1 a) Compare reflection and refraction of plane waves in any medium 1M

Reflection is the act of light reflecting back when it hits a medium on a plane. Refraction is the process by which light shifts its path as it travels through a material, causing the light to bend. Thus, this is the key difference between reflection and refraction. This phenomenon usually occurs in mirrors.

b) What is perpendicular polarization 1M

The electric vector is parallel to the boundary surface or perpendicular to the plane of incidence.

c) Recall Snell's law of refraction 1M

$$\frac{\sin\theta_1}{\sin\theta_2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

d) For perfect match $|\Gamma_L|$ is _____0____ and the return loss is ___∞____ 1M

e) Define critical angle. 1M

The angle of incidence at which the angle of refraction becomes 90 degrees

f) Find out the attenuation for lossless transmission line. 1M

Zero

g) Write the Helmholtz equation. 1M

$$\nabla^2\psi = \gamma^2\psi$$

The Helmholtz equation in rectangular coordinates is

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \gamma^2\psi$$

h) What is the dominant mode for the TM waves in the rectangular waveguide? 1M

TM_{11}

i) What are degenerate modes in a rectangular waveguide 1M

If any two modes of propagation share the same cut-off frequency, such modes are called degenerate modes. The modes TE_{mn} and TM_{mn} are degenerate modes in a rectangular waveguide.

j) What is dominant mode? 1M

The mode with the lowest cutoff frequency in a particular guide is called the *dominant mode*.

k) Which mode in a circular waveguide has attenuation effect decreasing with increase in frequency? 1M

TE01 mode in circular wave guide has attenuation effect decreasing with increase in frequency.

l) What is the cutoff frequency for TE_{mn}mode in a circular guide. 1M

$$f_c = \frac{X'_{np}}{2\pi a\sqrt{\mu\epsilon}}$$

m) Mention the dominant modes in circular waveguide. 1M

TE_{11} & TM_{01}

n) Mention the Helmholtz equation for E_z in a circular waveguide. 1M

$$\nabla^2 E_z = \gamma^2 E_z$$

Unit-I

2 a) Show that, when a given uniform plane wave is incident normally on a good conductor, the linear current density J_s is essentially independent of the conductivity (σ). 7M

when an electromagnetic wave traveling in one medium impinges upon a second medium having a different dielectric constant, permeability, or conductivity, the wave in general will be partially transmitted and partially reflected. In the case of a plane wave in air incident normally upon the surface of a perfect conductor, the wave is entirely reflected. For fields that vary with time neither **E** nor **H** can exist within a perfect conductor so that none of the energy of the incident wave can be transmitted. Since there can be no loss within a perfect conductor, none of the energy is absorbed. As a result the amplitudes of **E** and **H** in the reflected wave are the same as in the incident wave, and the only difference is in the direction of power flow. If the expression for the electric field of the *incident* wave is

$$E_i e^{-j\beta x}$$

and the surface of the perfect conductor is taken to be the $x = 0$ plane as shown in Fig. the expression for the reflected wave will be

$$E_r e^{j\beta x}$$

where E_r must be determined from the boundary conditions. Inasmuch as the tangential component of **E** must be continuous across the boundary and **E** is zero within the conductor, the tangential component of **E** just outside the conductor must also be zero. This requires that the sum of the electric field strengths in the incident and reflected waves add to give zero resultant field strength in the plane $x = 0$. Therefore

$$E_r = -E_i$$

The amplitude of the reflected electric field strength is equal to that of the incident electric field strength, but its phase has been reversed on reflection.

The resultant electric field strength at any point a distance $-x$ from the $x = 0$ plane will be the sum of the field strengths of the

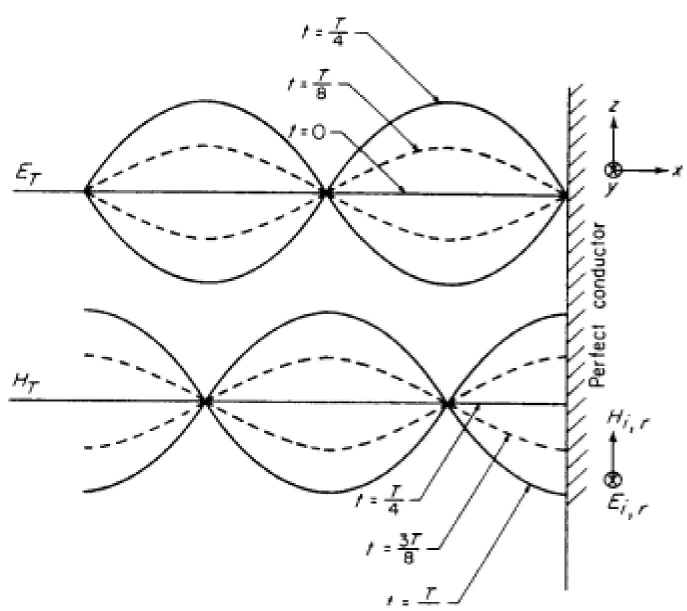


Fig. Standing waves of E and H.

incident and reflected waves at that point and will be given by

$$\begin{aligned} E_T(x) &= E_i e^{-j\beta x} + E_r e^{j\beta x} \\ &= E_i(e^{-j\beta x} - e^{j\beta x}) \\ &= -2jE_i \sin \beta x \end{aligned}$$

$$\tilde{E}_T(x, t) = \mathrm{Re}\{-2jE_i \sin \beta x \, e^{j\omega t}\}$$

If E_i is chosen to be real,

$$\tilde{E}_T(x, t) = 2E_i \sin \beta x \sin \omega t$$

The expression for the resultant magnetic field will be

$$H_T(x) = H_i e^{-j\beta x} + H_r e^{+j\beta x}$$
$$= H_i(e^{-j\beta x} + e^{+j\beta x})$$
$$= 2H_i \cos \beta x$$

H_i is real since it is in phase with E_i.

$$\tilde{H}_T(x, t) = \text{Re}\{H_T(x)e^{j\omega t}\}$$
$$= 2H_i \cos \beta x \cos \omega t$$

b) Derive the reflection of a plane wave by a perfect Dielectric at normal incidence. 7M

when

. ... When a plane electromagnetic wave is incident normally on the surface of a perfect dielectric, part of the energy is transmitted and part of it is reflected. A *perfect* dielectric is one with zero conductivity, so that there is no loss or absorption of power in propagation through the dielectric.

As before, consider the case of a plane wave traveling in the x direction incident on a boundary that is parallel to the $x = 0$ plane. Let E_i be the electric field strength of the incident wave striking the boundary, E_r be the electric field strength of the reflected wave leaving the boundary in the first medium, and E_t be the electric field strength of the transmitted wave propagated into the second medium. Similar subscripts will be applied to the magnetic field strength H. Let ϵ_1 and μ_1 be the constants of the first medium and ϵ_2 and μ_2 be the constants of the second medium. Designating by η_1 and η_2 the ratios $\sqrt{\mu_1/\epsilon_1}$ and $\sqrt{\mu_2/\epsilon_2}$, the following relations will hold

$$E_i = \eta_1 H_i$$
$$E_r = -\eta_1 H_r$$
$$E_t = \eta_2 H_t$$

The continuity of the tangential components of **E** and **H** require that

$$H_i + H_r = H_t$$
$$E_i + E_r = E_t$$

Combining these

$$H_i + H_r = \frac{1}{\eta_1}(E_i - E_r) = H_t = \frac{1}{\eta_2}(E_i + E_r)$$
$$\eta_2(E_i - E_r) = \eta_1(E_i + E_r)$$
$$E_i(\eta_2 - \eta_1) = E_r(\eta_2 + \eta_1)$$
$$\frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

Also

$$\frac{E_t}{E_i} = \frac{E_i + E_r}{E_i} = 1 + \frac{E_r}{E_i}$$
$$= \frac{2\eta_2}{\eta_2 + \eta_1}$$

Furthermore

$$\frac{H_r}{H_i} = -\frac{E_r}{E_i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$$
$$\frac{H_t}{H_i} = \frac{\eta_1}{\eta_2}\frac{E_t}{E_i} = \frac{2\eta_1}{\eta_1 + \eta_2}$$

The permeabilities of all known insulators do not differ appreciably from that of free space, so that $\mu_1 = \mu_2 = \mu_v$. Inserting this relation the above expressions can be written in terms of the dielectric constants as follows:

$$\frac{E_r}{E_i} = \frac{\sqrt{\mu_v/\epsilon_2} - \sqrt{\mu_v/\epsilon_1}}{\sqrt{\mu_v/\epsilon_2} + \sqrt{\mu_v/\epsilon_1}}$$
$$\frac{E_r}{E_i} = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

Similarly

$$\frac{E_t}{E_i} = \frac{2\sqrt{\epsilon_1}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$
$$\frac{H_r}{H_i} = \frac{\sqrt{\epsilon_2} - \sqrt{\epsilon_1}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$
$$\frac{H_t}{H_i} = \frac{2\sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

(OR)

3 a) Derive the conditions for reflection by a perfect insulator-oblique incidence for perpendicular polarization. 7M

In this case the electric vector **E** is perpendicular to the plane of incidence and parallel to the reflecting surface. Let the electric field strength E_i of the incident wave be in the positive x direction [outward in Fig. and let the assumed positive directions for E_r and E_t in the reflected and transmitted waves also be in the positive x direction. Then, applying the boundary condition that the tangential component of **E** is continuous across the boundary,

$$E_i + E_r = E_t$$

$$\frac{E_t}{E_i} = 1 + \frac{E_r}{E_i}$$

Insert this in eq. $\frac{E_r^2}{E_i^2} = 1 - \frac{\sqrt{\epsilon_2}\, E_t^2 \cos\theta_2}{\sqrt{\epsilon_1}\, E_i^2 \cos\theta_1}$

$$\frac{E_r^2}{E_i^2} = 1 - \sqrt{\frac{\epsilon_2}{\epsilon_1}}\left(1 + \frac{E_r}{E_i}\right)^2 \frac{\cos\theta_2}{\cos\theta_1}$$

$$1 - \left(\frac{E_r}{E_i}\right)^2 = \sqrt{\frac{\epsilon_2}{\epsilon_1}}\left(1 + \frac{E_r}{E_i}\right)^2 \frac{\cos\theta_2}{\cos\theta_1}$$

$$1 - \frac{E_r}{E_i} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}\left(1 + \frac{E_r}{E_i}\right) \frac{\cos\theta_2}{\cos\theta_1}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\epsilon_1}\cos\theta_1 - \sqrt{\epsilon_2}\cos\theta_2}{\sqrt{\epsilon_1}\cos\theta_1 + \sqrt{\epsilon_2}\cos\theta_2}$$

Now from eq.

$$\sqrt{\epsilon_2}\cos\theta_2 = \sqrt{\epsilon_2(1 - \sin^2\theta_2)} = \sqrt{\epsilon_2 - \epsilon_1 \sin^2\theta_1}$$

therefore

$$\frac{E_r}{E_i} = \frac{\sqrt{\epsilon_1}\cos\theta_1 - \sqrt{\epsilon_2 - \epsilon_1\sin^2\theta_1}}{\sqrt{\epsilon_1}\cos\theta_1 + \sqrt{\epsilon_2 - \epsilon_1\sin^2\theta_1}}$$

$$= \frac{\cos\theta_1 - \sqrt{(\epsilon_2/\epsilon_1) - \sin^2\theta_1}}{\cos\theta_1 + \sqrt{(\epsilon_2/\epsilon_1) - \sin^2\theta_1}}$$

This equation gives the ratio of reflected to incident electric field strength for the case of a perpendicularly polarized wave.

b) Explain the reflection of a plane wave by a perfect insulator at oblique incidence for parallel polarization. 7M

In this case **E** is parallel to the plane of incidence and **H** is parallel to the reflecting surface. Again applying the boundary condition that the tangential component of **E** is continuous across the boundary in this case gives

$$(E_i - E_r)\cos\theta_1 = E_t \cos\theta_2$$

$$\frac{E_t}{E_i} = \left(1 - \frac{E_r}{E_i}\right)\frac{\cos\theta_1}{\cos\theta_2}$$

Insert this in eq. $\frac{E_r^2}{E_i^2} = 1 - \frac{\sqrt{\epsilon_2}\, E_t^2 \cos\theta_2}{\sqrt{\epsilon_1}\, E_i^2 \cos\theta_1}$

$$\left(\frac{E_r}{E_i}\right)^2 = 1 - \sqrt{\frac{\epsilon_2}{\epsilon_1}}\left(1 - \frac{E_r}{E_i}\right)^2 \frac{\cos\theta_1}{\cos\theta_2}$$

$$1 - \frac{E_r^2}{E_i^2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}\left(1 - \frac{E_r}{E_i}\right)^2 \frac{\cos\theta_1}{\cos\theta_2}$$

$$1 + \frac{E_r}{E_i} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}\left(1 - \frac{E_r}{E_i}\right) \frac{\cos\theta_1}{\cos\theta_2}$$

$$\frac{E_r}{E_i}\left(1 + \sqrt{\frac{\epsilon_2}{\epsilon_1}}\frac{\cos\theta_1}{\cos\theta_2}\right) = \sqrt{\frac{\epsilon_2}{\epsilon_1}}\frac{\cos\theta_1}{\cos\theta_2} - 1$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\epsilon_2}\cos\theta_1 - \sqrt{\epsilon_1}\cos\theta_2}{\sqrt{\epsilon_2}\cos\theta_1 + \sqrt{\epsilon_1}\cos\theta_2}$$

$$= \frac{\sqrt{\epsilon_2}\cos\theta_1 - \sqrt{\epsilon_1(1 - \sin^2\theta_2)}}{\sqrt{\epsilon_2}\cos\theta_1 + \sqrt{\epsilon_1(1 - \sin^2\theta_2)}}$$

Recall that $\sin^2\theta_2 = \epsilon_1/\epsilon_2 \sin^2\theta_1$

$$\frac{E_r}{E_i} = \frac{(\epsilon_2/\epsilon_1)\cos\theta_1 - \sqrt{(\epsilon_2/\epsilon_1) - \sin^2\theta_1}}{(\epsilon_2/\epsilon_1)\cos\theta_1 + \sqrt{(\epsilon_2/\epsilon_1) - \sin^2\theta_1}}$$

The above Equation gives the reflection coefficient for parallel or vertical polarization, that is, the ratio of reflected to incident electric field strength when **E** is parallel to the plane of incidence.

Unit-II

4 a) In a transmission line the VSWR is given as 2.5. The characteristic impedance is 50Ω and the line is to transmit a power of 25 watts. Compute the magnitudes of the maximum and the minimum voltage and current. Also determine the 7M

magnitude of the receiving end voltage when load is (100-j80) Ω

4(a) In a transmission line the VSWR is given as 2.5. The characteristic impedance is 50 Ω and the line is to transmit a power of 25 watts. Compute the magnitudes of the maximum and the minimum voltage and current. Also determine the magnitude of the receiving end voltage when load is (100-j80) Ω.

Sol:- Given the standing wave ratio, $S = 2.5$

characteristic impedance $Z_0 = 50\,\Omega$

Power $P = 25$ Watts.

We know that $P = \frac{|V_{max}|^2}{Z_{max}} = \frac{|V_{max}|^2}{SZ_0}$

$25 = \frac{|V_{max}|^2}{2.5 \times 50}$ $\therefore \boxed{|V_{max}| = 56 \text{ volts}}$

$|V_{min}| = \frac{|V_{max}|}{S} = \frac{56}{2.5} = 22.6$ volts (∵ since $S = \frac{|V_{max}|}{|V_{min}|}$)

$\therefore \boxed{|V_{min}| = 22.6 \text{ volts}}$

$I_{max} = \frac{|V_{max}|}{Z_0} = \frac{56}{50} = 1.12$ Amp $\boxed{I_{max} = 1.12 \text{ Amp}}$ since $V_{max} = I_{max} Z_0$

$I_{min} = \frac{|V_{min}|}{Z_0} = \frac{22.6}{50} = 0.45$ amp as $|V_{min}| = |I_{min}| Z_0$

$\therefore \boxed{I_{min} = 0.45 \text{ Amp}}$

receiving end

$P = |I_R|^2 R_R = 25$

$\therefore Z_R = 100 - j80$ & $R_R = 100\,\Omega$

Since the powers at the sending end and receiving end are the same.

$\therefore 25 = |I_R|^2 \times 100$

$I_R^2 = \frac{25}{100}$ $\therefore |I_R| = \sqrt{\frac{25}{100}} = 0.5$ amp

$\therefore \boxed{|I_R| = 0.5 \text{ amp}}$

$\therefore |V_R| = |I_R||Z_R| = 0.5\sqrt{100^2 + 80^2} = 0.5\sqrt{10000 + 6400}$

$= 0.5 \times 128.06 = 64.03$ volts

b) What are the properties and applications of smith chart? 7M

The properties of smith chart

1. The constant r and constant x loci form two families of orthogonal circles in the chart.
2. The constant r and constant x circles all pass through the point ($\Gamma_r = 1$, $\Gamma_i = 0$).
3. The upper half of the diagram represents $+jx$.
4. The lower half of the diagram represents $-jx$.
5. For admittance the constant r circles become constant g circles, and the constant x circles become constant susceptance b circles.
6. The distance around the Smith chart once is one-half wavelength ($\lambda/2$).
7. At a point of $z_{min} = 1/\rho$, there is a V_{min} on the line.
8. At a point of $z_{max} = \rho$, there is a V_{max} on the line.
9. The horizontal radius to the right of the chart center corresponds to V_{max}, I_{min}, z_{max}, and ρ (SWR).
10. The horizontal radius to the left of the chart center corresponds to V_{min}, I_{max}, z_{min}, and $1/\rho$.
11. Since the normalized admittance **y** is a reciprocal of the normalized impedance **z**, the corresponding quantities in the admittance chart are 180° out of phase

Applications of Smith Charts

Smith charts find applications in all areas of RF Engineering. Some of the most popular application includes;

1. Impedance calculations on any transmission line, on any load.
2. Admittance calculations on any transmission line, on any load.
3. Calculation of the length of a short circuited piece of transmission line to provide a required capacitive or inductive reactance.
4. Impedance matching.
5. Measurement of $V_{max}, V_{min}, I_{max}, I_{min}$
6. Location calculation of Zmax and Zmin

(OR)

5 a) Derive the reflection coefficient in terms of load impedance 7M

In the analysis of the solutions of transmission-line equations, the traveling wave along the line contains two components: one traveling in the positive z direction and the other traveling the negative z direction. If the load impedance is equal to the line characteristic impedance, however, the reflected traveling wave does not exist.

Figure, shows a transmission line terminated in an impedance Zl. It is usually more convenient to start solving the transmission-line problem from the receiving rather than the sending end, since the voltage-to-current relationship at the load point is fixed by the load impedance. The incident voltage and current waves traveling along the transmission line are given by

$$\mathbf{V} = \mathbf{V}_{+}e^{-\gamma z} + \mathbf{V}_{-}e^{+\gamma z}$$

$$\mathbf{I} = \mathbf{I}_{+}e^{-\gamma z} + \mathbf{I}_{-}e^{+\gamma z}$$

in which the current wave can be expressed in terms of the voltage by

$$\mathbf{I} = \frac{\mathbf{V}_{+}}{\mathbf{Z}_0}e^{-\gamma z} - \frac{\mathbf{V}^{-}}{\mathbf{Z}_0}e^{\gamma z}$$

If the line has a length of l, the voltage and current at the receiving end become

$$\mathbf{V}_\ell = \mathbf{V}_{+}e^{-\gamma\ell} + \mathbf{V}_{-}e^{\gamma\ell}$$

$$\mathbf{I}_\ell = \frac{1}{\mathbf{Z}_0}(\mathbf{V}_{+}e^{-\gamma\ell} - \mathbf{V}_{-}e^{\gamma\ell})$$

The ratio of the voltage to the current at the receiving end is the load impedance. That is,

$$\mathbf{Z}_\ell = \frac{\mathbf{V}_\ell}{\mathbf{I}_\ell} = \mathbf{Z}_0\frac{\mathbf{V}_{+}e^{-\gamma\ell} + \mathbf{V}_{-}e^{\gamma\ell}}{\mathbf{V}_{+}e^{-\gamma\ell} - \mathbf{V}_{-}e^{\gamma\ell}}$$

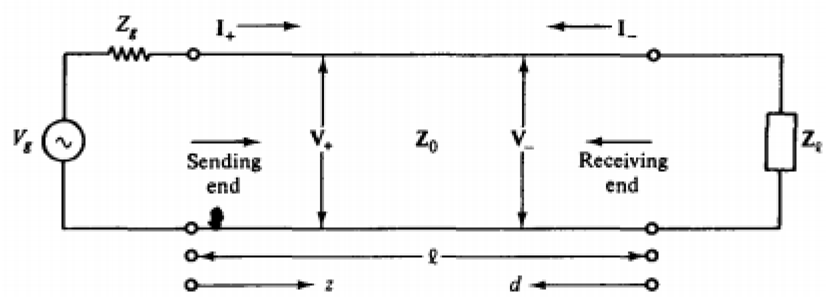


Fig. Transmission line terminated in a load impedance.

The reflection coefficient, which is designated by Γ (gamma), is defined as

$$\text{Reflection coefficient} \equiv \frac{\text{reflected voltage or current}}{\text{incident voltage or current}}$$

$$\Gamma \equiv \frac{\mathbf{V}_{ref}}{\mathbf{V}_{inc}} = \frac{-\mathbf{I}_{ref}}{\mathbf{I}_{inc}}$$

If Eq. is solved for the ratio of the reflected voltage at the receiving end which is $\mathbf{V}_{-}e^{\gamma\ell}$, to the incident voltage at the receiving end, which is $\mathbf{V}_{+}e^{\gamma\ell}$, the result is the reflection coefficient at the receiving end:

$$\Gamma_\ell = \frac{\mathbf{V}_{-}e^{\gamma\ell}}{\mathbf{V}_{+}e^{-\gamma\ell}} = \frac{\mathbf{Z}_\ell - \mathbf{Z}_0}{\mathbf{Z}_\ell + \mathbf{Z}_0}$$

$$\mathbf{T} \equiv \frac{\text{transmitted voltage or current}}{\text{incident voltage or current}} = \frac{\mathbf{V}_{tr}}{\mathbf{V}_{inc}} = \frac{\mathbf{I}_{tr}}{\mathbf{I}_{inc}}$$

$$\mathbf{T} = \frac{\mathbf{V}_{tr}}{\mathbf{V}_{+}} = \frac{2\mathbf{Z}_\ell}{\mathbf{Z}_\ell + \mathbf{Z}_0}$$

b) A transmission line has the propagation constant $\gamma = 0.1 + j10$ and characteristic impedance of $Z_0 = 50 + j5\ \Omega$. The line is terminated in an impedance of $100 - j30\ \Omega$. Find the impedance at a distance of 1.5 m from the load 7M

5(b) A Transmission line has the propagation constant $\gamma = 0.1 + j10$ and characteristic Impedance of $Z_0 = 50 + j5\,\Omega$. The line is terminated in an impedance of $100 - j30\,\Omega$. Find the impedance at a distance of 1.5 m from the Load.

Sol.. Given data

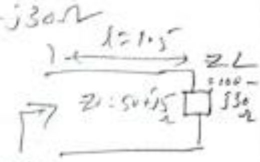
propagation constant $\gamma = 0.1 + j10 = \alpha + j\beta$

$\therefore \boxed{\beta = 10}$

characteristic Impedance $Z_0 = 50 + j5\,\Omega$

Load Impedance $Z_L = 100 - j30\,\Omega$

length of the line $l = 1.5$ m



$\therefore$ The expression for impedance at a distance l from the load end is

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

$\beta l = 10 \times 1.5 = 15$

$\tan\beta l = 0.27$

$$= 50+j5 \left[\frac{100 - j30 + j[50+j5]\,0.27}{[50+j5] + j[100-j30]\,0.27}\right]$$

$$= 50+j5 \left[\frac{100 - j30 + j13.5 - 1.35}{50 + j5 + j27 + 8.1}\right]$$

$$= 50+j5 \left[\frac{98.65 - j16.5}{58.1 + j32}\right]$$

$$= \frac{50.24\angle 5.71^\circ \cdot 99.97\angle -9.49^\circ}{66.32\angle 28.84}$$

$$= 75.73\angle -32.62^\circ$$

$$= 63.78 - j40.82\,\Omega$$

Unit-III

6 a) Derive the field equations for the electric and magnetic fields for TM_{mn} mode in rectangular wave guide? 7M

It has been previously assumed that the waves are propagating in the positive z direction in the waveguide.

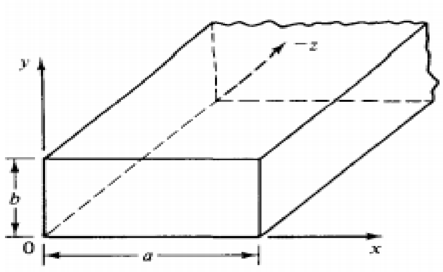


Fig. Co-ordinates of rectangular waveguides

The TM_{mn} modes in a rectangular guide are characterized by $H_z = 0$. In other words, the z component of an electric field E must exist in order to have energy transmission in the guide. Consequently, the Helmholtz equation for E in the rectangular coordinates is given by

$$\nabla^2 E_z = \gamma^2 E_z$$

A solution of the Helmholtz equation is in the form of

$$E_z = \left[A_m \sin\left(\frac{m\pi x}{a}\right) + B_m \cos\left(\frac{m\pi x}{a}\right)\right]\left[C_n \sin\left(\frac{n\pi y}{b}\right) + D_n \cos\left(\frac{n\pi y}{b}\right)\right]e^{-j\beta_g z}$$

which must be determined according to the given boundary conditions. The procedures for doing so are similar to those used in finding the TE-mode wave.

The boundary conditions on E_z require that the field vanishes at the waveguide walls, since the tangent component of the electric field E_z is zero on the conducting surface. This requirement is that for $E_z = 0$ at $x = 0$, a, then $B_m = 0$, and for $E_z = 0$ at $y = 0$, b, then $D_n = 0$. Thus the solution as shown in Eq. reduces to E

$$E_z = E_{0z} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z}$$

where $m = 1, 2, 3, \ldots$
$n = 1, 2, 3, \ldots$

If either $m = 0$ or $n = 0$, the field intensities all vanish. So there is no TM_{01} or TM_{10} mode in a rectangular waveguide, which means that TE_{10} is the dominant mode in a rectangular waveguide for $a > b$. For $H_z = 0$, the field equations, after expanding $\nabla \times \mathbf{H} = j\omega\epsilon \mathbf{E}$, are given by

$$\frac{\partial E_z}{\partial y} + j\beta_g E_y = -j\omega\mu H_x$$

$$j\beta_g E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0$$

$$\beta_g H_y = \omega\epsilon E_x$$

$$-\beta_g H_x = \omega\epsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z$$

These equations can be solved simultaneously for E_x, E_y, H_x, and H_y in terms of E_z.

The resultant field equations for TM modes are

$$E_x = \frac{-j\beta_g}{k_c^2}\frac{\partial E_z}{\partial x}$$

$$E_y = \frac{-j\beta_g}{k_c^2}\frac{\partial E_z}{\partial y}$$

$$E_z = \text{Eq. (4-1-61)}$$

$$H_x = \frac{j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial y}$$

$$H_y = \frac{-j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial x}$$

$$H_z = 0$$

where $\beta_g^2 - \omega^2\mu\epsilon = -k_c^2$ is replaced.

The TM_{mn} mode field equations in rectangular waveguides are

$$E_x = E_{0x} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z}$$

$$E_y = E_{0y} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z}$$

$$E_z = \text{Eq. (4-1-61)}$$

$$H_x = H_{0x} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z}$$

$$H_y = H_{0y} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z}$$

$$H_z = 0$$

Some of the TM-mode characteristic equations are identical to those of the TE modes, but some are different. For convenience, all are shown here:

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

$$\beta_g = \omega\sqrt{\mu\epsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1 - (f_c/f)^2}}$$

$$v_g = \frac{v_p}{\sqrt{1 - (f_c/f)^2}}$$

$$Z_g = \frac{\beta_g}{\omega\epsilon} = \eta\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

b) When TE_{10} mode is propagated through a standard rectangular wave guide, the guide wave length measured is 8cm and when TE_{11} mode is propagated, the guide wave length is increased to 12cm. If the operating frequency for both the modes is 6 GHz. Calculate "a" and "b" for the guide? 7M

6(b) When TE_{10} mode is propagated through a standard rectangular wave guide, the guide wave length measured is 8cm and when TE_{11} mode is propagated, the guide wavelength λ_g increased to 12 cm. If the operating frequency for both the modes is 6GHz. Calculate "a" and "b" for the guide?

Sol:- For TE_{10} mode: $\lambda_{g10} = 8\,cm = 8\times10^{-2}\,m$

For TE_{11} mode: $\lambda_{g11} = 12\,cm = 12\times10^{-2}\,m$

operating frequency $f = 6GHz = 6\times10^{9}\,Hz$, $\lambda_0 = \frac{3\times10^8}{6\times10^9} = c$

For standard waveguide, $a = 2b$ $\boxed{b = \frac{a}{2}}$

For dominant Mode TE_{10} mode: $\boxed{\lambda_{c10} = 2a}$

$$\lambda_{c11} = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}} = \frac{2a\left(\frac{a}{2}\right)}{\sqrt{(1)^2\left(\frac{a}{2}\right)^2+(1)^2a^2}}, \quad m=1, n=1$$

$$= \frac{a^2}{\sqrt{\frac{5a^2}{4}}} \qquad \therefore \lambda_{c11} = 0.894\,a$$

For TE_{10} mode $\lambda_{g10} = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_{c10}}\right)^2}} = 8 \rightarrow (1)$

For TE_{11} mode $\lambda_{g11} = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_{c11}}\right)^2}} = 12 \rightarrow (2)$

Dividing eq (1) by eq (2) $\frac{\sqrt{1-\left(\frac{\lambda_0}{\lambda_{c11}}\right)^2}}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_{c10}}\right)^2}} = \frac{8}{12}$

Hence substituting $\lambda_0 = \frac{c}{f} = \frac{3\times10^8}{6\times10^9} = 5\,cm$ $\qquad \lambda_0 = 5\,cm$

$$\therefore \frac{\sqrt{1-\left(\frac{5}{0.894a}\right)^2}}{\sqrt{1-\left(\frac{5}{2a}\right)^2}} = \frac{8}{12}$$

solving we get $a = 7.826\,cm$

Hence $b = \frac{a}{2} = \frac{7.826}{2}\,cm = 3.913\,cm$

(OR)

7 a) Derive electric and magnetic field components for TE_{mn} modes in rectangular waveguide. 7 M

It has been previously assumed that the waves are propagating in the positive z direction in the waveguide.

Below figure shows the coordinates of a rectangular waveguide**.**

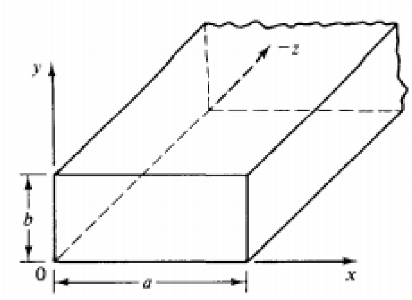


Fig.Co-ordinates of rectangular waveguides

The TE_{mn} modes in a rectangular guide are characterized by $E_z = 0$. In other words, the z component of the magnetic field, H_z, must exist in order to have energy transmission in the guide. Consequently, from a given Helmholtz equation,

$$\nabla^2 H_z = \gamma^2 H_z$$

a solution in the form of

$$H_z = \left[A_m \sin\left(\frac{m\pi x}{a}\right) + B_m \cos\left(\frac{m\pi x}{a}\right)\right] \times \left[C_n \sin\left(\frac{n\pi y}{b}\right) + D_n \cos\left(\frac{n\pi y}{b}\right)\right] e^{-j\beta_g z}$$

will be determined in accordance with the given boundary conditions, where $k_x = m\pi/a$ and $k_y = n\pi/b$ are replaced. For a lossless dielectric, Maxwell's curl equations in frequency domain are

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E}$$

In rectangular coordinates, their components are

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\epsilon E_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z$$

With the substitution $\partial/\partial z = -j\beta_g$ and $E_z = 0$, the foregoing equations are simplified to

$$\beta_g E_y = -\omega\mu H_x$$

$$\beta_g E_x = \omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_z}{\partial y} = -j\omega\mu H_z$$

$$\frac{\partial H_z}{\partial y} + j\beta_g H_y = j\omega\epsilon E_x$$

$$-j\beta_g H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = 0$$

Solving these six equations for E_x, E_y, H_x, and H_y in terms of H_z will give the TE-mode field equations in rectangular waveguides as

$$E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y}$$

$$E_y = \frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x}$$

$$E_z = 0$$

$$H_x = \frac{-j\beta_g}{k_c^2}\frac{\partial H_z}{\partial x}$$

$$H_y = \frac{-j\beta_g}{k_c^2}\frac{\partial H_z}{\partial y}$$

conditions are applied to the newly found field equations in such a manner that either the tangent **E** field or the normal **H** field vanishes at the surface of the conductor. Since $E_x = 0$, then $\partial H_z/\partial y = 0$ at $y = 0, b$. Hence $C_n = 0$. Since $E_y = 0$, then $\partial H_z/\partial x = 0$ at $x = 0, a$. Hence $A_m = 0$.

It is generally concluded that the normal derivative of H_z must vanish at the conducting surfaces—that is,

$$\frac{\partial H_z}{\partial n} = 0$$

at the guide walls. Therefore the magnetic field in the positive z direction is given by

$$H_z = H_{0z}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta_g z}$$

whereHoz is the amplitude constant.

TE_{mn} field equations in rectangular waveguides as

$$E_x = E_{0x}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta_g z}$$

$$E_y = E_{0y}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta_g z}$$

$$E_z = 0$$

$$H_x = H_{0x}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta_g z}$$

$$H_y = H_{0y}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta_g z}$$

$$H_z = H_{0z}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta_g z}$$

b) A TE_{10} mode is propagated through a wave guide with a = 10cm at a frequency 2.5 GHz, Find λ_c, V_p, V_g, λ_g, Z_g and β_g. 7 M

7(b) A TE10 mode is propagated through a wave guide with a = 10cm at a frequency 2.5 GHz, Find λ, v_p, v_g, λ_g, Z_g and β_g.

Sol:- Given data

Given Mode is TE_{10} - dominant mode

$m = 1, n = 0$

operating frequency $f = 2.5\,GHz = 2.5\times10^9\,Hz$

Broader dimension of W/G $a = 10cm = 10\times10^{-2}\,m$

(a) cut off wave length $= \lambda_c = 2a$ $\qquad \lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} = 2a$

$= 2\times10\times10^{-2} = 0.2\,m.$

(b) cut off frequency $f_c = \frac{c}{\lambda_c} = \frac{3\times10^8}{0.2} = 1.5\,GHz$

(c) $v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{3\times10^8}{\sqrt{1-\left(\frac{1.5}{2.5}\right)^2}} = 3.75\times10^8\,m/sec$

(d) $v_g = \frac{c^2}{v_p} = \frac{3\times10^8\times3\times10^8}{3.75\times10^8} = \frac{9}{3.75}\times10^8\,m/s = 2.4\times10^8\,m/s$

(d) $\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$ $\qquad \lambda_0 = \frac{3\times10^8}{2.5\times10^9} = \frac{3}{25} = 0.12\,m$

$= \frac{0.12}{\sqrt{1-\left(\frac{1.5}{2.5}\right)^2}} = 1.25\times0.12 = 0.15\,m$ $\qquad \frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = 1.25$

(e) $Z_g = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1-\left(\frac{1.5}{2.5}\right)^2}} = 377\times1.25 = 471.25\,\Omega$

(f) $\beta_g = \omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{\omega}{v_p} = \frac{2\pi\times2.5\times10^9}{3.75\times10^8} = 41.88\,rad/m$

Unit-IV

8 a) Derive the field equations for the electric and magnetic fields for TM_{mn} mode in Circular wave guide? 7M

The TM_{np} modes in a circular guide are characterized by $H_z = 0$. However, the z component of the electric field E_z must exist in order to have energy transmission in the guide. Consequently, the Helmholtz equation for E_z in a circular waveguide is given by

$$\nabla^2 E_z = \gamma^2 E_z$$

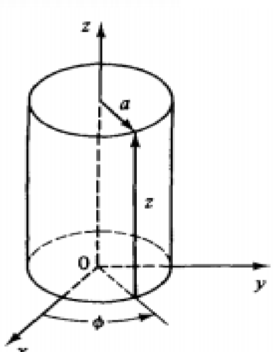


Fig.Co-ordinates of circular waveguide

Its solution is given by

$$E_z = E_{0z}J_n(k_c r)\cos(n\phi)e^{-j\beta_g z}$$

which is subject to the given boundary conditions.

The boundary condition requires that the tangential component of electric field E_z at $r = a$ vanishes. Consequently,

$$J_n(k_c a) = 0$$

Since $J_n(k_c r)$ are oscillatory functions, there are infinite numbers of roots of $J_n(k_c r)$.

For $H_z = 0$ and $\partial/\partial z = -j\beta_g$, the field equations in the circular guide, after expanding $\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$ and $\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E}$, are given by

$$E_r = \frac{-j\beta_g}{k_c^2}\frac{\partial E_z}{\partial r}$$

$$E_\phi = \frac{-j\beta_g}{k_c^2}\frac{1}{r}\frac{\partial E_z}{\partial \phi}$$

$$E_z = \text{Eq. (4-2-48)}$$

$$H_r = \frac{j\omega\epsilon}{k_c^2}\frac{1}{r}\frac{\partial E_z}{\partial \phi}$$

$$H_\phi = \frac{j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial r}$$

$$H_z = 0$$

where $k_c^2 = \omega^2\mu\epsilon - \beta_g^2$ has been replaced.

The field equations of TMnp modes in a circular waveguide:

$$E_r = E_{0r}J_n'\left(\frac{X_{np}r}{a}\right)\cos(n\phi)e^{-j\beta_g z}$$

$$E_\phi = E_{0\phi}J_n\left(\frac{X_{np}r}{a}\right)\sin(n\phi)e^{-j\beta_g z}$$

$$E_z = E_{0z}J_n\left(\frac{X_{np}r}{a}\right)\cos(n\phi)e^{-j\beta_g z}$$

$$H_r = \frac{E_{0\phi}}{Z_g}J_n\left(\frac{X_{np}r}{a}\right)\sin(n\phi)e^{-j\beta_g z}$$

$$H_\phi = \frac{E_{0r}}{Z_g}J_n'\left(\frac{X_{np}r}{a}\right)\cos(n\phi)e^{-j\beta_g z}$$

$$H_z = 0$$

where $Z_g = E_r/H_\phi = -E_\phi/H_r = \beta_g/(\omega\epsilon)$ and $k_c = X_{np}/a$ have been replaced and where $n = 0, 1, 2, 3, \ldots$ and $p = 1, 2, 3, 4, \ldots$.

b) Analyse the solutions of wave equations in cylindrical coordinates. 7M

Solutions of Wave Equations in Cylindrical Coordinates

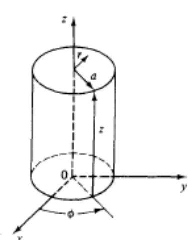


Figure 4-2-1 Cylindrical coordinates.

The scalar Helmholtz equation in cylindrical coordinates is given by

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\phi^2} + \frac{\partial^2\psi}{\partial z^2} = \gamma^2\psi \qquad (4\text{-}2\text{-}1)$$

Using the method of separation of variables, the solution is assumed in the form of

$$\Psi = R(r)\Phi(\phi)Z(z) \qquad (4\text{-}2\text{-}2)$$

where $R(r)$ = a function of the r coordinate only
$\Phi(\phi)$ = a function of the ϕ coordinate only
$Z(z)$ = a function of the z coordinate only

Substitution of Eq. (4-2-2) in (4-2-1) and division of the resultant by (4-2-2) yield

$$\frac{1}{rR}\frac{d}{dr}\left(r\frac{dR}{dr}\right) + \frac{1}{r^2\Phi}\frac{d^2\Phi}{d\phi^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} = \gamma^2 \qquad (4\text{-}2\text{-}3)$$

Since the sum of the three independent terms is a constant, each of the three terms must be a constant. The third term may be set equal to a constant γ_g^2:

$$\frac{d^2Z}{dz^2} = \gamma_g^2 Z \qquad (4\text{-}2\text{-}4)$$

The solutions of this equation are given by

$$Z = Ae^{-\gamma_g z} + Be^{\gamma_g z} \qquad (4\text{-}2\text{-}5)$$

where γ_g = propagation constant of the wave in the guide.

Inserting γ_g^2 for the third term in the left-hand side of Eq. (4-2-3) and multiplying the resultant by r^2 yield

$$\frac{r}{R}\frac{d}{dr}\left(r\frac{dR}{dr}\right) + \frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2} - (\gamma^2 - \gamma_g^2)\,r^2 = 0 \qquad (4\text{-}2\text{-}6)$$

The second term is a function of ϕ only; hence equating the second term to a constant ($-n^2$) yields

$$\frac{d^2\Phi}{d\phi^2} = -n^2\Phi \tag{4-2-7}$$

The solution of this equation is also a harmonic function:

$$\Phi = A_n \sin(n\phi) + B_n \cos(n\phi) \tag{4-2-8}$$

Replacing the Φ term by ($-n^2$) in Eq. (4-2-6) and multiplying through by R, we have

$$r\frac{d}{dr}\left(r\frac{dR}{dr}\right) + [(k_c r)^2 - n^2]R = 0 \tag{4-2-9}$$

This is Bessel's equation of order n in which

$$k_c^2 + \gamma^2 = \gamma_g^2 \tag{4-2-10}$$

This equation is called the *characteristic equation* of Bessel's equation. For a lossless guide, the characteristic equation reduces to

$$\beta_g = \pm\sqrt{\omega^2\mu\epsilon - k_c^2} \tag{4-2-11}$$

The solutions of Bessel's equation are

$$R = C_n J_n(k_c r) + D_n N_n(k_c r) \tag{4-2-12}$$

where $J_n(k_c r)$ is the nth-order Bessel function of the first kind, representing a standing wave of cos ($k_c r$) for $r < a$ as shown in Fig. 4-2-2. $N_n(k_c r)$ is the nth-order Bessel function of the second kind, representing a standing wave of sin ($k_c r$) for $r > a$ as shown in Fig. 4-2-3.

Therefore the total solution of the Helmholtz equation in cylindrical coordinates is given by

$$\Psi = [C_n J_n(k_c r) + D_n N_n(k_c r)][A_n \sin(n\phi) + B_n \cos(n\phi)]e^{\pm j\beta_g z} \tag{4-2-13}$$

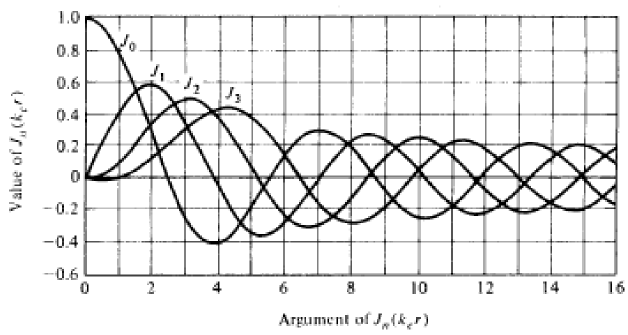


Figure 4-2-2 Bessel functions of the first kind.

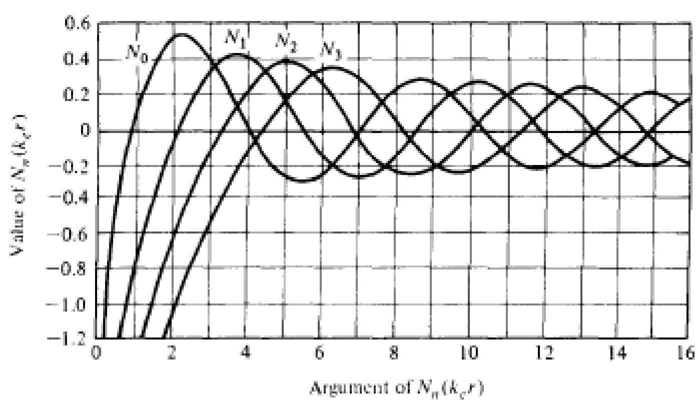


Figure 4-2-3 Bessel functions of the second kind.

At $r = 0$, however, $k_c r = 0$; then the function N_n approaches infinity, so $D_n = 0$. This means that at $r = 0$ on the z axis, the field must be finite. Also, by use of trigonometric manipulations, the two sinusoidal terms become

$$A_n \sin(n\phi) + B_n \cos(n\phi) = \sqrt{A_n^2 + B_n^2}\cos\left[n\phi + \tan^{-1}\left(\frac{A_n}{B_n}\right)\right]$$
$$= F_n \cos(n\phi) \tag{4-2-14}$$

Finally, the solution of the Helmholtz equation is reduced to

$$\Psi = \Psi_0 J_n(k_c r)\cos(n\phi)e^{-j\beta_g z} \tag{4-2-15}$$

(OR)

9 a) List out the various characteristics of standard circular waveguides 7M

The inner diameter of a circular waveguide is regulated by the frequency of the signal being transmitted. For example: at X-band frequencies from 8 to 12 GHz, the inner diameter of a circular waveguide designated as EIA WC(94) by the Electronic Industry Association is 2.383 cm (0.938 in.). Table 4-2-8 tabulates the characteristics of the standard circular waveguides.

EIA[a] designation WC[b]( )	Inside diameter in cm (in.)	Cutoff frequency for air-filled waveguide in GHz	Recommended frequency range for TE_{11} mode in GHz
992	25.184 (9.915)	0.698	0.80–1.10
847	21.514 (8.470)	0.817	0.94–1.29
724	18.377 (7.235)	0.957	1.10–1.51
618	15.700 (6.181)	1.120	1.29–1.76
528	13.411 (5.280)	1.311	1.51–2.07
451	11.458 (4.511)	1.534	1.76–2.42
385	9.787 (3.853)	1.796	2.07–2.83
329	8.362 (3.292)	2.102	2.42–3.31
281	7.142 (2.812)	2.461	2.83–3.88
240	6.104 (2.403)	2.880	3.31–4.54
205	5.199 (2.047)	3.381	3.89–5.33
175	4.445 (1.750)	3.955	4.54–6.23
150	3.810 (1.500)	4.614	5.30–7.27
128	3.254 (1.281)	5.402	6.21–8.51
109	2.779 (1.094)	6.326	7.27–9.97
94	2.383 (0.938)	7.377	8.49–11.60
80	2.024 (0.797)	8.685	9.97–13.70
69	1.748 (0.688)	10.057	11.60–15.90
59	1.509 (0.594)	11.649	13.40–18.40
50	1.270 (0.500)	13.842	15.90–21.80
44	1.113 (0.438)	15.794	18.20–24.90
38	0.953 (0.375)	18.446	21.20–29.10
33	0.833 (0.328)	21.103	24.30–33.20
28	0.714 (0.281)	24.620	28.30–38.80
25	0.635 (0.250)	27.683	31.80–43.60
22	0.556 (0.219)	31.617	36.40–49.80
19	0.478 (0.188)	36.776	42.40–58.10
17	0.437 (0.172)	40.227	46.30–63.50
14	0.358 (0.141)	49.103	56.60–77.50
13	0.318 (0.125)	55.280	63.50–87.20
11	0.277 (0.109)	63.462	72.70–99.70
9	0.239 (0.094)	73.552	84.80–116.00

b) Find the related expressions for not propagation of TEM waves in hallow waveguides. 7M

The transverse electric and transverse magnetic (TEM) modes or transmission-line modes are characterized by

$$E_z = H_z = 0$$

This means that the electric and magnetic fields are completely transverse to the direction of wave propagation. This mode cannot exist in hollow waveguides, since it requires two conductors, such as the coaxial transmission line and two-open-wire line. Analysis of the TEM mode illustrates an excellent analogous relationship between the method of circuit theory and that of the field theory.

Maxwell's curl equations in cylindrical coordinates

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E}$$

become

$$B_g E_r = \omega\mu H_\phi$$

$$B_g E_\phi = \omega\mu H_r$$

$$\frac{\partial}{\partial r}(rE_\phi) - \frac{\partial Er}{\partial \phi} = 0$$

$$\beta_g H_r = -\omega\epsilon E_\phi$$

$$\beta_g H_\phi = \omega\epsilon E_r$$

$$\frac{\partial}{\partial r}(rH_\phi) - \frac{\partial H_r}{\partial \phi} = 0$$

where $\partial/\partial r = -j\beta_g$ and $E_z = H_z = 0$ are replaced.

The propagation constant of the TEM mode in a coaxial line:

$$\beta_g = \omega\sqrt{\mu\epsilon}$$

which is the phase constant of the wave in a lossless transmission line with a dielectric.

In comparing the preceding equation with the characteristic equation of the Helmholtz equation in cylindrical coordinates as given in

$$\beta_g = \sqrt{\omega^2\mu\epsilon - k_c^2}$$

it is evident that

$$k_c = 0$$

This means that the cutoff frequency of the TEM mode in a coaxial line is zero, which is the same as in ordinary transmission lines.

The phase velocity of the TEM mode can be expressed from Eq as

$$v_p = \frac{\omega}{\beta_g} = \frac{1}{\sqrt{\mu\epsilon}}$$

which is the velocity of light in an unbounded dielectric.

The wave impedance of the TEM mode is found from either Eqs.

$$\eta\,(\text{TEM}) = \sqrt{\frac{\mu}{\epsilon}}$$

which is the wave impedance of a lossless transmission line in a dielectric.

Ampère's law states that the line integral of **H** about any closed path is exactl equal to the current enclosed by that path. This is

$$\oint \mathbf{H} \cdot d\ell = I = I_0 e^{-j\beta_g z} = 2\pi r H_\phi$$

II B.Tech Regular DEGREE EXAMINATION
Department of ECE
Name of the Subject: EMWTL
Subject Code: 20EC403
Date of Exam: 03-08-2023

Signature of the External