SYLLABUS

The Relational Algebra and Relational Calculus:

Unary Relational Operations: SELECT and PROJECT - Relational Algebra Operations from Set Theory - Binary Relational Operations: JOIN and DIVISION – Additional Relational Operations - The Tuple Relational Calculus - The Domain Relational Calculus

SQL-99:

Schema Definition, Constraints, Queries, and Views: SQL Data Definition and Data Types -

Specifying Constraints in SQL - Schema Change Statements in SQL - Basic Queries in SQL - More Complex SQL Queries - INSERT, DELETE, and UPDATE Statements in SQL - Views (Virtual Tables) in SQL

The Relational Algebra and Relational Calculus

Relational Algebra:

- The basic set of operations for the relational model is the relational algebra.
- A sequence of relational algebra operations forms a relational algebra expression, whose result will also be a relation that represents the result of a database query (or retrieval request).
- The relational algebra is very important for several reasons.
	- 1. It provides a formal foundation for relational model operations.
	- 2. It is used as a basis for implementing and optimizing queries in the query processing and optimization modules that are integral parts of relational database management systems (RDBMSs).
	- 3. Third, some of its concepts are incorporated into the SQL standard query language for RDBMSs.
- The relational algebra operations can be divided into two groups.
	- \triangleright One group includes set operations from mathematical set theory. Set operations include UNION, INTERSECTION, SET DIFFERENCE, and CARTESIAN PRODUCT (also known as CROSS PRODUCT).
	- \triangleright The other group consists of operations developed specifically for relational databases—these include SELECT, PROJECT, and JOIN, among others

Tables:

DEPARTMENT

WORKS ON Essn

123456789

123456789

666884444

453453453

453453453

333445555

333445555

333445555

333445555

999887777

999887777

987987987

987987987

987654321

987654321

888665555

 Pno

 $\overline{2}$

 $\overline{3}$

 $\overline{2}$

 $\overline{2}$

 10

 20

30

 10

 10

30

30

 $20[°]$

 20

Hours

32.5

40.0

20.0

 20.0

10.0

10.0

 10.0

10.0

30.0

 10.0

35.0

5.0

 20.0

15.0

NULL

 7.5

DEPENDENT

Stafford 5 **Bellaire** 5

5

Sugarland Houston

DEPT_LOCATIONS Dnumber Dlocation Houston

1.1 Unary Relational Operations: SELECT and PROJECT

The SELECT and PROJECT operator are **unary**; that is, it is applied to a **single relation (table)**.

1.1.1 SELECT Operation:

- The SELECT operation is used to **retrieve a subset of the tuples from a relation that satisfies a selection condition**.
- **Syntax:**

σ **< selection condition>** (R)

Where the symbol **σ (sigma)** is used to denote the SELECT operator,

selection condition is a Boolean expression (condition) specified on the attributes of relation R.

For example, to select the EMPLOYEE tuples whose department is 4, SELECT operation as follows:

σ Dno= 4 (EMPLOYEE)

- The SELECT operation can also be visualized as a horizontal partition of the relation into two sets of tuples:
	- \triangleright Tuples that satisfy the condition and are selected.
	- \triangleright Tuples that do not satisfy the condition and are discarded.
	- The **<selection condition>** is applied **independently to each individual tuple t in R**.
- The Boolean expression specified in **<selection condition**> is made up of a number of clauses of the form.

<attribute name> <comparison op> <constant value>

Or

<attribute name> <comparison op> <attribute name>

Where **<attribute name>** is the name of an attribute of R

<comparison op> is normally one of the operators $\{=, \leq, \leq, \geq, \neq\}$

For **ordered values** (numeric or date), the comparison operators are $\{=, \leq, \leq, \geq, \neq\}$

For **unordered values** (names, colors), the comparison operators are $\{=\neq\}$

<constant value> is a constant value from the attribute domain.

Example:

σ (Dno= 4 AND Salary> 25000) OR (Dno= 5 AND Salary> 30000) (EMPLOYEE)

Fname	Minit	Lname	<u>Ssn</u>	B date	Address	Sex	Salary	Super_ssn	Dno
Franklin		Wong	333445555		1955-12-08 638 Voss, Houston, TX	M		40000 888665555	
Jennifer		Wallace	987654321		1941-06-20 291 Berry, Bellaire, TX			43000 888665555	
Ramesh		Naravan			666884444 1962-09-15 975 Fire Oak, Humble, TX	M		38000 333445555	

 The number of tuples in the resulting relation is always less than or equal to the number of tuples in R. That is, $|\sigma_c(R)| \leq |R|$ for any condition C.

The **degree of the relation** is the **number of attributes in a relation**.

Notice that the SELECT operation is **commutative**; that is,

 σ < cond1> $(\sigma$ < cond2> (R)) = σ < cond2> $(\sigma$ < cond1> (R))

 we can always combine a **cascade** (or **sequence**) of SELECT operations into a single SELECT operation with a conjunctive (AND) condition; that is,

 σ < cond1> (σ < cond2> (...(σ < condn> (R)) ...)) = σ < cond1> AND< cond2> AND...AND < condn> (R)

In SQL, the SELECT condition is typically specified in the WHERE clause of a query.

For example, the following operation:

σ Dno= 4 AND Salary> ²⁵⁰⁰⁰ (EMPLOYEE)

would correspond to the following SQL query:

SELECT * FROM EMPLOYEE

WHERE Dno=4 AND Salary>25000;

1.1.2 PROJECT Operation:

- The PROJECT operation is used to **retrieve only certain attributes (columns) in a relation (table).**
- The PROJECT operation is display the attributes **in the same order as they appear in the list.**
- **Syntax:**

 π < attribute list> (R)

Where π (**pi**) is the symbol used to represent the PROJECT operation,

<attribute list> is the desired sub-list of attributes from the relation R.

 For example, to list each employee's first and last name and salary, we can use the PROJECT operation as follows:

π Lname, Fname, Salary(EMPLOYEE)

- The PROJECT operation can be **visualized as a vertical partition** of the relation into two relations:
	- \triangleright One has the needed columns (attributes) and contains the result of the operation.
		- \triangleright The other contains the discarded columns.
- Degree is equal to the number of attributes in \leq attribute list \geq .
- If the attribute list includes only non-key attributes of R, duplicate tuples are likely to occur.
- The PROJECT operation removes any duplicate tuples from relation and retrieve a valid set of distinct tuples is known as **duplicate elimination**.
- For example, consider the following PROJECT operation:

 π Sex, Salary(EMPLOYEE)

Notice that the tuple <'F', 25000> appears only once in above Figure, even though this combination of values appears twice in the EMPLOYEE relation.

 π < list1> (π < list2> (R)) = π < list1> (R)

- As long as \langle list2 $>$ contains the attributes in \langle list1 $>$; otherwise, the left-hand side is an incorrect expression.
- It is also noteworthy that **commutativity does not hold** on PROJECT.
- In SQL, the PROJECT attribute list is specified in the SELECT clause of a query.
- For example, the following operation:

 π Sex, Salary(EMPLOYEE)

would correspond to the following SQL query:

SELECT DISTINCT Sex, Salary

FROM EMPLOYEE

Write a relational algebra expression in single line known as an **in-line expression**.

Example: $π$ Fname, Lname, Salary($σ$ Dno= 5 (EMPLOYEE))

1.1.3 RENAME Operation

- To rename the attributes in a relation, we simply list the new attribute names in parentheses.
- Rename operation is denoted with the symbol ρ (rho)
- Example:

TEMP \leftarrow σ _{Dno= 5} (EMPLOYEE)

R (First_name, Last_name, Salary) $\leftarrow \pi$ Frame, Lname, Salary(TEMP)

TEMP

- If no renaming is applied, the names of the attributes in the resulting relation of a SELECT operation are the same as those in the original relation and in the same order.
- We can rename either the **relation name** or the **attribute names**, or **both**—as a unary operator.
- The general RENAME operation when applied to a relation R of degree n is denoted by any of the following three forms:
	- $\rho_S(R)$ or $\rho_{(B 1, B 2, ..., B n)}(R)$ or $\rho_{S (B 1, B 2, ..., B n)}(R)$
- Where the symbol ρ (rho) is used to denote the RENAME operator, S is the new relation name, and B1, B2, ..., Bn are the new attribute names.
- The first expression renames the relation only, the second renames the attributes only, and the third renames both the relation and its attributes.
- Renaming in SQL is accomplished by aliasing using AS, as in the following example:

SELECT E.Fname AS First_name, E.Lname AS Last_name, E.Salary AS Salary

FROM EMPLOYEE AS E

WHERE E.Dno=5;

1.2 Relational Algebra Operations from Set Theory

1.2.1 The UNION, INTERSECTION, and MINUS Operations

- Set theoretic operations are used to merge the elements of two sets in various ways, including UNION, INTERSECTION, and SET DIFFERENCE (also called MINUS or EXCEPT).
- These are binary operations; that is, each is applied to two relations.
- The two relations on which any of these three operations are applied must follow two rules.
	- Two relations must have **the same number of attributes**.
	- \checkmark Two relations must have **union compatibility** or **type compatibility** (i.e., same type of tuples).
- Two relations R(A1 , A2 , ..., An) and S(B1 , B2 , ..., Bn) are said to be union compatible (or type compatible) if they have the same degree n and if $dom(Ai) = dom(Bi)$.
- We can define the three operations **UNION**, **INTERSECTION**, and **SET DIFFERENCE** on two union-compatible relations R and S as follows:
	- **UNION:** The result of this operation, **denoted by R** ∪ **S**, is a relation that includes **all tuples that are either in R or in S or in both R and S**. **Duplicate tuples are eliminated**.
	- **INTERSECTION:** The result of this operation, **denoted by R ∩ S**, is a relation that includes **all tuples that are in both R and S**.
	- **SET DIFFERENCE (or MINUS):** The result of this operation, **denoted by R – S**, is a relation that includes **all tuples that are in R but not in S**.

Figure: The set operations UNION, INTERSECTION, and MINUS. (a) Two union-compatible relations. (b) STUDENT ∪ INSTRUCTOR. (c) STUDENT ∩ INSTRUCTOR. (d) STUDENT − INSTRUCTOR. (e) INSTRUCTOR − STUDENT

 (a)

 (c)

INSTRUCTOR Lname Fname John Smith Ricardo **Browne** Susan Yao Francis Johnson Ramesh Shah

Fn

Johnny

Barbara

Amy Jimmy

Ernest

Ln Kohler

Jones

Ford

Wang

Gilbert

 (b)

Notice that both UNION and INTERSECTION are **commutative operations**.

 (d)

 $R \cup S = S \cup R$ and $R \cap S = S \cap R$

Ln

Yao

Shah

- The MINUS operation is **not commutative**.
	- $R S \neq S R$

Fn

Susan

Ramesh

- Note that INTERSECTION can be expressed in terms of union and set difference as follows: $R \cap S = ((R \cup S) - (R - S)) - (S - R)$
- There are multiset operations (UNION ALL, INTERSECT ALL, and EXCEPT ALL) that **do not eliminate duplicates.**

1.2.2 The CARTESIAN PRODUCT (CROSS PRODUCT) Operation:

- CROSS PRODUCT or CROSS JOIN—which is **denoted by ×.**
- This is also a **binary set operation**, but the relations on which it is **applied do not have to be union compatible or type compatibility**.
- The CARTESIAN PRODUCT creates tuples with the combined attributes of two relations.
- CROSS PRODUCT operation produces a new element by **combining every member (tuple) from one relation (set) with every member (tuple) from the other relation (set).**
- The result of R(A1, A2, ..., An) \times S(B1, B2, ..., Bm) is a relation Q with degree n + m attributes Q(A1 , A2 , ..., An , B1 , B2 , ..., Bm), in that order.

Example:

Salgrade

Dept X Salgrade

1.3 Binary Relational Operations: JOIN and DIVISION

6.3.1 The JOIN Operation

- The JOIN operation, denoted by \boxtimes , is used to combine related tuples from two relations into single "longer" tuples.
- There are several join operations: **THETA JOIN, EQUIJOIN, NATURAL JOIN**

THETA JOIN:

- A JOIN operation with general join condition is called a **THETA JOIN**.
- Syntax:

 $R \bowtie_{\text{cjoin condition}} S$ or $R \bowtie_{\theta} S$

- Where R and S are two relations R $(A1, A2, ..., An)$ and $S(B1, B2, ..., Bm)$.
- where each <condition> is of the form $A_i \theta B_i$, Ai is an attribute of R, B_i is an attribute of S, A_i and B_j have the same domain, and θ (theta) is one of the comparison operators $\{\equiv, \leq, \leq, \geq, \neq\}$.
- Example: to retrieve the employee names and their grades

 π Ename, Grade(Emp \bowtie emp.sal>=salgrade.losal AND emp.sal>=salgrade.hisal Salgrade Salgrade)

 Tuples whose join attributes are NULL or for which the join condition is FALSE do not appear in the result.

EQUIJOIN:

- A JOIN, where the only comparison operator used is =, is called an EQUIJOIN.
- EQUIJOIN we **always have one or more pairs of attributes that have identical values in every tuple**.
- Example: π Ename, Deptno, Dname(Emp \bowtie emp.deptno=dept.deptno Dept)

NATURAL JOIN:

- NATURAL JOIN denoted by *****.
- A NATURAL JOIN requires that the two join **attributes** (or **each pair of join attributes**) **have the same name** in both relations.
- NATURAL JOIN was created to get rid of the second (superfluous) attribute in an EQUIJOIN condition.
- **Example:** π Ename, Deptno, Dname(Emp \bowtie emp.deptno=dept.deptno Dept)
- If this is not the case, a renaming operation is applied first. Example: e1 EMP e2 EMP mgr ϵ e2.Empno π e1.Ename, e2.Ename (e1 \bowtie e1.mgr=mgr e2)
- **Join attribute:** an **attribute is used to join the both relations** is called join attribute.
- In JOIN, only combinations of tuples satisfying the join condition appear in the result, whereas in the CARTESIAN PRODUCT all combinations of tuples are included in the result.
- The expected size of the join result divided by the maximum size $n_R * n_S$ leads to a ratio called **join** selectivity, which is a property of each join condition.

1.3.2 The DIVISION Operation

- The DIVISION operation, denoted by \div
- The DIVISION include the tuples appear in R in combination with every tuple in S.
- Note that in the formulation of the DIVISION operation, the tuples in the denominator relation S restrict the numerator relation R by selecting those tuples in the result that match all values present in the denominator.
- The DIVISION operation is applied to two relations $R(Z) \div S(X)$, where the attributes of R are a subset of the attributes of S.
- **Example** is Retrieve the names of employees who work on all the projects that 'John Smith' works on.

 $\text{SMITH} \leftarrow \sigma_{\text{Frame}^= \text{ 'John' AND Lname}^= \text{ 'Smith' (EMPLOYEE)} }$ SMITH_PNOS ← π_{Pno} (WORKS_ON \bowtie _{Essn= Ssn}SMITH) $SSN_PNOS \leftarrow \pi_{Essn,\;Pno}(WORKS_ON)$

 Apply the DIVISION operation to the two relations, which gives the desired employees' Social Security numbers:

Figure 1.8 The DIVISION operation. (a) Dividing SSN_PNOS by SMITH_PNOS. (b) $T \leftarrow R \div S$

The DIVISION operation can be expressed as a sequence of π , \times , and – operations as follows:

$$
T1 \leftarrow \pi Y \text{ (R)}T2 \leftarrow \pi Y \text{ ((S × T1) - R)}T \leftarrow T1 - T2
$$

1.3.3 Notation for Query Trees

- **Query tree** is a tree data structure that corresponds to a relational algebra expression.
- It represents the input relations of the query as leaf nodes of the tree, and rep-resents the relational algebra operations as internal nodes.
- An execution of the query tree consists of executing an internal node operation whenever its operands (represented by its child nodes) are available, and then replacing that internal node by the relation that results from executing the operation. The execution terminates when the root node is executed and produces the result relation for the query.
- Example:

For every project located in 'Stafford', list the project number, the controlling department number, and the department manager's last name, address, and birth date.

 π Pnumber, Dnum, Lname, Address, Bdate $(($

 $(\sigma$ Plocation= 'Stafford'(PROJECT)) \bowtie Dnum= Dnumber (DEPARTMENT)) \bowtie $_{Mgr_ssn= Ssn}$ (EMPLOYEE))

1.4 Additional Relational Operations

1.4.1 Generalized Projection

- The generalized projection operation extends the projection operation by allowing functions of attributes to be included in the projection list.
- Syntax:

 $\pi_{F_{1}, F_{2}, ..., F_{n}}(R)$

- Where F1, F2, ..., Fn are functions over the attributes in relation R and may involve arithmetic operations and constant values.
- As an example, consider the relation EMPLOYEE (Ssn, Salary, Deduction, Years_service)
- A report may be required to show Net Salary = Salary – Deduction, Bonus $= 2000 * Years_service, and$
	- Tax = $0.25 *$ Salary.

Then a generalized projection combined with renaming may be used as follows:

 $REPORT \leftarrow \rho_{(Sm, Net_salary, Bonus, Tax)}$ ($\pi_{Sm, Salary - Deduction, 2000}$ * Years_service, 0.25 * Salary($EMPLOYEE$)).

1.4.2 Aggregate Functions and Grouping

- Aggregate functions are **SUM, AVERAGE, MAXIMUM, MINIMUM, and COUNT**.
- **SUM:** function is used to find sum of tuples or values.
- **AVERAGE:** function is used to find average of tuples or values.
- **MAXIMUM:** function is used to find maximum value from the tuples or values.
- **MINIMUM:** function is used to find minimum value from the tuples or values.
- **COUNT:** function is used for counting tuples or values.
- **GROUP:** grouping the tuples in a relation by the value of some of their attributes and then applying an aggregate function independently to each group.
- We can define an AGGREGATE FUNCTION operation, using the symbol **3** (pronounced **script F**)
- Syntax:

<grouping attributes> ℑ **<function list> (R)**

- Where **<grouping attributes>** is a list of attributes of the relation specified in R, **<function list>** is a list of (<function> <attribute>) pairs.
- **Example:** List Count of employees in each department and their average salary.

(Dno ℑ COUNT Ssn, AVERAGE Salary (EMPLOYEE))

- If **no renaming is applied**, then the attribute names be the **concatenation of the function name with the attribute name in the form <function>_<attribute>**.
- If **rename is applied** then corresponding rename attributes are displayed.

 ρ R (Dno, No_of_employees, Average_sal) (Dno \Im COUNT Ssn, AVERAGE Salary (EMPLOYEE))

- If **no grouping attributes are specified**, the functions are applied to all the tuples in the relation, so the resulting relation has a **single tuple only**.
- Example:

Note: duplicates are not eliminated when an aggregate function is applied.

1.4.3 OUTER JOIN Operations

- **Outer joins** were developed for the case where the user wants to **keep all the tuples in R, or all those in S, or all those in both relations** in the result of the JOIN, regardless of whether or not they have matching tuples in the other relation.
- This satisfies the need of queries in which tuples from two tables are to be combined by matching corresponding rows, but without losing any tuples for lack of matching values.
- There are three type of outer joins are there
	- **1. LEFT OUTER JOIN** The LEFT OUTER JOIN operation keeps every tuple in the *first*, or *left*, relation R in $R \bowtie S$.
	- **2. RIGHT OUTER JOIN** The RIGHT OUTER JOIN operation keeps every tuple in the *second*, or *right*, relation S in $R \bowtie S$.
	- **3. FULL OUTER JOIN** The FULL OUTER JOIN operation keeps both tuple in the *left and right*, relations in $R \mathbb{M}S$.
- **If no matching tuple** is found in relations then **filled or padded with NULL values**.
- **For example**, list of all employee names as well as the name of the departments they manage.
- If employee manage a department then department name will be result; if they do not manage placed with NULL value. We can apply an operation LEFT OUTER JOIN

TEMP \leftarrow (EMPLOYEE \bowtie _{Ssn= Mgr_ssn}DEPARTMENT)

 $RESULT \leftarrow \pi$ Fname, Minit, Lname, Dname(TEMP)

1.4.4 The OUTER UNION Operation

- The OUTER UNION operation was developed to take the **union of tuples** from two relations that have some common attributes, but are **not union (type) compatible**.
- This operation will take the UNION of tuples in two relations R(X, Y) and S(X, Z) that are **partially compatible**, meaning that only some of their attributes, say X, are union compatible.
- The attributes that are union compatible are represented only once in the result, and those attributes that are not union compatible from either relation are also kept in the result relation $T(X, Y, Z)$.
- It is therefore the same as a **FULL OUTER JOIN** on the common attributes.

1.5 Examples of Queries in Relational Algebra

Query 1. Retrieve the name and address of all employees who work for the 'Research' department.

Ans: RESEARCH_DEPT ← σ_{Dname= 'Research'(DEPARTMENT)}

RESEARCH_EMPS ← (RESEARCH_DEPT $\bowtie_{\text{Dnumber}= \text{Dno}}$ EMPLOYEE)

 $\text{RESULT} \leftarrow \pi_{\text{Frame}, \text{ Lname}, \text{ Address}}(\text{ RESEARCH_EMPS})$

As a single in-line expression, this query becomes:

 $π$ Fname, Lname, Address ($σ$ Dname= 'Research' (DEPARTMENT \bowtie Dnumber= Dno (EMPLOYEE))

Query 2. For every project located in 'Stafford', list the project number, the controlling department number, and the department manager's last name, address, and birth date.

Ans: $STAFORD PROJS \leftarrow \sigma_{Plocation= 'Stafford'}(PROJECT)$

CONTR_DEPTS ← (STAFFORD PROJS \bowtie $_{Dnum= Dnumber}$ DEPARTMENT) ${\tt PROJ_DEPT_MGRS} \leftarrow (\textrm{CONTR_DEPTS} \boxtimes_{\textrm{Mgr_SSn}} \textrm{Ssn} \textrm{EMPLOYEE})$ $RESULT \leftarrow \pi_{Pnumber, Dnum, Lname, Address, Bdate}$ (PROJ_DEPT_MGRS)

Query 3. Find the names of employees who work on all the projects controlled by department number 5.

Ans: DEPT5 PROJS \leftarrow $\rho_{(Pno)}$ (π Pnumber(σ Dnum= 5 (PROJECT))) $EMP_PROJ \leftarrow \rho_{(Ssn, Pno)} (\pi_{Essn, Pno}(WORKS_ON))$ RESULT_EMP_SSNS ← EMP_PROJ ÷ DEPT5_PROJS $RESULT \leftarrow \pi_{Lname, France}$ RESULT_EMP_SSNS * EMPLOYEE)

Query 4. Make a list of project numbers for projects that involve an employee whose last name is 'Smith', either as a worker or as a manager of the department that controls the project.

Ans: SMITHS(Essn) $\leftarrow \pi_{\text{Ssn}}$ (σ_{Lname} 'Smith'(EMPLoyEE)) SMITH_WORKER_PROJS $\leftarrow \pi_{\text{Pno}}$ WORKS_ON * SMITHS) $\text{MGRS} \gets \pi_{\text{Lname, Dnumber}}(\text{ EMPLOYEE}\otimes_{\text{Ssn= Mgr_ssn}}\text{DEPARTMENT})$ SMITH_MANAGED_DEPTS(Dnum) $\leftarrow \pi_{D\text{number}}$ (σ Lname= 'Smith'(MGRS)) SMITH_MGR_PROJS(Pno) $\leftarrow \pi_{Pnumber}$ (SMITH_MANAGED_DEPTS * PROJECT) RESULT ← (SMITH_WORKER_PROJS ∪ SMITH_MGR_PROJS)

 π Pno (WORKS_ON M_{Essen} = Ssn(π Ssn (σ Lname= 'Smith'(EMPLOYEE))) U

 π Pno $((\pi$ Dnumber $(\sigma$ Lname= 'Smith' $(\pi$ Lname, Dnumber(EMPLOYEE))) \bowtie Ssn= mgr_ssn DEPARTMENT)) \bowtie Dnumber= DnumPROJECT)

Query 5. List the names of all employees with two or more dependents.

Ans: $T1(Ssn, No of dependents) \leftarrow_{Essn} \mathfrak{I}$ COUNT Dependent_name(DEPENDENT) $T2 \leftarrow \sigma_{No_of_depends \ge 2} (T1)$ $RESULT \leftarrow \pi_{Lname, France}$ (T2 * EMPLOYEE)

Query 6. Retrieve the names of employees who have no dependents.

This is an example of the type of query that uses the MINUS (SET DIFFERENCE) operation.

Ans: $ALL_EMPS \leftarrow \pi_{Ssn}(\text{EMPLoyEE})$

 $EMPS_WITH_DEPS(Ssn) \leftarrow \pi_{Essn}$ (DEPENDENT) $EMPS$ _WITHOUT_DEPS \leftarrow (ALL_EMPS – EMPS_WITH_DEPS) $\text{RESULT} \leftarrow \pi_{\text{Lname, Frame}}(\text{ EMPS_WITHOUT_DEPS * EMPLOYEE})$

As a single in-line expression, this query becomes:

 $π$ Lname, Fname(($π$ Ssn(EMPLOYEE) – $ρ$ Ssn($π$ Essn(DEPENDENT))) * EMPLOYEE)

Query 7. List the names of managers who have at least one dependent.

Ans: MGRS(Ssn) $\leftarrow \pi_{Mgr_ssn}$ (DEPARTMENT) EMPS WITH DEPS(Ssn) $\leftarrow \pi_{\text{Essn}}($ DEPENDENT) MGRS_WITH_DEPS ← (MGRS ∩ EMPS_WITH_DEPS) $\text{RESULT} \leftarrow \pi_{\text{Lname, Frame}}(\text{MGRS_WITH_DEPS * EMPLOYE})$

Relational Calculus:

- The relational calculus is a formal language, based on the branch of **mathematical logic called predicate calculus**.
- The relational calculus provides a *higher-level declarative language* for specifying relational queries.
- In a relational calculus expression, there is *no order of operations* to specify how to retrieve the query result—only *what information the result should contain*.
- This is the main distinguishing feature between relational algebra and relational calculus.
- There are two variations of relational calculus**-tuple relational calculus** and **domain relational calculus**.

1.6 The Tuple Relational Calculus

- The relational calculus is considered to be a **nonprocedural language** i.e., **no need to write a sequence of operations to specify a retrieval request in a particular order** of applying the operations.
- A calculus expression specifies **what is to be retrieved rather than how to retrieve it**.
- Relational calculus is important for two reasons-
	- \triangleright First, it has a firm basis in mathematical logic.
		- \triangleright Second, the standard query language (SQL) for RDBMSs has some of its foundations in the tuple relational calculus.

1.6.1 Tuple Variables and Range Relations

- The tuple relational calculus is based on specifying a **number of tuple variables**.
- **Each tuple variable usually ranges over a particular database relation**, meaning that the variable may take as its value any individual tuple from that relation.
- Syntax:

{t | COND(t)}

- Where **t** is a tuple variable and **COND(t**) is a conditional (Boolean) expression involving t that evaluates to either TRUE or FALSE for different assignments of tuples to the variable t.
- For example, to find all employees whose salary is above \$50,000, we can write the following tuple calculus expression:

{t | EMPLOYEE(t) AND t .Salary>50000}

We can also retrieve only some of the attributes(say the first and last names) as

{t.Fname, t.Lname | EMPLOYEE(t) AND t .Salary>50000}

- we need to specify the following information in a tuple relational calculus expression:
	- For each tuple variable t, the **range relation** R of t means R(t). **If we do not specify a range relation,** then the variable t will **range over all possible tuples "in the universe"** as it is not restricted to any one relation.
	- \triangleright A condition to select particular combinations of tuples from range relations, the condition is evaluated for every possible combination of tuples to identify the **selected combinations** for which the condition evaluates to TRUE.
	- A **set of attributes** to be retrieved, the **requested attributes**. The values of these attributes are retrieved for each selected combination of tuples.

1.6.2 Expressions and Formulas in Tuple Relational Calculus

A general **expression of the tuple relational calculus** is of the form

 $\{t_1. A_j, t_2. A_k, ..., t_n. A_m | COND(t1, t2, ..., t n, t n +1, t n +2, ..., t n+m)\}\$

- where t_1 , t_2 , ..., t_n , t_{n+1} , ..., t_{n+m} are tuple variables, each A_i is an attribute of the relation on which t_i ranges, and **COND is a condition or formula**.
- A formula is **made up of predicate calculus atoms**, which can be one of the following:
	- 1. An atom of the form $R(t_i)$, where R is a relation name and t_i is a tuple variable.
	- 2. An atom of the form t_i . A op t_j . B. Where **op** is one of the comparison operators in the set $\{=\, \leq, \leq, \geq, \neq\}$, t_i and t_i are tuple variables,

A is an attribute of the relation on which t_i ranges, and

B is an attribute of the relation on which t_i ranges.

- 3. An atom of the form ti. A op c or c op tj. B, where c is a constant value.
- A formula (Boolean condition) is made up of one or more atoms connected via the logical operators AND, OR, and NOT and is defined recursively by Rules 1 and 2 as follows:
	- \triangleright Rule 1: Every atom is a formula.
	- \triangleright Rule 2: If F1 and F2 are formulas, then so are (F1 AND F 2), (F1 OR F 2), NOT (F1).
		- a. (F1 AND F 2) is TRUE if both F1 and F2 are TRUE; otherwise, it is FALSE.
			- b. (F1 OR F 2) is FALSE if both F1 and F2 are FALSE; otherwise, it is TRUE.
			- c. NOT (F1) is TRUE if F1 is FALSE; it is FALSE if F1 is TRUE.
			- d. NOT (F2) is TRUE if F2 is FALSE; it is FALSE if F2 is TRUE.

1.6.3 The Existential and Universal Quantifiers

- **Quantifiers** are two types, they are the **universal quantifier (**∀**)** and the **existential quantifier (**∃**)**.
- We need to define the concepts of **free** and **bound tuple variables** in a formula.
- Informally, a tuple variable t is **bound if it is quantified**, meaning that it appears in an **(**∃**t)** or **(**∀**t)** clause; **otherwise, it is free.**
- we define a tuple variable in a formula as **free** or **bound** according to the following rules:
	- An occurrence of a tuple variable in a formula F that is **an atom is free in F**.
	- \triangleright An occurrence of a tuple variable t is free or bound in a formula made up of logical connectives (F1 **AND** F 2), (F1 **OR** F 2), **NOT**(F1), and **NOT**(F2) -- depending on whether it is free or bound in F1 or F2 (if it occurs in either).
- **Truth values** for formulas with quantifiers are described below-
	- The formula (∃t)(F) is TRUE if the formula **F evaluates to TRUE for some (at least one) tuple assigned to free occurrences of t in F**; otherwise, (∃t)(F) is FALSE.
	- \triangleright The formula ($\forall t$)(F) is TRUE if the formula F evaluates to TRUE for every tuple (in the universe) assigned to free occurrences of t in F; otherwise, $(\forall t)(F)$ is FALSE.

1.6.4 Sample Queries in Tuple Relational Calculus

Query 1. List the name and address of all employees who work for the 'Research' department.

Ans: { t.Fname, t.Lname, t.Address | EMPLOYEE(t) AND (∃d)(DEPARTMENT(d) AND

d .Dname='Research' AND d .Dnumber= t.Dno)}

Query 2. For every project located in 'Stafford', list the project number, the controlling department number, and the department manager's last name, birth date, and address.

Ans: { p.Pnumber, p.Dnum, m.Lname, m.Bdate, m.Address | PROJECT(p) AND EMPLOYEE(m) AND

p .Plocation='Stafford' AND ((∃d)(DEPARTMENT(d) AND p .Dnum= d.Dnumber AND

d .Mgr_ssn= $m(Ssn)$ }

- **Query 3**. List the name of each employee who works on **some** project controlled by department number 5. This is a variation of Q3 in which all is changed to **some**. In this case we need two join conditions and two existential quantifiers.
- **Ans:** { e. Lname, e. Fname | EMPLOYEE(e) AND ((∃x)(∃w)(PROJECT(x) AND WORKS_ON(w) AND x.Dnum=5 AND w .Essn= e.Ssn AND x.Pnumber= w.Pno))}

Query 4. Make a list of project numbers for projects that involve an employee whose last name is 'Smith', either as a worker or as manager of the controlling department for the project.

{ p.Pnumber | PROJECT(p) AND (((∃e)(∃w)(EMPLOYEE(e)

AND WORKS_ON(w) AND w .Pno= p.Pnumber

AND e.Lname='Smith' AND e.Ssn= w.Essn))

OR ((∃m)(∃d)(EMPLOYEE(m) AND DEPARTMENT(d) AND p .Dnum= d.Dnumber AND d .Mgr_ssn= m.Ssn AND m .Lname='Smith')))}

1.6.5 Notation for Query Graphs

- Graphical representation of a query is called a query graph.
- F Relations in the query are represented by **relation nodes**, which are displayed as **single circles**.
- Constant values, typically from the query selection conditions, are represented by **constant nodes**, which are displayed as **double circles or ovals**.
- **Selection and join conditions are represented by the graph edges** (the lines that connect the nodes), as shown in below figure.
- The attributes to be retrieved from each relation are displayed in square brackets above each relation.
- Example: For every project located in 'Stafford', list the project number, the controlling department number, and the department manager's last name, birth date, and address.
- **Ans:** { p.Pnumber, p.Dnum, m.Lname, m.Bdate, m.Address | PROJECT(p) AND EMPLOYEE(m) AND p .Plocation='Stafford' AND ((∃d)(DEPARTMENT(d) AND p .Dnum= d.Dnumber AND d . $Mgr_ssn= m.Ssn))$

1.6.6 Transforming the Universal and Existential Quantifiers

- It is possible to transform a universal quantifier into an existential quantifier, and vice versa, to get an equivalent expression.
- One general transformation can be described informally as follows:
- Transform one type of quantifier into the other with negation (preceded by NOT); AND and OR replace one another; a negated formula becomes unnegated; and an unnegated formula becomes negated.
- Some special cases of this transformation can be stated as follows, where the ≡ symbol stands for equivalent to:

$$
(\forall x) (P(x)) \equiv NOT (\exists x) (NOT (P(x)))
$$

$$
(\exists x) (P(x)) \equiv NOT (\forall x) (NOT (P(x)))
$$

 $(\forall x)$ $(P(x)$ AND Q $(x)) \equiv NOT$ ($\exists x)$ $(NOT$ ($P(x)$) OR NOT ($Q(x)$))

 $(\forall x) (P(x) \text{ OR } Q(x)) \equiv \text{NOT} (\exists x) (\text{NOT} (P(x)) \text{ AND NOT} (Q(x)))$

 $(\exists x) (P(x)) \text{ OR } Q(x)) \equiv \text{NOT} (\forall x) (\text{NOT} (P(x)) \text{ AND NOT} (Q(x)))$

$$
(\exists x) (P(x) AND Q(x)) \equiv NOT (\forall x) (NOT (P(x)) OR NOT (Q(x)))
$$

Notice also that the following is TRUE, where the \Rightarrow symbol stands for implies:

 $(\forall x)(P(x)) \Rightarrow (\exists x)(P(x))$

NOT ($\exists x)(P(x)) \Rightarrow NOT (\forall x)(P(x))$

1.6.7 Using the Universal Quantifier in Queries

Query 3. List the names of employees who work on all the projects controlled by department number 5. One way to specify this query is to use the universal quantifier as shown:

Ans: { e. Lname, e. Fname | EMPLOYEE(e) AND ((∀x)(NOT(PROJECT(x)) OR NOT

 $(x.$ Dnum=5) OR $((\exists w)(WORKS_ON(w)AND w .Essn=e.Ssn AND x .Pnumber=w. Pno))))$

Query 6. List the names of employees who have no dependents.

Ans: { e.Fname, e.Lname | EMPLOYEE(e) AND (NOT (∃d)(DEPENDENT(d) AND e .Ssn= d.Essn))} Using the general transformation rule, we can rephrase Q6 as follows:

Ans: { e.Fname, e.Lname | EMPLOYEE(e) AND ((∀d)(NOT(DEPENDENT(d)) OR NOT(e.Ssn= $d.Essn))$ }

Query 7. List the names of managers who have at least one dependent.

Ans: { e.Fname, e.Lname | EMPLOYEE(e) AND ((∃d)(∃ρ) (DEPARTMENT(d) AND DEPENDENT(ρ) AND e .Ssn= d.Mgr_ssn AND ρ.Essn= e.Ssn))}

This query is handled by interpreting managers who have at least one dependent as managers for whom there exists some dependent.

1.6.8 Safe Expressions

- A **safe expression** in relational calculus is one that is guaranteed to yield a finite number of tuples as its result; otherwise, the expression is called **unsafe**.
- For example, the expression
	- ${t | NOT (EMPLOYEE(t))}$
- Above expression is **unsafe** because it yields **all tuples in the universe that are not EMPLOYEE tuples**, which are infinitely numerous.