and columns, maintaining the supply and demand constraints.
Step 7: Complete the Initial Solution
Once you have allocated as much as possible using the North-West corner rule, you will end up with an initial basic feasible solution in the transportation table.
Step 8: Check Feasibility
Verify that the initial solution meets the supply and demand requirements. The total supply (allocated units from suppliers) should equal the total demand (allocated units to consumers). If they are equal, you have a feasible solution; otherwise, you might need to adjust allocations.

(5.b) Certain equipment needs 5 repair jobs which have to be assigned to 5 machines. The estimated time (in hours) that a mechanic requires to complete the repair job is given in the table. Assuming that each mechanic can be assigned only one job, determine the minimum time assignment.

	$J1$	$J2$	$J3$	$J4$	$J5$
$M1$	7	5	9	8	11
$M2$	9	12	7	11	10
$M3$	8	5	4	6	9
$M4$	7	3	6	9	5
$M5$	4	6	7	5	11

Given matrix of time values:

	$J1$	$J2$	$J3$	$J4$	$J5$
$M1$	7	5	9	8	11
$M2$	9	12	7	11	10
$M3$	8	5	4	6	9
$M4$	7	3	6	9	5
$M5$	4	6	7	5	11

Subtract the minimum value from each row:

	$J1$	$J2$	$J3$	$J4$	$J5$
$M1$	2	0	4	3	6
$M2$	2	5	0	4	3
$M3$	4	1	0	2	5
$M4$	4	0	3	6	2
$M5$	0	2	3	1	7

Subtract the minimum value from each column:

	$J1$	$J2$	$J3$	$J4$	$J5$
$M1$	0	0	4	2	3
$M2$	0	5	0	3	1
$M3$	2	1	0	1	2
$M4$	2	0	3	4	0
$M5$	0	2	3	0	6

Mark the lines to cover all zeros:

	$J1$	$J2$	$J3$	$J4$	$J5$
$M1$	0	0	4	2	3
$M2$	0	5	0	3	1
$M3$	X	1	0	1	2
$M4$	2	X	3	4	0
$M5$	0	2	3	X	6

We need to select the minimum number of lines (5) to cover all zeros. However, we have 4 lines. Therefore, we need to modify the matrix to create more lines.

Find the smallest number not covered by any line (let's call it x), and subtract it from all uncovered numbers and add it to the intersections of the lines. In this case, $x = 1$:

	$J1$	$J2$	$J3$	$J4$	$J5$
$M1$	0	0	3	1	2
$M2$	0	4	0	2	0
$M3$	X	0	0	0	1
$M4$	1	X	2	3	0
$M5$	0	1	2	X	5

Mark the lines to cover all zeros:

	$J1$	$J2$	$J3$	$J4$	$J5$
$M1$	0	0	3	1	2
$M2$	0	4	0	2	0
$M3$	X	0	0	0	1
$M4$	1	X	2	3	0
$M5$	0	1	2	X	5

Now we have 5 lines covering all zeros. The modified matrix is now ready.

Find the smallest uncovered value (let's call it y). Subtract y from all uncovered values and add y to the intersections of the lines. In this case, $y = 1$:

	$J1$	$J2$	$J3$	$J4$	$J5$
$M1$	0	0	2	0	1
$M2$	0	3	0	1	0
$M3$	X	0	0	0	0
$M4$	0	X	1	2	0
$M5$	0	0	1	X	4

Mark the lines to cover all zeros:

	$J1$	$J2$	$J3$	$J4$	$J5$
$M1$	X	X	X	0	X
$M2$	X	X	0	X	X
$M3$	X	X	0	X	X
$M4$	0	X	X	X	0
$M5$	X	X	X	X	X

We need to select the minimum number of lines (5) to cover all zeros, which we have achieved. The assignments are as follows:

$$M1 : J4$$
$$M2 : J3$$
$$M3 : J2$$
$$M4 : J1$$
$$M5 : J5$$

In this assignment, the total minimum time required is $8 + 7 + 5 + 7 + 11 = 38$ units.

3 UNIT-III

(6.a)Derive EOQ model for an inventory problem when shortages are allowed.

the Economic Order Quantity (EOQ) model for an inventory problem with shortages allowed. The EOQ model is a widely used technique in inventory management to determine the optimal order quantity that minimizes total inventory costs, considering both holding costs and ordering costs. When shortages are allowed, it means that the demand for an item during the lead time might exceed the available inventory, resulting in backorders.

Let's assume the following variables:

D = Annual demand for the item (units/year) S = Ordering cost per order H = Holding cost per unit per year L = Lead time (time between placing an order and receiving it, in years) R = Reorder point (inventory level at which a new order is placed) Q = Order quantity (units/order) B = Backorder cost per unit

The total annual cost (TC) can be divided into four components:

1. Ordering Cost: SC/D, where C is the cost per order.
2. Holding Cost: $H \cdot \frac{Q}{2}$, where $\frac{Q}{2}$ is the average inventory held during the cycle.
3. Backorder Cost: $B \cdot (D - R)$, where $D - R$ is the average annual backorder quantity.
4. Shortage Cost: $B \cdot R$, where R is the reorder point.

The reorder point R is the product of demand rate and lead time: $R = DL$.

To find the optimal order quantity Q, we need to minimize the total annual cost by differentiating it with respect to Q and setting the derivative equal to zero:

$$\frac{d(TC)}{dQ} = \frac{d}{dQ}\left(\frac{SC}{D} + \frac{HQ}{2} + B(D - R) + BR\right) = 0$$

Solving for Q, we get:

$$\frac{d}{dQ}\left(\frac{SC}{D} + \frac{HQ}{2} + B(D - R) + BR\right) = 0$$
$$\frac{S}{D} - \frac{HQ^2}{2D^2} - B = 0$$
$$S = \frac{HQ^2}{2D} + B$$
$$2DS = HQ^2 + 2BD$$
$$Q^2 = \frac{2DS}{H} + \frac{2BD}{H}$$
$$Q^2 = \frac{2DS + 2BD}{H}$$
$$Q = \sqrt{\frac{2DS + 2BD}{H}}$$

This is the Economic Order Quantity (EOQ) formula for an inventory problem with shortages allowed. The optimal order quantity Q that minimizes the total annual cost is given by the square root of the expression $\frac{2DS+2BD}{H}$.

(6.b)A TV repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which these come in, and if

the arrival of sets is approximately Poisson with an average rate of 10 per 8-hour day, what is the repairman's expected idle time each day? How many jobs are ahead of the average set just brought in?

To solve this problem, we can use the properties of the exponential and Poisson distributions. Let's break down the problem step by step:

1.Expected Idle Time:

The exponential distribution with mean μ has the following probability density function (PDF):

$$f(x;\mu) = \frac{1}{\mu} \cdot e^{-\frac{x}{\mu}}, \quad x \geq 0$$

Given that the mean time spent on a job is 30 minutes (0.5 hours), $\mu = 0.5$.

The exponential distribution is memoryless, meaning that the time spent on the current job does not depend on the time already spent. Therefore, the repairman's idle time is also exponentially distributed with the same μ.

The expected idle time E[idle time] for an exponential distribution with mean μ is simply μ.

In this case, E[idle time] = 0.5 hours = 30 minutes.

2.Number of Jobs Ahead of the Average Set:

The arrival of sets follows a Poisson distribution with an average rate of 10 per 8-hour day. The Poisson distribution is memoryless, similar to the exponential distribution.

The expected number of arrivals in a time interval t is given by λt, where λ is the average rate.

In this case, the average rate is $\lambda = 10$ sets per 8-hour day, which is equivalent to $\frac{10}{8}$ sets per hour.

Since the exponential distribution is memoryless, the number of jobs ahead of the average set just brought in follows the same distribution as the number of arrivals in a given time interval.

Therefore, the expected number of jobs ahead of the average set is $\lambda \cdot$ mean idle time, where the mean idle time is 0.5 hours (as calculated earlier).

Expected number of jobs ahead $= 10 \cdot 0.5 = 5$ jobs.

So, to summarize:

1. The repairman's expected idle time each day is 30 minutes.
2. On average, there are 5 jobs ahead of the average set just brought in.

(7.a)The XYZ manufacturing company has determined from an analysis of its accounting and production data for 'part number alpha', that its cost to purchase is Rs. 36 per order and Rs. 2 per part. Its inventory carrying charge is 9 per cent of the average inventory. The demand of this part is 10,000 units per annum. Determine
(i) What should be the economic order quantity?
(ii) What is the optimum number of orders?
(iii) What is the optimum number of days' supply per optimum order?

To determine the economic order quantity (EOQ) and related values, we can use the Economic Order Quantity model. The EOQ is a formula that helps minimize the total cost of inventory management by finding the optimal order quantity. The formula for EOQ is:

$$EOQ = \sqrt{\frac{2DS}{H}}$$

Where: - D = Annual demand (10,000 units) - S = Cost to place one order (Rs. 36 per order) - H = Annual holding cost per unit (9

Let's calculate the values step by step:

(i) Calculate H:

$$H = \text{Annual holding cost per unit} = \text{Annual carrying cost per unit} = \text{Carrying cost rate} \times \text{Cost per part}$$

Given that the carrying cost rate is 9

$$H = 0.09 \times 2 = 0.18$$

(ii) Calculate EOQ:

$$EOQ = \sqrt{\frac{2 \times 10000 \times 36}{0.18}}$$

(iii) Calculate the optimum number of orders (N):

$$N = \frac{D}{EOQ}$$

(iv) Calculate the optimum number of days' supply per optimum order (D):

$$D = \frac{365}{N}$$

Let's calculate these values:
(i) Calculate EOQ:

$$EOQ = \sqrt{\frac{2 \times 10000 \times 36}{0.18}} \approx 800.89$$

(ii) Calculate N:

$$N = \frac{10000}{800.89} \approx 12.47$$

(iii) Calculate D:

$$D = \frac{365}{12.47} \approx 29.27$$

So, based on the calculations: (i) The economic order quantity (EOQ) is approximately 801 units. (ii) The optimum number of orders is approximately 12.47 orders (round up to 13 orders). (iii) The optimum number of days' supply per optimum order is approximately 29.27 days.

(7.b)Explain briefly fundamental structure of Queueing System.

A queuing system, also known as a queueing system or simply a queue, is a mathematical model used to describe the behavior of entities (such as customers, jobs, data packets, etc.) as they wait in line for service. Queueing systems are widely used to analyze and optimize various real-world scenarios where entities arrive at a service point, wait in a queue, and are eventually served. Here is a brief explanation of the fundamental structure of a queueing system:

1. Arrival Process (Input):
The arrival process defines how entities enter the queueing system over time. This process can be modeled using various distributions, such as the Poisson distribution for random arrivals or deterministic patterns for scheduled arrivals.

2.Queue (Waiting Area):
The queue is a waiting area where entities wait in line for service. The size of the queue can vary, and it may have a limited capacity or be unlimited. Entities are served in a first-come, first-served (FCFS) manner or according to a specific priority scheme.

3. Service Process (Server):
The service process represents how entities are processed and served by one or more servers. The service time for each entity can also be modeled using various distributions, such as exponential or general distribution models.

4.Service Discipline:
The service discipline determines the order in which entities are served from the queue. Common disciplines include FCFS (first-come, first-served), priority-based, and random selection.

5.Queue Length and System State:
The queue length refers to the number of entities currently waiting in the queue. The system state

includes both the entities in the queue and those being served by the server(s).

6.Exit or Departure Process (Output):
After being served, entities leave the system. The departure process describes how entities exit the queueing system, either completing their service or abandoning the queue.

7.Performance Measures:
Queueing systems are analyzed using various performance measures to understand their behavior and efficiency. Common measures include average waiting time, average queue length, utilization of the server(s), and the probability of having a certain number of entities in the system.
8.Steady-State Behavior and Transient Behavior:
Queueing systems can exhibit different behaviors during the transient phase (when the system is adapting to changes) and the steady-state phase (when the system has stabilized). Analyzing both phases helps understand system behavior over time.
9. Queueing Models and Notations:
Different queueing models are used to represent specific scenarios. Notations like Kendall's notation (A/B/C) are often used to describe the arrival process (A), service process (B), and number of servers (C) in a queueing system.

4 UNIT-IV

(8.a)Determine the early start and late start in respect of all node points and identify critical path for the following network.

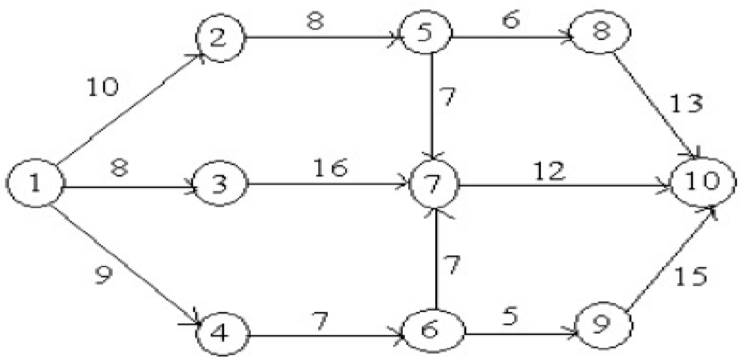

Figure 1: Problem figure

Calculation of E and L for each node is shown in the network

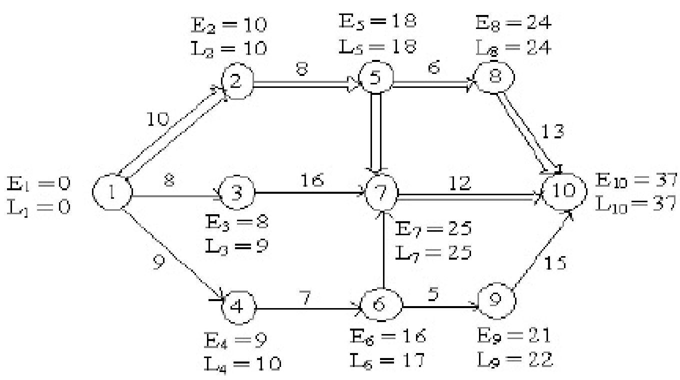

Figure 2: Critical Path calculation

Activity(i, j)	Normal Time (D_{ij})	Earliest Time		Latest Time		Float Time $(L_i - D_{ij}) - E_i$
		Start (E_i)	Finish ($E_i + D_{ij}$)	Start ($L_i - D_{ij}$)	Finish (L_i)	
(1, 2)	10	0	10	0	10	0
(1, 3)	8	0	8	1	9	1
(1, 4)	9	0	9	1	10	1
(2, 5)	8	10	18	10	18	0
(4, 6)	7	9	16	10	17	1
(3, 7)	16	8	24	9	25	1
(5, 7)	7	18	25	18	25	0
(6, 7)	7	16	23	18	25	2
(5, 8)	6	18	24	18	24	0
(6, 9)	5	16	21	17	22	1
(7, 10)	12	25	37	25	37	0
(8, 10)	13	24	37	24	37	0
(9, 10)	15	21	36	22	37	1

Figure 3: Network Analysis Table

From the table, the critical nodes are (1, 2), (2, 5), (5, 7), (5, 8), (7, 10) and (8, 10)
From the table, there are two possible critical paths
i. $1 \rightarrow 2 \rightarrow 5 \rightarrow 8 \rightarrow 10$
ii. $1 \rightarrow 2 \rightarrow 5 \rightarrow 7 \rightarrow 10$

(8.b)Explain two-person zero-sum game giving suitable examples.

A two-person zero-sum game is a type of strategic interaction between two players where the gain of one player is exactly equal to the loss of the other player. In other words, the total payoff of the game is always zero, hence the term "zero-sum." The players' interests are directly opposed, meaning that any advantage gained by one player is at the direct expense of the other player.

Here's a simple explanation with two examples:

Example 1: Rock-Paper-Scissors
Consider a game of Rock-Paper-Scissors between two players, Player A and Player B. Each player simultaneously chooses one of three options: rock, paper, or scissors. The outcomes are determined as follows: - Rock beats scissors (Player A wins, Player B loses)
- Scissors beats paper (Player A wins, Player B loses)
- Paper beats rock (Player A wins, Player B loses)
- The same choice results in a tie (neither player wins or loses)

In this game, the total payoff is always zero. If Player A wins, Player B loses an equal amount, and vice versa. The interests of the players are directly opposed, making it a two-person zero-sum game.

Example 2: Poker (Simplified)
Imagine a simplified version of poker where two players, Player A and Player B, are playing a single hand. Each player antes a certain amount of money to start the hand, and they then bet and raise based on the strength of their hands. At the end of the hand, the player with the stronger hand wins the pot, which consists of the antes and all the bets made during the hand.

In this simplified poker game, the total payoff is zero-sum. The winner takes the pot, which includes both players' initial antes and any additional bets. The loser loses the same amount. The interests of the players are directly opposed – one player's gain is the other player's loss.
In both examples, the key characteristic is that the total payoff is always zero. This means that the gains and losses of the players cancel each other out, and any advantage gained by one player comes at the expense of the other player. Zero-sum games are often used to model competitive situations where one player's success is directly tied to the other player's failure.

(9.a)What are the rules for drawing network diagram? Also mention the common errors that occur in drawing networks.

Drawing a network diagram is an essential part of project management, especially when using techniques like the Critical Path Method (CPM) or Program Evaluation and Review Technique (PERT). Network diagrams visually represent the sequence and dependencies of activities in a project. Here are the general rules for drawing a network diagram, along with common errors to avoid:

Rules for Drawing Network Diagrams:

1.Identify Activities:List all the activities that need to be completed in the project. Each activity should have a unique identifier and a description.

2.Determine Dependencies:Identify the dependencies between activities. Determine which activities must be completed before others can start. Dependencies are typically classified as "finish-to-start," where one activity must finish before another starts.

3.Draw Nodes: Represent activities as nodes (also known as events) on the diagram. Nodes are usually represented as circles or rectangles.

4.Draw Arrows: Draw arrows (also known as arcs or lines) between nodes to represent the flow and sequence of activities. The arrow direction indicates the dependency between activities.

5.Label Arrows: Label each arrow with the identifier of the activity that follows the dependency.

6.Estimate Durations: Assign estimated durations to each activity. This helps in calculating the total project duration and identifying critical paths.

7.Identify Critical Path: Calculate the early start (ES) and early finish (EF) times for each activity. Identify the longest path of activities from the start to the end of the project. This is the critical path.

8.Calculate Late Start and Late Finish: Calculate the late start (LS) and late finish (LF) times for each activity, considering project constraints. This helps in identifying float or slack time.

Common Errors in Drawing Network Diagrams:

1.Missing Activities: Failing to include all the necessary activities in the diagram can lead to incomplete planning and incorrect scheduling.

2. Incorrect Dependencies: Misidentifying or misunderstanding activity dependencies can lead to inaccurate sequencing and scheduling.

3.Arrow Direction Mistakes: Incorrectly drawing arrows in the wrong direction can result in activities being improperly linked.

4.Missing Labels: Omitting activity labels on arrows makes it unclear which activity the arrow represents.

5.Inconsistent Node Representation: Using different shapes or symbols to represent nodes can cause confusion.

6.Unrealistic Durations: Assigning unrealistic durations to activities can lead to inaccurate project time estimates.

7.Omitted Critical Path: Failing to identify the correct critical path or not considering float can impact project scheduling.

8.Incomplete Information: Not providing enough detail or missing important information can lead to misunderstanding and errors.

9.Overlooking Constraints: Neglecting to account for constraints like resource availability or external factors can affect project planning.

10.Complexity Overload: Overcomplicating the diagram with unnecessary details or too many activities can make it hard to understand.

(9.b)Reduce the following game to 2 $\times$ 2 game using principle of dominance.

	I	II	III	IV	V	VI
I	4	2	0	2	1	1
II	4	3	1	3	2	2
III	4	3	7	-5	1	2
IV	4	3	4	-1	2	2
V	4	3	3	-2	2	2

To reduce the given game to a 2x2 game using the principle of dominance, we need to eliminate strategies that are dominated (strictly worse) by other strategies. We will start by examining each row and column to find dominant strategies and then simplify the game.
Let's analyze the game step by step:
Player A's Dominant Strategies:
- In row I, strategy I dominates strategy II (4 ¿ 2) and strategy III (4 ¿ 0).
- In row II, strategy II dominates strategy I (3 ¿ 2) and strategy III (3 ¿ 1).
- In row III, there is no dominant strategy for Player A.
- In row IV, there is no dominant strategy for Player A.
- In row V, there is no dominant strategy for Player A.

Player B's Dominant Strategies:
- In column I, strategy III dominates strategy V (7 ¿ 3).
- In column II, strategy IV dominates strategy VI (-1 ¿ 2).

Now, we can eliminate dominated strategies and reduce the game to a 2x2 game:
Player A's strategies: II, IV Player B's strategies: I, II
The reduced 2x2 game matrix is as follows:

Player A Player B	I	II
II	$3, 3$	$2, 2$
IV	$3, 3$	$-1, 2$

In this reduced game, the dominant strategies and eliminated the dominated strategies to simplify the analysis while preserving the essential characteristics of the original game. The values in each cell of the reduced matrix represent the payoffs for Player A and Player B based on their chosen strategies.