Scheme- August, 2023 - Six Semester

III/IV B.Tech (Regular) DEGREE EXAMINATION

Linear Control Systems

Maximum: 70 Marks

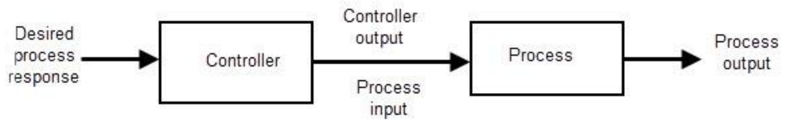
Answer question 1 compulsory. **(14X1 = 14Marks)**
Answer one question from each unit. **(4X14=56 Marks)**

			CO	BL	M
1	a)	What is time-varying control system? A system whose output response depends on moment of observation as well as moment of input signal application.	CO1	L1	1
	b)	State Mason's Gain formula. $T = \frac{C(s)}{R(s)} = \frac{\Sigma_{i=1}^{N} P_i \Delta_i}{\Delta}$	CO1	L2	1
	c)	What is the effect of positive feedback on stability? Positive feedback will increase in gain and make a stable system to become unstable.	CO1	L1	1
	d)	What does the term 'type' of a system indicate? What is its significance? Type number represents the number of poles of the system at the origin of the s plane.	CO2	L1	1
	e)	Define Peak over shoot. The deviation of the response at peak time from the final value of response.	CO2	L1	1
	f)	List only the dominant poles of the system with characteristic equation $s^3 + 6s^2 + 10s + 8 = 0$ A pole which is more near to origin than other poles in the system (1)	CO2	L4	1
	g)	Define BIBO stability. A system is called BIBO stable (or bounded-input, bounded-output stable) system, if and only if every bounded input to the system produces a bounded output.	CO3	L1	1
	h)	Define Gain margin. The gain margin is the difference between 0 dB and the gain at the phase cross-over frequency that gives a phase of −180°.	CO3	L1	1
	i)	What is Nyquist contour? Give its limits. A stability test for time-invariant linear systems can also be derived in the frequency domain. It is known as the Nyquist stability criterion. It is based on the complex analysis result known as Cauchy's principle of argument.	CO4	L2	1
	j)	What are the corner frequencies of the system, $G(s)H(s) = \frac{10}{S(S+1)(S+5)}$ W_1=1, w_2=0.2	CO4	L2	1
	k)	If a system has 2 open loop poles and 4 open loop zeros, then the root locus will have how many branches. Two branches	CO1	L2	1
	l)	Define 'state vector' of a system. state vector is a mathematical representation of the system's current state	CO1	L1	1
	m)	How to find stability from state model? Numerically, we can determine the stability of a state space model by finding the eigenvalues of the state space A matrix. If all of the eigenvalues are negative, then the system is stable	CO4	L2	1
	n)	Define Observability of a system. Observability is a measure of how well internal states of a system can be inferred from knowledge of its external outputs.	CO2	L1	1

Unit-I

2 a) Explain open loop control system with an example. CO1 L1 7M

In an open loop system, there is no feedback to the controller about the current state of the system. An example of an open loop control would be to run the heater for 10 minutes every hour, no matter how hot or cold the air temperature is.



In an open loop system, the output can be adjusted / varied by varying the input but the output has no effect on the input. The output of the open loop system can be determined only by its present state input. If the output is affected due to some external noise / disturbance, the open loop system cannot correct it. Also, there is no chance to correct the transition errors in open loop systems.

Applications:

Washing Machine
Electric Bulb
Electric Hand Drier
Time based Bread Toaster
Automatic Water Faucet
TV Remote Control
Electric Clothes Drier
Shades or Blinds on a window
Stepper Motor or Servo Motor
Inkjet Printers
Door Lock System
Traffic Control System
Traffic Control System

Most automated Traffic Control Systems are time-based open loop control systems i.e., each signal is allotted with a specific time slot during which it operates irrespective of the amount of traffic.

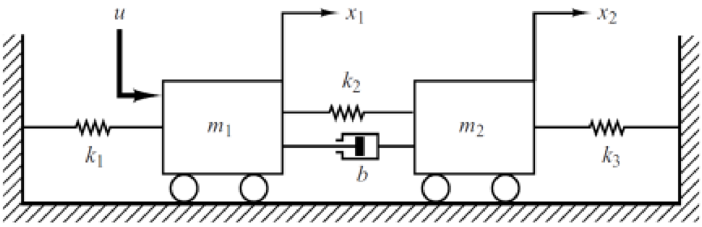
Advantages of Open Loop Control System:

The main advantages of the open loop control system are listed below:
Open Loop Control Systems are very simple and easy to design.
These are considerably cheaper than other types of control systems.
Maintenance of an open loop control system is very simple.
Generally, open loop systems are stable up to some extent.
These types of systems are easy to construct and are convenient to use.

Disadvantages of Open Loop control System:

The disadvantages of open loop system are:
The bandwidth of open loop control system is less.
The non-feedback system does not facilitate the process of automation.
Open loop systems are inaccurate in nature and unreliable.
If their output is affected by some external disturbances, there is no way to correct them automatically as these are non-feedback systems.

b) Find the transfer function $\frac{X_1(S)}{U(S)}$ for the given mechanical system shown. CO1 L4 7M



At node x1

$U(t)= m_1d^2x_1/dt^2+k_1x_1+k_2(x_1-x_2)+b(dx_1/dt-dx_2/dt)$ -----(1)

At node x2

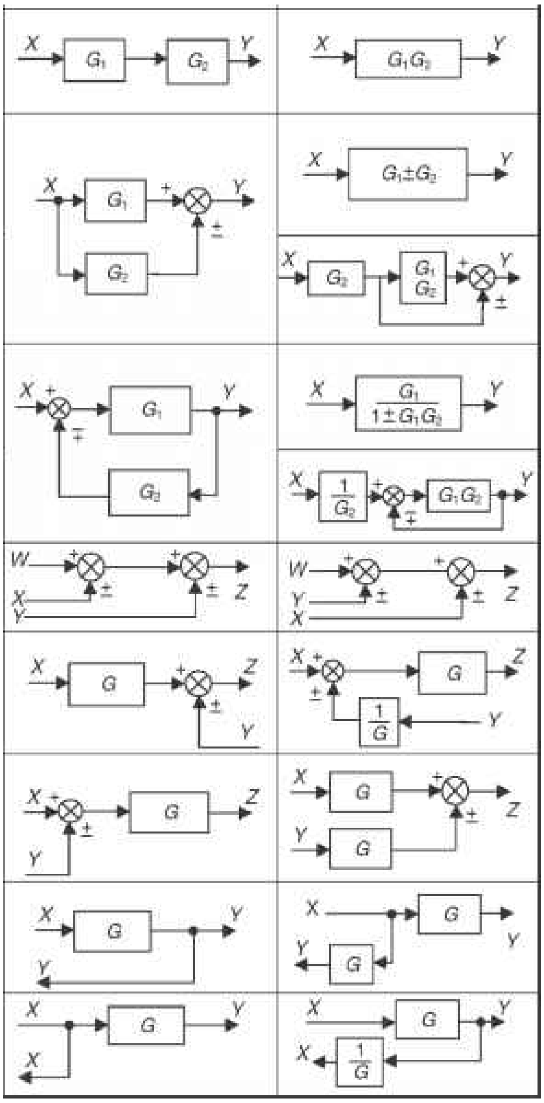
$0=m_2d^2x/dt^2+k_3x_2+K_2(x_2-x_1)+b(d^2x_2/dt^2-d^2x_1/dt^2)$ ------- (2)

Sub 2 equation of x2 in 1 equation

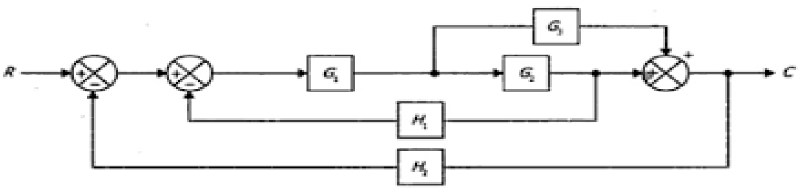
$X_1(s)/U(s)=m_2s^2+k_2+k_3+bs/(m_1s^2+k_1+k_2+b_1)(m_2s^2+k_3+k_2+bs)-(k_2+bs)^2$

(OR)

3 a) Explain the block diagram reduction techniques. CO1 L1 7M



b) Determine the transfer function of the block diagram show below using Signal flow graph approach. CO1 L4 7M



Step1: move the takeoff point before G_2 to after G_2 then G_3 is parallel with unity gain $(1+G_3/G_2)$.
Step2: G_1 and G_2 are cascaded--→G_1G_2
Step3: G_1G_2 feedback with H_1--→ $G_1G_2/1+G_1G_2H_1$.
Step4: cascaded blocks-→ $(1+G_3/G_2)(G_1G_2/1+G_1G_2H_1)=x$
Step5: Above block is feedback with H_2 → $x/1+xH_2$

Unit-II

4	a)	Determine an expression for the transient response of a second order system subjected to unit step input.	CO2	L3	7M

The standard form of closed loop transfer function of second order system is,

$$\frac{C(s)}{R(s)}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

For underdamped system, $0<\zeta<1$ and roots of the denominator (characteristic equation) are complex conjugate.

The roots of the denominator are, $s=-\zeta\omega_n\pm\omega_n\sqrt{\zeta^2-1}$

Since $\zeta<1$, ζ^2 is also less then 1, and so $1-\zeta^2$ is always positive.

$$\therefore\ s=-\zeta\omega_n\pm\omega_n\sqrt{(-1)(1-\zeta^2)}=-\zeta\omega_n\pm j\omega_n\sqrt{1-\zeta^2}$$

The damped frequency of oscillation, $\omega_d=\omega_n\sqrt{1-\zeta^2}$

$$\therefore s=-\zeta\omega_n\pm j\omega_d$$

The response in s-domain, $C(s)=R(s)\,\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

For unit step input, $r(t)=1$ and $R(s)=1/s$.

$$\therefore C(s)=\frac{\omega_n^2}{s\,(s^2+2\zeta\omega_n s+\omega_n^2)}$$

By partial fraction expansion, $C(s)=\dfrac{\omega_n^2}{s\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}=\dfrac{A}{s}+\dfrac{Bs+C}{s^2+2\zeta\omega_n s+\omega_n^2}$

$$\therefore\ C(s)=\frac{1}{s}-\frac{s+2\zeta\omega_n}{s^2+2\zeta\omega_n s+\omega_n^2+\zeta^2\omega_n^2-\zeta^2\omega_n^2}=\frac{1}{s}-\frac{s+2\zeta\omega_n}{(s^2+2\zeta\omega_n s+\zeta^2\omega_n^2)+(\omega_n^2-\zeta^2\omega_n^2)}$$

$$=\frac{1}{s}-\frac{s+2\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)}=\frac{1}{s}-\frac{s+2\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}\qquad\boxed{\omega_d=\omega_n\sqrt{1-\zeta^2}}$$

$$=\frac{1}{s}-\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}-\frac{\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}\qquad .....(2.27)$$

Let us multiply and divide by ω_d in the third term of the equation (2.27).

$$\therefore\ C(s)=\frac{1}{s}-\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}-\frac{\zeta\omega_n}{\omega_d}\,\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}$$

$\mathcal{L}\{1\}=\frac{1}{s}$
$\mathcal{L}\{e^{-at}\sin\omega t\}=\frac{\omega}{(s+a)^2+\omega^2}$
$\mathcal{L}\{e^{-at}\cos\omega t\}=\frac{s+a}{(s+a)^2+\omega^2}$

The response in time domain is given by,

$$c(t)=\mathcal{L}^{-1}\{C(s)\}=\mathcal{L}^{-1}\left\{\frac{1}{s}-\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}-\frac{\zeta\omega_n}{\omega_d}\,\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}\right\}$$

$$=1-e^{-\zeta\omega_n t}\cos\omega_d t-\frac{\zeta\omega_n}{\omega_d}e^{-\zeta\omega_n t}\sin\omega_d t=1-e^{-\zeta\omega_n t}\left(\cos\omega_d t+\frac{\zeta\omega_n}{\omega_n\sqrt{1-\zeta^2}}\sin\omega_d t\right)\qquad\boxed{\omega_d=\omega_n\sqrt{1-\zeta^2}}$$

$$=1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\sqrt{1-\zeta^2}\ \cos\omega_d t+\zeta\ \sin\omega_d t\right)=1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\sin\omega_d t\times\zeta+\cos\omega_d t\times\sqrt{1-\zeta^2}\right)$$

b) What are the limitations of Routh-Hurwitz Criterion? CO2 L1 7M

- This criterion is applicable only for a linear system.
- It does not provide the exact location of poles on the right and left half of the S plane.
- In case of the characteristic equation, it is valid only for real coefficients.

(OR)

5 a) Derive expressions for time domain specifications Peak time and Rise time. CO2 L4 7M

Rise time (t_r)

The unit step response of second order system for underdamped

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t_r}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \theta)$$

$$\therefore c(t_r) = 1 - \frac{e^{-\zeta\omega_n t_r}}{\sqrt{1-\zeta^2}}\sin(\omega_d t_r + \theta) = 1$$

$$\therefore \frac{-e^{-\zeta\omega_n t_r}}{\sqrt{1-\zeta^2}}\sin(\omega_d t_r + \theta) = 0$$

Since $-e^{-\zeta\omega_n t_r} \neq 0$, the term, $\sin(\omega_d t_r + \theta) = 0$

When, $\phi = 0, \pi, 2\pi, 3\pi \ldots$, $\quad \sin\phi = 0$

$$\therefore \omega_d t_r + \theta = \pi$$

$$\omega_d t_r = \pi - \theta$$

$$\boxed{\therefore \text{ Rise Time, } t_r = \frac{\pi - \theta}{\omega_d}}$$

Peak time (t_p)

To find the expression for peak time, t_p, differentiate c(t) with respect to t

$$\text{i.e., } \frac{d}{dt}c(t)\Big|_{t=t_p} = 0$$

The unit step response of under damped second order system is given by,

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \theta)$$

Differentiating c(t) with respect to **t**.

$$\frac{d}{dt}c(t) = \frac{-e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}(-\zeta\omega_n)\sin(\omega_d t + \theta) + \left(\frac{-e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\right)\cos(\omega_d t + \theta)\omega_d$$

Put, $\omega_d = \omega_n\sqrt{1-\zeta^2}$

$$\therefore \frac{d}{dt}c(t) = \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}(\zeta\omega_n)\sin(\omega_d t + \theta) - \frac{\omega_n\sqrt{1-\zeta^2}}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\cos(\omega_d t + \theta)$$

$$= \frac{\omega_n e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left[\zeta\sin(\omega_d t + \theta) - \sqrt{1-\zeta^2}\cos(\omega_d t + \theta)\right]$$

at $t = t_p$, $\frac{d}{dt}c(t) = 0$

$$\therefore \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t_p} \sin(\omega_d t_p) = 0$$

Since, $e^{-\zeta\omega_n t_p} \neq 0$, the term, $\sin(\omega_d t_p) = 0$

When $\phi = 0, \pi, 2\pi, 3\pi$, $\sin\phi = 0$

$\therefore \omega_d t_p = \pi$

$$\boxed{\therefore \text{Peak time, } t_p = \frac{\pi}{\omega_d}}$$

The damped frequency of oscillation, $\omega_d = \omega_n\sqrt{1-\zeta^2}$

$$\boxed{\therefore \text{Peak time, } t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}}$$

b)	Determine range of values of K for the system to be stable, whose characteristic equation is given by $S^4 + 20KS^3 + 5S^2 + 10S + 15 = 0$.	CO2	L3	7M

Calculation:

$s^4 + 20Ks^3 + 5s^2 + 10s + 15 = 0$

By applying the Routh tabulation method,

s^4	1	5	15
s^3	20K	10	
s^2	$\frac{100K-10}{20K}$	15	
s^1	$\frac{(1000K-100-6000K^2)}{100K-10}$		
s^0	15		

For the system to become stable, the sign changes in the first column of the Routh table must be zero.

$$\frac{(1000K-100-6000K^2)}{100K-10} > 0$$

$$K = \frac{1000 \pm j1183.21}{12000}$$

As K should always be a real quantity,

So the system will remain unstable at every value of K

Unit-III

6	a)	A unit step response test conducted on a second order system yielded peak overshoot M_p=0.12 and peak time t_p=0.2 sec. Obtain the corresponding frequency response specifications.	CO3	L4	7M

$$\boxed{\text{Resonant peak, } M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}}$$

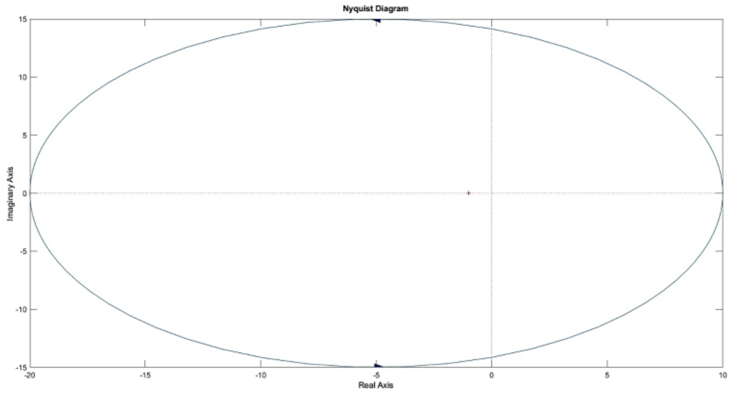
$$\boxed{\text{The resonant frequency, } \omega_r = \omega_n\sqrt{1-2\zeta^2}}$$

$$\therefore \boxed{\text{Bandwidth, } \omega_b = \omega_n u_b = \omega_n\left[1-2\zeta^2+\sqrt{2-4\zeta^2+4\zeta^4}\right]^{\frac{1}{2}}}$$

M_r=1.079 w_r=10.88 rad/sec, BW=21.36 rad/sec

b) Sketch the Nyquist plot for the system $G(s) = \frac{10(S+4)}{S(S-2)}$ and comment on stability. CO3 L4 7M

A pole is at the origin so there are 4 sections in the Nyquist plot
Section C1 is simple polar plot C2 section is a semicircle and C3 is inverse polar plot and C4 is



(OR)

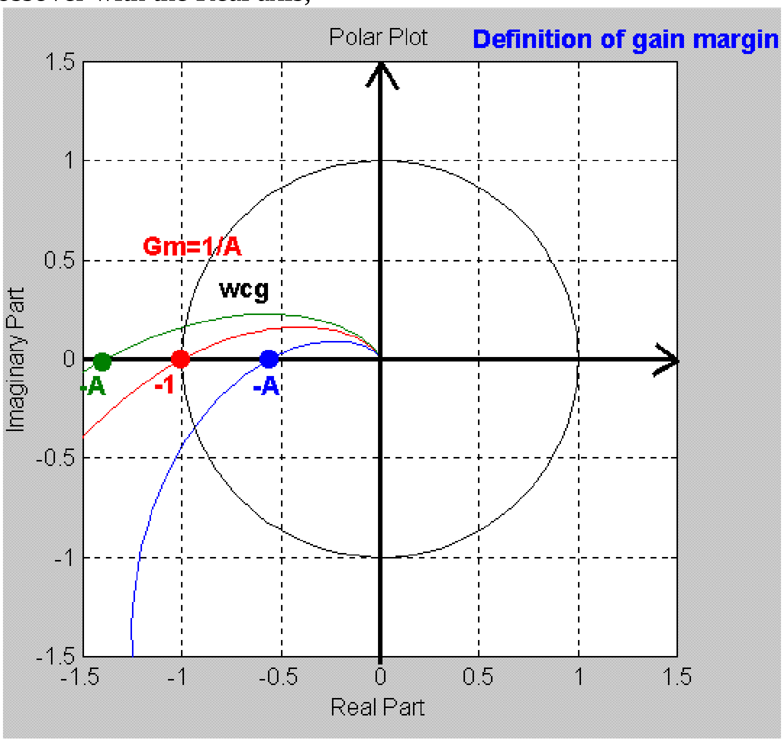
7 a) How do we calculate the gain margin and phase margin from the polar plot. Explain. CO3 L2 7M

<u>Gain Margin from Polar Plot:</u>
Let the crossover frequency be defined as ωcg, the frequency at which the phase plot crosses over the −180∘ line. On the polar plot, this corresponds to the plot crossing the negative part of Real axis.

Gm>1 system stable

Gm<1 system unstable

On the polar plot, Gain Margin Gm can be found as an inverse of the coordinate A of the polar plot crossover with the Real axis,

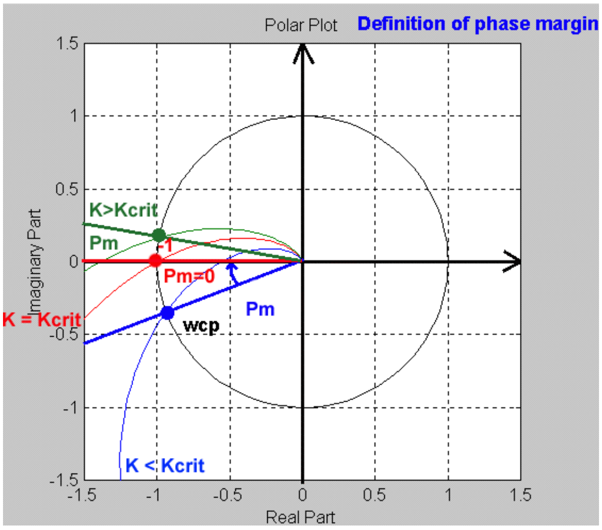


$G_m = 1/|A|$

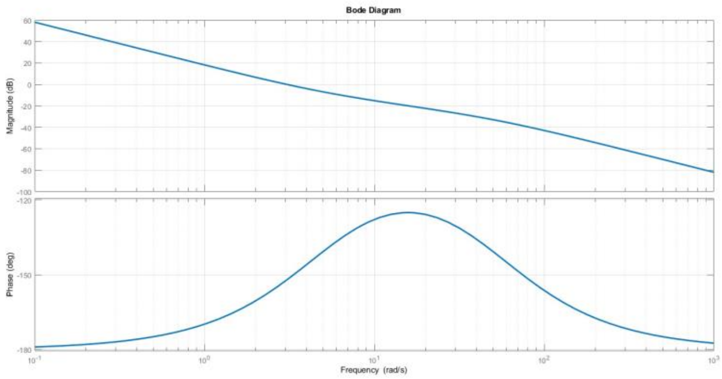
|A|<1, Gm>1, and the system is stable. If the crossover is to the left of (-1, j0) point

|A|>1,G m <1 , and the system is unstable.

<u>Phase Margin from Polar Plot:</u>
The crossover frequency defined as ωcp, is the frequency at which the polar plot crosses over the unit circle. Phase Margin Φm is defined as Φm=180∘+∠GH(ωcp). Therefore, on the polar plot Phase Margin Φm can be found as the angle between the Real axis and the crossover of the polar plot with the unit circle. If this angle is above the Real axis, the system is unstable, if this angle is below the Real axis, the system is stable.

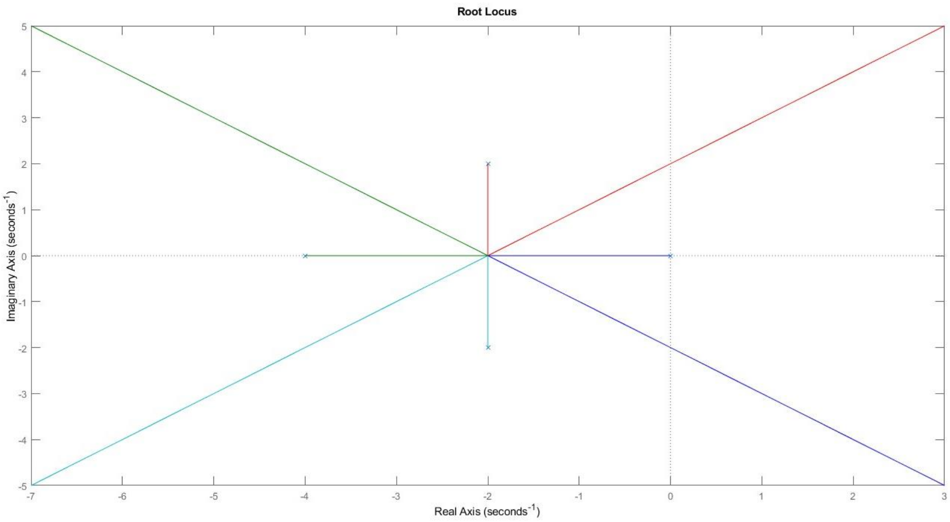


b) Sketch the bode plot for the following system $G(s) = \frac{80(S+5)}{S^2\,(S+50)}$ CO3 L3 7M



Unit-IV

8 a) Sketch the root locus for the system having OLTF, $GH(S) = \frac{K}{S(S+4)(S^2+4S+8)}$ CO4 L3 7M



b) Derive the transfer functions corresponding to the following state models. CO4 L4 7M

$\dot{X} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ And $y = \begin{bmatrix} 1 & 0 \end{bmatrix} X$

$[SI-A]^{-1}$= S 1

-2 S+3 /(s2+3S+2)

$$\boxed{T(s) = \frac{Y(s)}{U(s)} = C\,[sI - A]^{-1}\,B + D}$$

TF=1/(s^2+3S+2)

(OR)

9	a)	Explain the procedural steps to draw the root locus sketch. Rule 1 − Locate the open loop poles and zeros in the‘s’ plane. Rule 2 − Find the number of root locus branches. Rule 3 − Identify and draw the real axis root locus branches. Rule 4 − Find the centroid and the angle of asymptotes. Rule 5 − Find the intersection points of root locus branches with an imaginary axis. Rule 6 − Find Breakaway and Break-in points. Rule 7 − Find the angle of departure and the angle of arrival.	CO4	L2	7M
	b)	Determine the controllability and Observability for the following system.	CO4	L3	7M

$\dot{X} = \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ And $y = [1 \quad 0]X$

Controllability

$$Q_c = [B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B]$$

AB=1 -3

Det=-1 completely controllable

Observability

$$Q_o = [C^T : A^T C^T : \dots : (A^T)^{n-1} C^T]$$

Det=1 Completely observable.