UNIT – II

STEADY STATE ANALYSIS OF A.C CIRCUITS

Introduction

The resistance, inductance and capacitance are three basic elements of any electrical network. In order to analyze any electric circuit, it is necessary to understand the following three cases,

1) A.C. through pure resistive circuit.
2) A.C. through pure inductive circuit.
3) A.C. through pure capacitive circuit.

In each case, it is assumed that a purely sinusoidal alternating voltage given by the equation $v = V_m \sin(\omega t)$ is applied to the circuit. The equation for the current, power and phase shift are developed in each case.

A.C. Through Pure Resistance

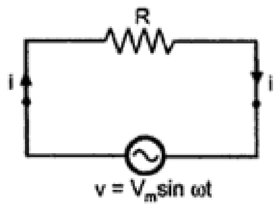


Fig. 4.1 Pure resistive circuit

Consider a simple circuit consisting of a pure resistance 'R' ohms connected across a voltage $v = V_m \sin \omega t$. The circuit is shown in the Fig. 4.1.

According to Ohm's law, we can find the equation for the current i as,

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R}$$

i.e.

$$i = \left(\frac{V_m}{R}\right) \sin(\omega t)$$

This is the equation giving instantaneous value of the current.

Comparing this with standard equation,

$$i = I_m \sin(\omega t + \phi)$$

$$\boxed{I_m = \frac{V_m}{R} \quad \text{and} \quad \phi = 0}$$

So, maximum value of alternating current, i is $I_m = \frac{V_m}{R}$ while, as $\phi = 0$, it indicates that it is in phase with the voltage applied. There is no phase difference between the two. The current is going to achieve its maximum (positive and negative) and zero whenever voltage is going to achieve its maximum (positive and negative) and zero values.

In purely resistive circuit, the current and the voltage applied are in phase with each other.

The waveforms of voltage and current and the corresponding phasor diagram is shown in the Fig. 4.2 (a) and (b).

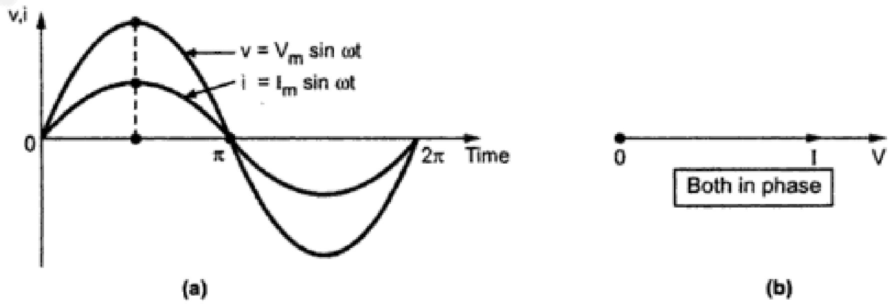


Fig. 4.2 A.C. through purely resistive circuit

In the phasor diagram, the phasors are drawn in phase and there is no phase difference in between them. Phasors represent the **r.m.s. values** of alternating quantities.

Power

The instantaneous power in a.c. circuits can be obtained by taking product of the instantaneous values of current and voltage.

$$P = v \times I$$

$$= V_m \sin(\omega t) \times I_m \sin \omega t$$

$$= V_m I_m \sin^2(\omega t)$$

$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$\therefore \quad \boxed{P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos(2\omega t)}$$

From the above equation, it is clear that the instantaneous power consists of two components,

1) Constant power component $\left(\frac{V_m I_m}{2}\right)$

2) Fluctuating component $\left[\frac{V_m I_m}{2}\cos(2\omega t)\right]$ having frequency, double the frequency of the applied voltage.

Now, the average value of the fluctuating cosine component of double frequency is zero, over one complete cycle. So, average power consumption over one cycle is equal to the constant power component i.e. $\frac{V_m I_m}{2}$.

$$\therefore \quad P_{av} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$\therefore \quad \boxed{P_{av} = V_{rms} \times I_{rms} \quad \text{watts}}$$

Generally, r.m.s. values are indicated by capital letters

$$\therefore \quad \boxed{P_{av} = V \times I \quad \text{watts} = I^2 R \quad \text{watts}}$$

The Fig. 4.3 shows the waveforms of voltage, current and power.

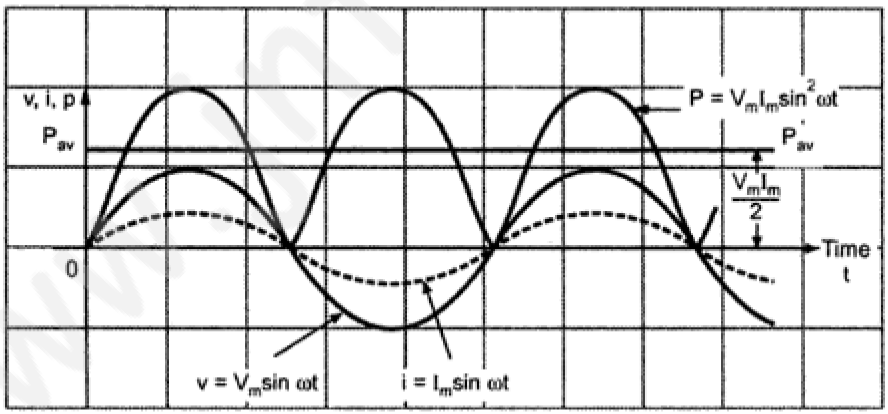


Fig. 4.3 v, i and p for purely resistive circuit

A.C. Through Pure Inductance

Consider a simple circuit consisting of a pure inductance of L henries, connected across a voltage given by the equation, $v = V_m \sin \omega t$. The circuit is shown in the Fig. 4.4.

Pure inductance has zero ohmic resistance. Its internal resistance is zero. The coil has pure inductance of L henries (H).

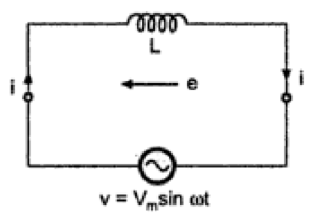


Fig. 4.4 Purely inductive circuit

When alternating current 'i' flows through inductance 'L', it sets up an alternating magnetic field around the inductance. This changing flux links the coil and due to self inductance, e.m.f. gets induced in the coil. This e.m.f. opposes the applied voltage.

The self induced e.m.f. in the coil is given by,

$$\text{Self induced e.m.f.,} \quad e = -L\frac{di}{dt}$$

At all instants, applied voltage, v is equal and opposite to the self induced e.m.f., e

$$\therefore \quad v = -e = -\left(-L\frac{di}{dt}\right)$$

$$\therefore \quad v = L\frac{di}{dt}$$

$$\therefore \quad V_m \sin \omega t = L\frac{di}{dt}$$

$$\therefore \quad di = \frac{V_m}{L}\sin \omega t \, dt$$

$$\therefore \quad i = \int di = \int \frac{V_m}{L}\sin \omega t \, dt = \frac{V_m}{L}\left(\frac{-\cos \omega t}{\omega}\right)$$

$$= -\frac{V_m}{\omega L}\sin\left(\frac{\pi}{2}-\omega t\right) \quad \text{as } \cos \omega t = \sin\left(\frac{\pi}{2}-\omega t\right)$$

$$\therefore \quad i = \frac{V_m}{\omega L}\sin\left(\omega t-\frac{\pi}{2}\right) \quad \text{as } \sin\left(\frac{\pi}{2}-\omega t\right) = -\sin\left(\omega t-\frac{\pi}{2}\right)$$

$$\therefore \quad \boxed{i = I_m \sin\left(\omega t-\frac{\pi}{2}\right)}$$

$$\text{where} \quad I_m = \frac{V_m}{\omega L} = \frac{V_m}{X_L}$$

$$\text{where} \quad \boxed{X_L = \omega L = 2\pi f L \ \Omega}$$

The above equation clearly shows that the current is purely sinusoidal and having phase angle of $-\frac{\pi}{2}$ radians i.e. $-90°$. This means that **the current lags voltage applied by 90°**. The negative sign indicates lagging nature of the current. If current is assumed as a reference, we can say that the voltage across inductance leads the current passing through the inductance by 90°.

The Fig. 4.5 shows the waveforms and the corresponding phasor diagram

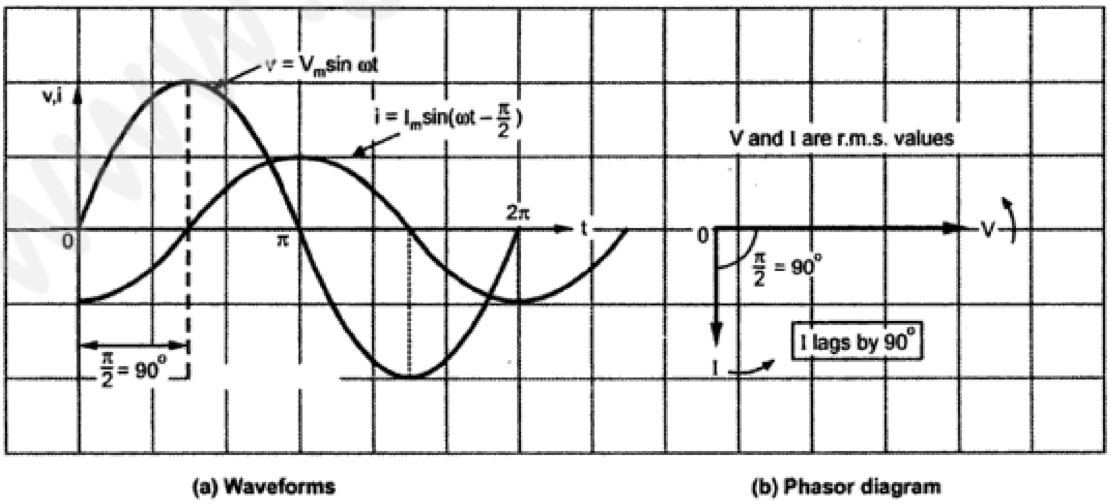


Fig. 4.5 A.C. through purely inductive circuit

In purely inductive circuit, current lags voltage by 90°.

Concept of Inductive Reactance

We have seen that in purely inductive circuit,

$$I_m = \frac{V_m}{X_L}$$

where $$\boxed{X_L = \omega L = 2\pi f L\ \Omega}$$

The term, X_L, is called **Inductive Reactance** and is measured in **ohms.**

So, **inductive reactance** is defined as the opposition offered by the inductance of a circuit to the flow of an alternating sinusoidal current.

It is measured in ohms and it depends on the frequency of the applied voltage.

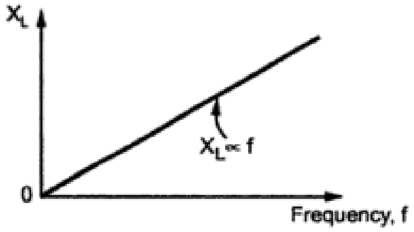


Fig. 4.6 X_L Vs f

The inductive reactance is directly proportional to the frequency for constant L.

$X_L \propto f$, for constant L

So, graph of X_L Vs f is a straight line passing through the origin as shown in the Fig. 4.6.

Key Point : *If frequency is zero, which is so for d.c. voltage, the inductive reactance is zero. Therefore, it is said that the inductance offers zero reactance for the d.c. or steady current.*

Power

The expression for the instantaneous power can be obtained by taking the product of instantaneous voltage and current.

$$\therefore \quad P = v \times i$$

$$= V_m \sin \omega t \times I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -V_m I_m \sin(\omega t)\cos(\omega t) \quad \text{as } \sin\left(\omega t - \frac{\pi}{2}\right) = -\cos \omega t$$

$$\therefore \quad \boxed{P = -\frac{V_m I_m}{2}\sin(2\omega t)} \quad \text{as } 2\sin \omega t \cos \omega t = \sin 2\omega t$$

This power curve is a sine curve of frequency double than that of applied voltage.

The average value of sine curve over a complete cycle is always zero.

$$P_{av} = \int_0^{2\pi} -\frac{V_m I_m}{2}\sin(2\omega t)\, d(\omega t) = 0$$

The Fig. 4.7 shows voltage, current and power waveforms.

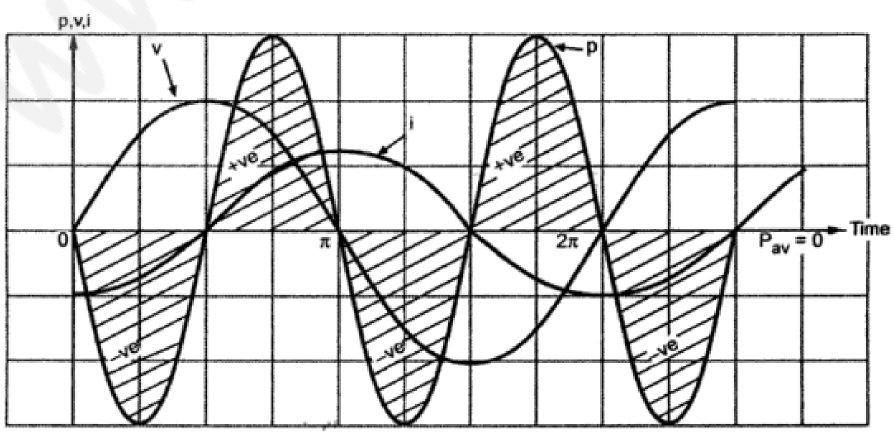


Fig. 4.7 Waveforms of voltage, current and power

when power curve is positive, energy gets stored in the magnetic field established due to the increasing current while during negative power curve, this power is returned back to the supply.

The areas of positive loop and negative loop are exactly same and hence, average power consumption is zero.

Pure inductance never consumes power.

A.C. Through Pure Capacitance

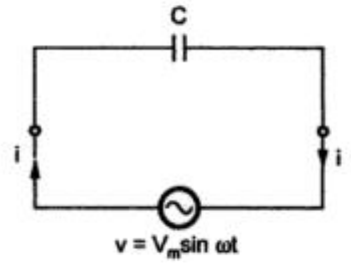


Fig. 4.8 Purely capacitive circuit

Consider a simple circuit consisting of a pure capacitor of C-farads, connected across a voltage given by the equation, $v = V_m \sin \omega t$. The circuit is shown in the Fig. 4.8.

The current i charges the capacitor C. The instantaneous charge 'q' on the plates of the capacitor is given by,

$$q = C\,v$$

$$\therefore \quad q = C\,V_m \sin \omega t$$

Now, current is rate of flow of charge.

$$\therefore \quad i = \frac{dq}{dt} = \frac{d}{dt}(C\,V_m \sin \omega t)$$

$$\therefore \quad i = C\,V_m \frac{d}{dt}(\sin \omega t) = C\,V_m\,\omega \cos \omega t$$

$$\therefore \quad i = \frac{V_m}{\left(\frac{1}{\omega C}\right)} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\therefore \quad i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

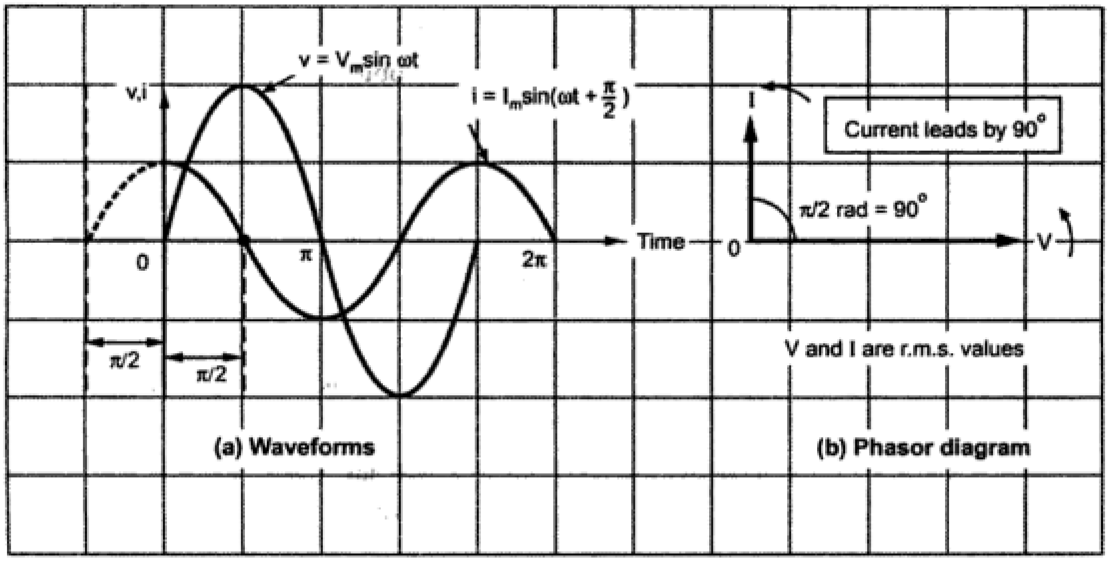
where $$I_m = \frac{V_m}{X_C}$$

where $$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \quad \Omega$$

The above equation clearly shows that the current is purely sinusoidal and having phase angle of $+\frac{\pi}{2}$ radians i.e. + 90°.

This means **current leads voltage applied by 90°**. The positive sign indicates leading nature of the current. If current is assumed reference, we can say that voltage across capacitor lags the current passing through the capacitor by 90°.

The Fig. 4.9 shows waveforms of voltage and current and the corresponding phasor diagram. The current waveform starts earlier by 90° in comparison with voltage waveform. When voltage is zero, the current has positive maximum value.



In purely capacitive circuit, current leads voltage by 90°.

Concept of Capacitive Reactance

We have seen while expressing current equation in the standard form that,

$$I_m = \frac{V_m}{X_C}$$

and
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}\ \Omega$$

The term X_C is called **Capacitive Reactance** and is measured in ohms.

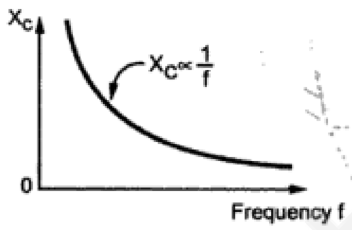


Fig. 4.10 X_C Vs f

So, **capacitive reactance** is defined as the opposition offered by the capacitance of a circuit to the flow of an alternating sinusoidal current.

X_C is measured in ohms and it depends on the frequency of the applied voltage.

The capacitive reactance is inversely proportional to the frequency for constant C.

$$X_C \propto \frac{1}{f} \quad \text{for constant C}$$

The graph of X_C Vs f is a rectangular hyperbola as shown in Fig. 4.10.

Key Point : *If the frequency is zero, which is so for d.c. voltage, the capacitive reactance is infinite. Therefore, it is said that the capacitance offers open circuit to the d.c. or it blocks d.c.*

Power

The expression for the instantaneous power can be obtained by taking the product of instantaneous voltage and current.

$$P = v \times i = V_m \sin(\omega t) \times I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin(\omega t)\cos(\omega t) \qquad \text{as } \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

$$\therefore \qquad \boxed{P = \frac{V_m I_m}{2}\sin(2\omega t)} \qquad \text{as } 2\sin\omega t\cos\omega t = \sin 2\omega t$$

Thus, power curve is a sine wave of frequency double that of applied voltage. The average value of sine curve over a complete cycle is always zero.

$$P_{av} = \int_0^{2\pi} \frac{V_m I_m}{2}\sin(2\omega t)\, d(\omega t) = 0$$

The Fig. 4.11 shows waveforms of current, voltage and power. It can be observed from the figure that when power curve is positive, in practice, an electrostatic energy gets stored in the capacitor during its charging while the negative power curve represents that the energy stored is returned back to the supply during its discharging. The areas of positive and negative loops are exactly the same and hence, average power consumption is zero.

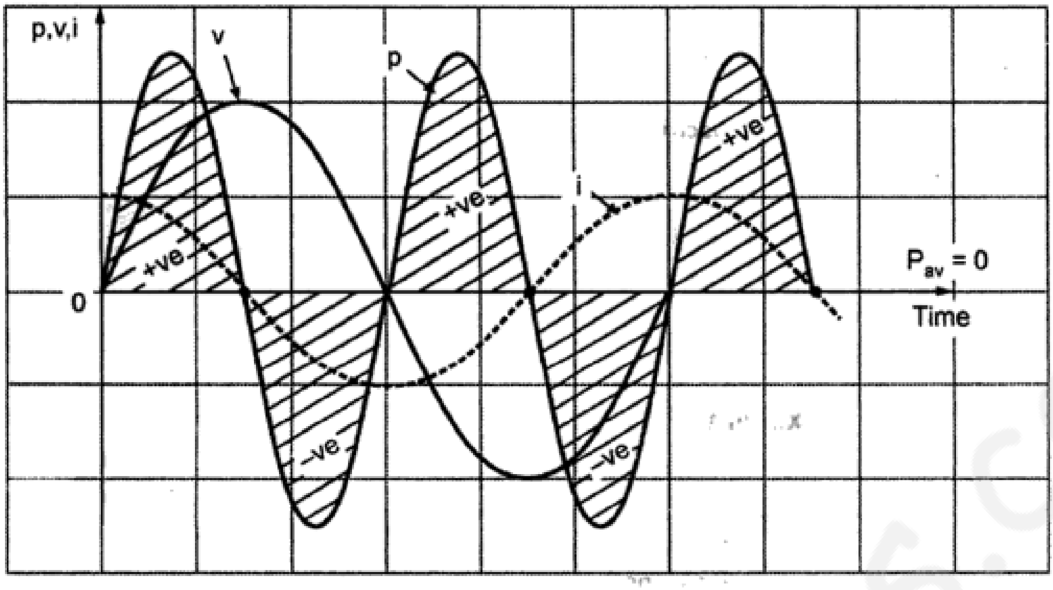


Fig. 4.11 Waveforms of voltage, current and power

Pure capacitance never consumes power.

Example 4.1 : *A 50 Hz, alternating voltage of 150 V (r.m.s.) is applied independently to (1) Resistance of 10 Ω (2) Inductance of 0.2 H (3) Capacitance of 50 μF*

Find the expression for the instantaneous current in each case. Draw the phasor diagram in each case.

Solution : Case 1 : R = 10 Ω

$$v = V_m \sin \omega t$$

$$V_m = \sqrt{2}\, V_{rms}$$

$$= \sqrt{2} \times 150 = 212.13 \text{ V}$$

$$I_m = \frac{V_m}{R} = \frac{212.13}{10}$$

$$= 21.213 \text{ A}$$

In pure resistive circuit, current is in phase with the voltage.

$$\therefore \quad \phi = \text{Phase Difference} = 0°$$

$$\therefore \quad i = I_m \sin \omega t$$

$$= I_m \sin (2 \pi f t)$$

$$\therefore \quad \mathbf{i = 21.213 \sin (100 \pi t) \text{ A}}$$

Fig. 4.12 (a)

The phasor diagram is shown in the Fig. 4.12 (a).

Case 2 : $L = 0.2\ \Omega$

Inductive reactance, $X_L = \omega L = 2\pi f L$

$\therefore \quad X_L = 2\pi \times 50 \times 0.2$

$= 62.83\ \Omega$

$\therefore \quad I_m = \frac{V_m}{X_L} = \frac{212.13}{62.83}$

$= 3.37\ A$

In pure inductive circuit, current lags voltage by 90°.

$\therefore \quad \phi = \text{Phase difference} = -90° = \frac{\pi}{2}\ \text{rad}$

$\therefore \quad i = I_m \sin(\omega t - \phi)$

$\therefore \quad i = 3.37 \sin\left(100\pi t - \frac{\pi}{2}\right) A$

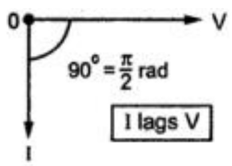


Fig. 4.12 (b)

The phasor diagram is shown in the Fig. 4.12 (b).

Case 3 : $C = 50\ \mu F$

Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$\therefore \quad X_C = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.66\ \Omega$

$\therefore \quad I_m = \frac{V_m}{X_C} = \frac{212.13}{63.66} = 3.33\ A$

In pure capacitive circuit, current leads voltage by 90°.

$\therefore \quad \phi = \text{Phase Difference} = 90° = \frac{\pi}{2}\ \text{rad}$

$\therefore \quad i = I_m \sin(\omega t + \phi)$

$\therefore \quad i = 3.33 \sin\left(100\pi t + \frac{\pi}{2}\right) A$

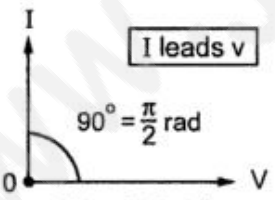


Fig. 4.12 (c)

The phasor diagram is shown in the Fig. 4.12 (c).

All the phasor diagrams represent r.m.s. values of voltage and current.

Example 4.2 : *A voltage $v = 141 \sin\{314t+\pi/3\}$ is applied to i) resistor of 20 ohms ii) inductance of 0.1 Henry iii) capacitance of 100 μF*

Find in each case r.m.s. value of current and power dissipated.

Draw the phasor diagram in each case.

Solution : Comparing given voltage with $v = V_m \sin(\omega t+\theta)$ we get,

$$V_m = 141 \text{ V and hence } V = V_{rms} = \frac{V_m}{\sqrt{2}} = 99.702 \text{ V}$$

$$\omega = 314 \text{ and hence } f = \frac{\omega}{2\pi} = 50 \text{ Hz}, \theta = \frac{\pi}{3} = 60°$$

Hence the polar form of applied voltage becomes,

$$V = 99.702 \angle 60° \text{ V}$$

Case 1 : $R = 20\ \Omega$

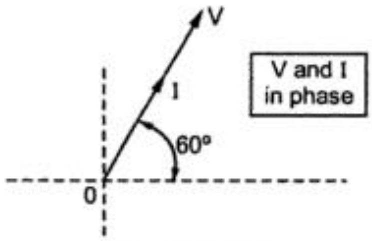


Fig. 4.13 (a)

$$I = \frac{V}{R} = \frac{99.702\angle 60°}{20\angle 0°} = 4.9851 \angle 60° \text{ A}$$

$\therefore$ I_{rms} = **4.9851 A**

The phase of both V and I is same for pure resistive circuit. Both are in phase.

P = VI = 99.702×4.9851 = **497.0244 W**

The phasor diagram is shown in the Fig. 4.13 (a).

Case 2 : $L = 0.1$ H

$\therefore$ $X_L = \omega L = 314 \times 0.1 = 31.4\ \Omega$

$\therefore$ $I = \frac{|V|}{X_L} = \frac{99.702}{31.4}$ = **3.1752 A**

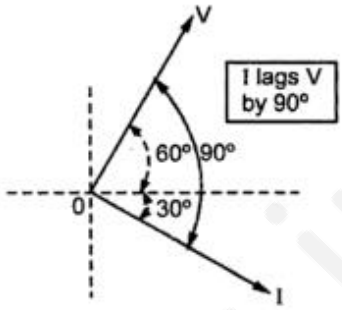


Fig. 4.13 (b)

This is r.m.s. value of current. It has to lag the applied voltage by 90° in case of pure inductor.

Hence phasor diagram is shown in the Fig. 4.13 (b).

The individual phase of I is − 30°.

In polar form I can be represented as $3.1752\angle -30°$ A.

Pure inductor never consumes power so **power dissipated is zero.**

Case 3 : $C = 100$ μF

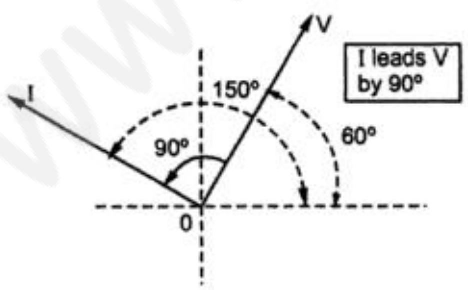


Fig. 4.13 (c)

$$\therefore X_C = \frac{1}{\omega C} = \frac{1}{314\times 100\times 10^{-6}} = 31.8471\ \Omega$$

$\therefore$ $I = \frac{|V|}{X_C} = \frac{99.702}{31.8471}$ = **3.1306 A**

This is r.m.s. value of current.

It has to lead the applied voltage by 90° in case of pure capacitor.

Hence phasor diagram is shown in the Fig. 4.13 (c).

The individual phase of I is 150°. In polar form I can be represented as $3.1306\angle +150°$ A. Pure capacitor never consumes power and hence **power dissipated is zero.**

A.C. Through Series R-L Circuit

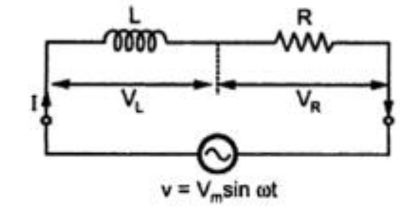


Fig. 4.14 (a) Series R-L circuit

Consider a circuit consisting of pure resistance R ohms connected in series with a pure inductance of L henries as shown in the Fig. 4.14 (a).

The series combination is connected across a.c. supply given by $v = V_m \sin \omega t$.

Circuit draws a current I then there are two voltage drops,

a) Drop across pure resistance, $V_R = I \times R$

b) Drop across pure inductance, $V_L = I \times X_L$ where $X_L = 2 \pi f L$

I = r.m.s. value of current drawn

V_R , V_L = r.m.s. values of the voltage drops.

The Kirchhoff's voltage law can be applied to the a.c. circuit but only the point to remember is the addition of voltages should be a phasor (vector) addition and no longer algebraic as in case of d.c.

$$\therefore \quad \overline{V} = \overline{V_R} + \overline{V_L} \quad \text{(phasor addition)}$$

$$\therefore \quad \overline{V} = \overline{IR} + \overline{I X_L}$$

Let us draw the phasor diagram for the above case.

Key Point : *For series a.c. circuits, generally, current is taken as the reference phasor as it is common to both the elements.*

Following are the **steps to draw the phasor diagram :**

1) Take current as a reference phasor.

2) In case of resistance, voltage and current are in phase, so V_R will be along current phasor.

3) In case of inductance, current lags voltage by 90°. But, as current is reference, V_L must be shown leading with respect to current by 90°.

4) The supply voltage being vector sum of these two vectors V_L and V_R obtained by law of parallelogram.

From the voltage triangle, we can write,

$$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (I \times X_L)^2}$$

$$= I\sqrt{(R)^2 + (X_L)^2}$$

$$\therefore \quad V = I Z$$

where $$\boxed{Z = \sqrt{(R)^2 + (X_L)^2}}$$ ... **Impedance** of the circuit.

The impedance Z is measured in ohms.

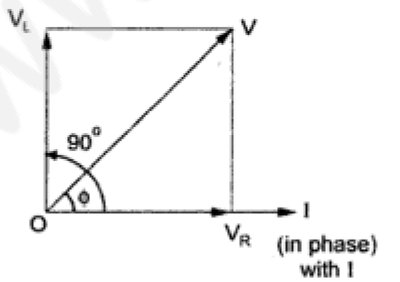


Fig. 4.14 (b) Phasor diagram

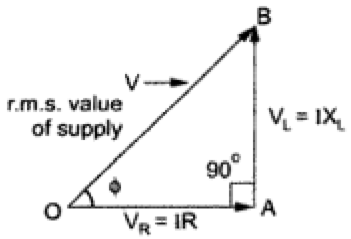


Fig. 4.14 (c) Voltage triangle

Impedance

Impedance is defined as the opposition of circuit to flow of alternating current. It is denoted by Z and its unit is ohms.

For the R-L series circuit, it can be observed from the phasor diagram that the current lags behind the applied voltage by an angle ϕ. From the voltage triangle, we can write,

$$\tan\phi = \frac{V_L}{V_R} = \frac{X_L}{R}, \quad \cos\phi = \frac{V_R}{V} = \frac{R}{Z}, \quad \sin\phi = \frac{V_L}{V} = \frac{X_L}{Z}$$

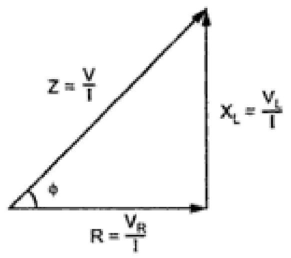


Fig. 4.15 Impedance triangle

If all the sides of the voltage triangle are divided by current, we get a triangle called **impedance triangle** as shown in the Fig. 4.15.

Sides of this triangle are resistance R, inductive reactance X_L and an impedance Z.

From this impedance triangle, we can see that the X component of impedance is R and is given by,

$$R = Z\cos\phi$$

and Y component of impedance is X_L and is given by,

$$X_L = Z\sin\phi$$

In rectangular form the impedance is denoted as,

$$\boxed{Z = R + j\,X_L \quad \Omega}$$

While in polar form, it is denoted as,

$$Z = |Z| \angle\phi \quad \Omega$$

where

$$|Z| = \sqrt{R^2 + X_L^2}, \qquad \phi = \tan^{-1}\left[\frac{X_L}{R}\right]$$

Key Point: *Thus ϕ is + ve for inductive impedance.*

Power and Power Triangle

The expression for the current in the series R-L circuit is,

$$i = I_m \sin(\omega t - \phi) \text{ as current lags voltage.}$$

The power is product of instantaneous values of voltage and current,

$$\therefore \quad P = v \times i = V_m \sin\omega t \times I_m \sin(\omega t - \phi)$$

$$= V_m I_m [\sin(\omega t) \cdot \sin(\omega t - \phi)]$$

$$= V_m I_m \left[\frac{\cos(\phi) - \cos(2\omega t - \phi)}{2}\right]$$

$$= \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)$$

Now, the second term is cosine term whose average value over a cycle is zero. Hence, average power consumed is,

$$P_{av} = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\cos\phi$$

$$\therefore \quad \boxed{P = V\,I\cos\phi \quad \text{watts}}$$ where V and I are r.m.s. values

If we multiply voltage equation by current I, we get the power equation.

$$\overline{VI} = \overline{V_R I} + \overline{V_L I}$$

$$\therefore \quad \overline{VI} = \overline{V\cos\phi I} + \overline{V\sin\phi I}$$

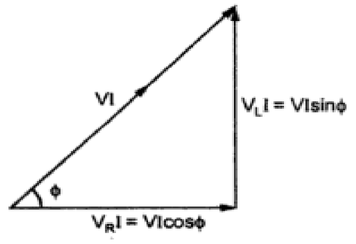


Fig. 4.16 Power triangle

From this equation, power triangle can be obtained as shown in the Fig. 4.16.

So, three sides of this triangle are,

1) VI, 2) VI cos ϕ, 3) VI sin ϕ

These three terms can be defined as below.

Apparent Power (S)

It is defined as the product of r.m.s. value of voltage (V) and current (I). It is denoted by S.

$$\therefore \quad \boxed{S = VI \quad VA}$$

It is measured in unit volt-amp (VA) or kilo volt-amp (kVA).

Real or True Power (P)

It is defined as the product of the applied voltage and the active component of the current.

It is real component of the apparent power. It is measured in unit watts (W) or kilowatts (kW).

$$\boxed{P = VI\cos\phi \quad \text{watts}}$$

Reactive Power (Q)

It is defined as product of the applied voltage and the reactive component of the current.

It is also defined as imaginary component of the apparent power. It is represented by 'Q' and it is measured in unit Volt-Amp Reactive (VAR) or kilo volt–Amp Reactive kVAR

$$\boxed{Q = VI\sin\phi \quad VAR}$$

Apparent power, $S = VI$ VA

True power $P = VI\cos\phi$ W (Average Power)

Reactive power $Q = VI\sin\phi$ VAR

Power Factor (cos ϕ)

It is defined as factor by which the apparent power must be multiplied in order to obtain the true power.

It is the ratio of true power to apparent power.

$$\boxed{\text{Power factor} = \frac{\text{True Power}}{\text{Apparent Power}} = \frac{VI\cos\phi}{VI} = \cos\phi}$$

The numerical value of cosine of the phase angle between the applied voltage and the current drawn from the supply voltage gives the power factor. It cannot be greater than 1.

It is also defined as the ratio of resistance to the impedance.

$$\boxed{\cos\phi = \frac{R}{Z}}$$

Key Point : *The nature of power factor is always determined by position of current with respect to the voltage.*

If current lags voltage power factor is said to be lagging. If current leads voltage power factor is said to be leading.

So, for pure inductance, the power factor is cos (90°) i.e. zero lagging while for pure capacitance, the power factor is cos (90°) i.e. zero but leading. For purely resistive circuit voltage and current are in phase i.e. $\phi = 0$. Therefore, power factor is cos (0°) = 1. Such circuit is called unity power factor circuit.

$$\boxed{\text{Power factor} = \cos\phi}$$

ϕ is the angle between supply voltage and current.

Key Point : *Nature of power factor always tells position of current with respect to voltage.*

Example 4.3 : *An alternating current, i = 1.414 Sin (2 π × 50 × t) A, is passed through a series circuit consisting of a resistance of 100 ohm and an inductance of 0.31831 henry. Find the expressions for the instantaneous values of the voltages across (i) the resistance, (ii) the inductance and (iii) the combination.*

Solution : The circuit is shown in the Fig. 4.17.

$$i = 1.414 \sin (2\pi \times 50\, t) \text{ A}$$

$$\therefore \quad \omega = 2\pi \times 50 = 2\pi f$$

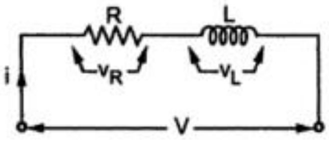


Fig. 4.17

$$\therefore \quad f = 50 \text{ Hz}, \quad R = 100\ \Omega, \quad L = 0.31831 \text{ H}$$

$$\therefore \quad X_L = 2\pi f L$$

$$= 2\pi \times 50 \times 0.31831 = 100\ \Omega$$

i) The voltage across the resistance is,

$$v_R = i R = 1.414 \sin (2\pi \times 50\, t) \times 100$$

$$= \mathbf{141.4 \sin (2\pi \times 50\, t)\ V}$$

ii) The voltage across L leads current by 90° as current lags by 90° with respect to voltage.

$$\therefore \quad v_L = i X_L \text{ but leading current by } 90^\circ$$

$$= \mathbf{141.4 \sin (2\pi \times 50\, t + 90^\circ)\ V}$$

iii) From the expression of V_R we can write,

$$\text{r.m.s. value of} \quad V_R = \frac{141.4}{\sqrt{2}} = 100 \text{ V}, \qquad \phi = 0^\circ$$

$$\therefore \quad V_R = 100 \angle 0^\circ = 100 + j0 \text{ V}$$

$$\text{r.m.s. value of } V_L = \frac{141.4}{\sqrt{2}} = 100 \text{ V}, \qquad \phi = 90^\circ$$

$$\therefore \quad V_L = 100 \angle 90^\circ = 0 + j\,100 \text{ V}$$

$$\therefore \quad V = \bar{V}_R + \bar{V}_L = 100 + j0 + 0 + j100$$

$$= 100 + j\,100 = 141.42 \angle 45^\circ \text{ V}$$

$$\therefore \quad V_m = \sqrt{2} \times 141.42 = 200 \text{ V}$$

Hence expression of instantaneous value of resultant voltage is,

$$v = \mathbf{200 \sin (2\pi \times 50\, t + 45^\circ)\ V}$$

Example. 4.4 : *A voltage e = 200 sin 100 π t is applied to a load having R = 200 Ω in series with L = 638 mH.*

Estimate :-

i) expression for current in $i = I_m \sin(\omega t \pm \phi)$ form ii) power consumed by the load iii) reactive power of the load iv) voltage across R and L.

Solution : The circuit is shown in the Fig. 4.18.

$$e_m = 200 \text{ V} \quad \therefore \ V = \frac{200}{\sqrt{2}} = 141.421 \text{ V (r.m.s.)}$$

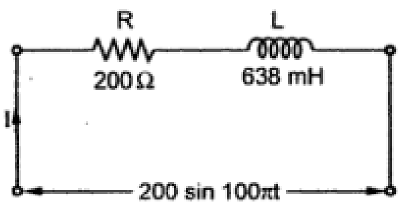


Fig. 4.18

$$\omega = 100\pi \quad \therefore \ f = 50 \text{ Hz}$$

$$\therefore \quad X_L = \omega L = 100\pi \times 638 \times 10^{-3}$$

$$= 200.433\ \Omega$$

$$Z = R + j X_L = 200 + j\,200.433\ \Omega$$

$$= 283.149 \angle 45.06^\circ\ \Omega$$

$$\therefore \quad I = \frac{V}{Z} = \frac{141.421\angle 0^\circ}{283.149\angle 45.06^\circ}$$

$$= 0.5\angle -45.06^\circ \text{ A}, \qquad \text{current lags voltage by } 45.06^\circ.$$

$$\therefore \quad I_m = \sqrt{2}\times 0.5 = 0.7071 \text{ A}, \phi = -45.06^\circ$$

i) $i = I_m \sin(\omega t - \phi)$

$= \mathbf{0.7071 \sin(100\,\pi\, t - 45.06^\circ)\ A}$

ii) $P = VI\cos\phi = 141.421 \times 0.5 \times \cos(45.06^\circ)$

$= 49.9474 \approx \mathbf{50\ W}$

iii) $Q = VI\sin\phi = 141.421 \times 0.5 \times \sin(45.06^\circ)$

$= \mathbf{50\ VAR}$

iv) $V_R = IR = 0.5 \times 200 = \mathbf{100\ V}$

$V_L = I\,X_L = 0.5 \times 200.433 = \mathbf{100.21\ V}$

A.C. Through Series R-C Circuit

Consider a circuit consisting of pure resistance R-ohms and connected in series with a pure capacitor of C-farads as shown in the Fig. 4.19.

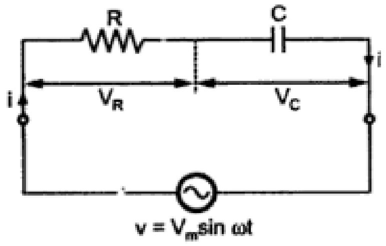


Fig. 4.19 Series R-C circuit

The series combination is connected across a.c. supply given by

$$v = V_m \sin \omega t$$

Circuit draws a current I, then there are two voltage drops,

a) Drop across pure resistance $V_R = I \times R$

b) Drop across pure capacitance $V_C = I \times X_C$

where $X_C = \frac{1}{2\pi f\, C}$ and I, V_R, V_C are the r.m.s. values

The Kirchhoff's voltage law can be applied to get,

$$V = \overline{V_R} + \overline{V_C} \qquad \text{... (Phasor Addition)}$$

$$\therefore \quad \overline{V} = \overline{IR} + \overline{IX_C}$$

Let us draw the phasor diagram. Current I is taken as reference as it is common to both the elements.

Following are the steps to draw the phasor diagram :-

1) Take current as reference phasor.
2) In case of resistance, voltage and current are in phase. So, V_R will be along current phasor.
3) In case of pure capacitance, current leads voltage by 90° i.e. voltage lags current by 90° so V_C is shown downwards i.e. lagging current by 90°.
4) The supply voltage being vector sum of these two voltages V_C and V_R obtained by completing parallelogram.

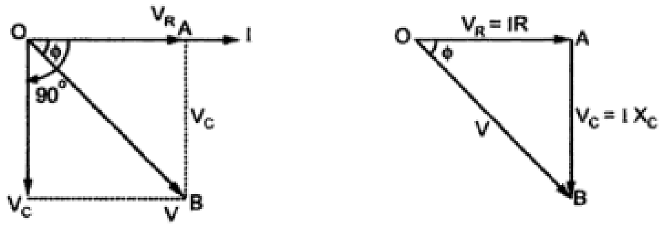


Fig. 4.20 Phasor diagram and voltage triangle

From the voltage triangles,

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (I\,X_C)^2}$$

$$= I\sqrt{(R)^2 + (X_C)^2}$$

$$\therefore \quad V = I\,Z$$

where $\boxed{Z = \sqrt{(R)^2 + (X_C)^2}}$ is the impedance of the circuit.

Impedance

It is measured in ohms given by $Z = \sqrt{(R)^2 + (X_C)^2}$ where $X_C = \frac{1}{2\pi f C}$ Ω called **capacitive reactance.**

In R-C series circuit, current leads voltage by angle ϕ or supply voltage V lags current I by angle ϕ as shown in the phasor diagram in Fig. 4.21.

From voltage triangle, we can write,

$$\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R}, \quad \cos\phi = \frac{V_R}{V} = \frac{R}{Z}, \quad \sin\phi = \frac{V_C}{V} = \frac{X_C}{Z}$$

If all the sides of the voltage triangle are divided by the current, we get a triangle called **impedance triangle.**

Two sides of the triangle are 'R' and 'X_C' and the third side is impedance 'Z'.

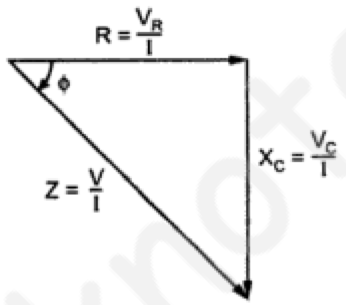


Fig. 4.21 Impedance triangle

The X component of impedance is R and is given by

$$R = Z\cos\phi$$

and Y component of impedance is X_C and is given by

$$X_C = Z\sin\phi$$

But, as direction of the X_C is the negative Y direction, the rectangular form of the impedance is denoted as,

$$Z = R - j\,X_C \ \Omega$$

While in polar form, it is denoted as,

where $\boxed{Z = |Z| \angle -\phi \ \Omega}$

$$Z = R - j\,X_C = |Z| \angle -\phi$$

where $|Z| = \sqrt{R^2 + X_C^2}$, $\phi = \tan^{-1}\left[\frac{-X_C}{R}\right]$

Key Point : *Thus ϕ is – ve for capacitive impedance.*

Power and Power Triangle

The current leads voltage by angle ϕ, hence its expression is,

$$i = I_m \sin(\omega t + \phi) \text{ as current leads voltage}$$

The power is the product of instantaneous values of voltage and current.

$$\therefore \quad P = v \times i = V_m \sin \omega t \times I_m \sin(\omega t + \phi)$$

$$= V_m I_m [\sin(\omega t) . \sin(\omega t + \phi)]$$

$$= V_m I_m \left[\frac{\cos(-\phi) - \cos(2\omega t + \phi)}{2}\right]$$

$$= \frac{V_m I_m \cos\phi}{2} - \frac{V_m I_m}{2} \cos(2\omega t + \phi) \qquad \text{as } \cos(-\phi) = \cos\phi$$

Now, second term is cosine term whose average value over a cycle is zero. Hence, average power consumed by the circuit is,

$$P_{av} = \frac{V_m I_m}{2} \cos\phi = \frac{V_m}{\sqrt{2}} . \frac{I_m}{\sqrt{2}} \cos\phi$$

$$\therefore \quad \boxed{P = V I \cos\phi \quad \text{watts}} \qquad \text{where V and I are r.m.s. values}$$

If we multiply voltage equation by current I, we get the power equation,

$$\overline{VI} = \overline{V_R I} + \overline{V_C I}$$

$$\therefore \quad \overline{VI} = \overline{VI \cos\phi} + \overline{VI \sin\phi}$$

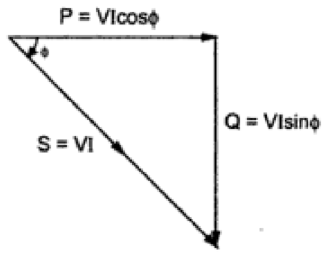


Fig. 4.22

Hence, the power triangle can be shown as in the Fig. 4.22.

Thus, the various powers are,

Apparent Power, $S = V I$ V A

True or average power, $P = V I \cos\phi$ W

Reactive Power, $Q = V I \sin\phi$ V A R

Remember that, X_L term appears positive in Z.

$$Z = R + j X_L = |Z| \angle \phi \qquad \phi \text{ is + ve for inductive Z}$$

While X_C term appears negative in Z.

$$Z = R - j X_C = |Z| \angle -\phi \qquad \phi \text{ is - ve for capacitive Z}$$

For any single phase a.c. circuit, the average power is given by,

$$P = V I \cos\phi \quad \text{watts}$$

where V, I are r.m.s. values

$$\cos\phi = \text{Power factor of circuit}$$

$\cos\phi$ **is lagging for inductive** circuit and $\cos\phi$ **is leading for capacitive** circuit.

Example 4.5 : *Calculate the resistance and inductance or capacitance in series for each of the following impedances. Assume the frequency to be 60 Hz.*

i) (12 + j 30) ohms ii) – j 60 ohms iii) 20 ∠ 60° ohms.

Solution : i) 12 + j 30 Ω

Comparing the value of impedance with,

$$Z = R + j X_L, \quad R = 12\ \Omega \quad \text{and } X_L = 30\ \Omega = 2\pi f L$$

$$\therefore \quad L = \frac{30}{2\pi f} = \frac{30}{2\pi \times 60} = \mathbf{79.58\ mH}$$

ii) 0 – j 60 Ω

Comparing with, $Z = R - j X_C$

$$R = \mathbf{0\ \Omega}$$

$$X_C = 60\ \Omega = \frac{1}{2\pi f C}$$

$$\therefore \quad C = \frac{1}{2\pi \times 60 \times 60} = \mathbf{44.209\ \mu F}$$

iii) 20 ∠60° Ω

Converting to rectangular form, Z = 10 + j 17.32

Comparing with, $Z = R + j X_L$

$$R = \mathbf{10\ \Omega}$$

$$X_L = 17.32\ \Omega = 2\pi f L$$

$$\therefore \quad L = \frac{1732}{2\pi \times 60} = \mathbf{45.94\ mH}$$

Example 4.6 : *A resistance of 120 ohms and a capacitive reactance of 250 ohms are connected in series across a AC voltage source. If a current of 0.9 A is flowing in the circuit find out (i) power factor, (ii) supply voltage (iii) voltages across resistance and capacitance (iv) Active power and reactive power.*

Solution : The circuit is shown in the Fig. 4.23.

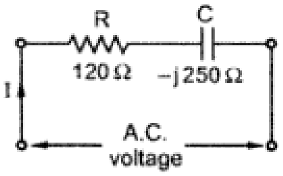


Fig. 4.23

$R = 120\ \Omega,\ X_C = 250\ \Omega,\ I = 0.9\ A$

$Z = R - j\,X_C = 120 - j250\ \Omega = 277.308\ \angle -64.358°$

Take current as reference.

$\therefore\ I = 0.9 \angle 0°\ A$

i) Power factor $\cos\phi = \cos(-64.358°) = \mathbf{0.4327\ leading}$

ii) Supply voltage $V = I \times Z = [0.9 \angle 0°] \times [277.308 \angle -64.358°]$

$\therefore\quad V = \mathbf{249.5772 \angle -64.358°\ V}$

iii) $V_R = I \times R = 0.9 \times 120 = \mathbf{108\ V}$ (magnitude)

$V_C = I \times X_C = 0.9 \times 250 = \mathbf{225\ V}$ (magnitude)

iv) P = Active power $= V\,I\cos\phi = 249.5772 \times 0.9 \times 0.4327$

$= \mathbf{97.1928\ W}$

Q = Reactive power $= VI\sin\phi$

$= 249.5772 \times 0.9 \times \sin(-64.358°)$

$= \mathbf{-202.498\ VAR}$

The negative sign indicates leading nature of reactive volt-amperes.

A.C. Through Series R-L-C Circuit

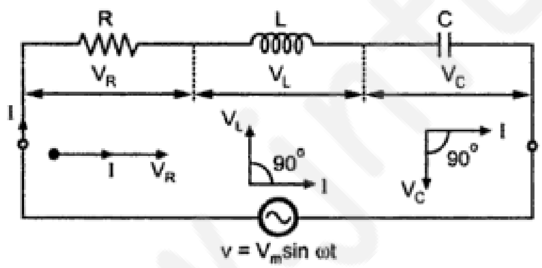


Fig. 4.24 R-L-C series circuit

Consider a circuit consisting of resistance R ohms pure inductance L henries and capacitance C farads connected in series with each other across a.c. supply. The circuit is shown in the Fig. 4.24.

The a.c. supply is given by,

$v = V_m \sin \omega t$. The circuit draws a current I. Due to current I, there are different voltage drops across R, L and C which are given by,

a) Drop across resistance R is $V_R = I\,R$

b) Drop across inductance L is $V_L = I\,X_L$

c) Drop across capacitance C is $V_C = I\,X_C$

The values of I, V_R, V_L and V_C are r.m.s. values

The characteristics of three drops are,

a) V_R is in phase with current I.

b) V_L leads current I by 90°.

c) V_C lags current I by 90°.

According to Kirchhoff's laws, we can write,

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C \quad \text{... Phasor addition}$$

Let us see the phasor diagram. Current I is taken as reference as it is common to all the elements.

Following are the steps to draw the phasor diagram :

1) Take current as reference. 2) V_R is in phase with I.

3) V_L leads current I by 90°. 4) V_C lags current I by 90°.

5) Obtain the resultant of V_L and V_C. Both V_L and V_C are in phase opposition (180° out of phase).

6) Add that with V_R by law of parallelogram to get the supply voltage.

The phasor diagram depends on the conditions of the magnitudes of V_L and V_C which ultimately depends on the values of X_L and X_C. Let us consider the different cases.

1. $X_L > X_C$

When $X_L > X_C$, obviously, I X_L i.e. V_L is greater than I X_C i.e. V_C. So, resultant of V_L and V_C will be directed towards V_L i.e. leading current I. Current I will lag the resultant of V_L and V_C i.e. $(V_L - V_C)$.

The circuit is said to be **inductive** in nature. The phasor sum of V_R and $(V_L - V_C)$ gives the resultant supply voltage, V. This is shown in the Fig. 4.25.

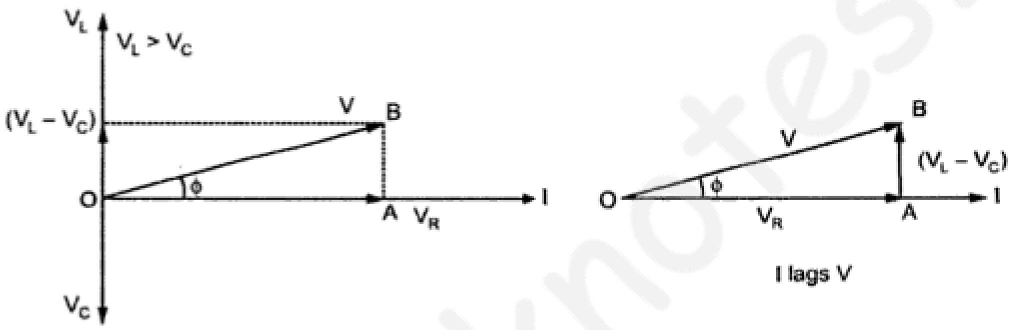


Fig. 4.25 Phasor diagram and voltage triangle for $X_L > X_C$

From the voltage triangle, $V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(I\,R)^2 + (I\,X_L - I\,X_C)^2}$

$$= I\sqrt{(R)^2 + (X_L - X_C)^2}$$

$\therefore$ $$V = I\,Z$$

where $$Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

So, if $v = V_m \sin \omega t$, then $i = I_m \sin(\omega t - \phi)$ as current lags voltage by angle ϕ.

2. $X_L < X_C$

When $X_L < X_C$, obviously, I X_L i.e. V_L is less than I X_C i.e. V_C. So, the resultant of V_L and V_C will be directed towards V_C. Current I will lead $(V_C - V_L)$.

The circuit is said to be **capacitive** in nature. The phasor sum of V_R and $(V_C - V_L)$ gives the resultant supply voltage V. This is shown in the Fig. 4.26.

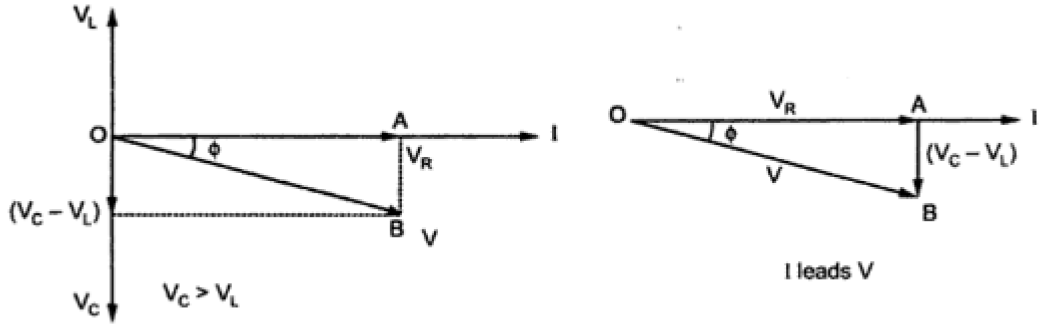


Fig. 4.26 Phasor diagram and voltage triangle for $X_L < X_C$

From the voltage triangle,

$$V = \sqrt{(V_R)^2 + (V_C - V_L)^2} = \sqrt{(I\,R)^2 + (I\,X_C - I\,X_L)^2}$$

$$= I\sqrt{(R)^2 + (X_C - X_L)^2}$$

$\therefore$ $$V = I\,Z$$

where $$Z = \sqrt{(R)^2 + (X_C - X_L)^2}$$

So, if $v = V_m \sin \omega t$, then $i = I_m \sin(\omega t + \phi)$ as current leads voltage by angle ϕ.

3. **$X_L = X_C$**

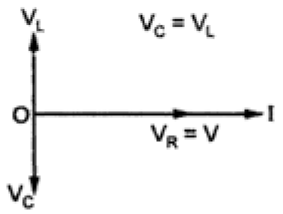


Fig. 4.27 Phasor diagram for $X_L = X_C$

When $X_L = X_C$, obviously, $V_L = V_C$. So, V_L and V_C will cancel each other and their resultant is zero. So, $V_R = V$ in such case and overall circuit is purely resistive in nature. The phasor diagram is shown in the Fig. 4.27.

From phasor diagram, $V = V_R$

$\therefore$ $$V = I\,R$$

$\therefore$ $$V = I\,Z$$

where $$Z = R$$

Impedance

In general, for RLC series circuit impedance is given by,

$$Z = R + j\,X$$

where $X = X_L - X_C$ = total reactance of circuit

If $X_L > X_C$, X is positive and circuit is inductive.

If $X_L < X_C$, X is negative and circuit is capacitive.

If $X_L = X_C$, X is zero and circuit is purely resistive.

$$\tan\phi = \left[\frac{X_L - X_C}{R}\right], \quad \cos\phi = \frac{R}{Z} \text{ and } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Impedance Triangle

The impedance is expressed as,

$$Z = R + jX \quad \text{where} \quad X = X_L - X_C$$

For $X_L > X_C$, ϕ is positive and the impedance triangle is as shown in the Fig. 4.28 (a).

For $X_L < X_C$, $X_L - X_C$ is negative, so ϕ is negative and the impedance triangle is as shown in Fig. 4.28 (b).

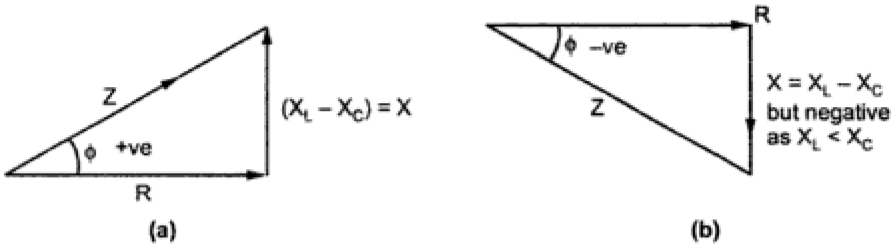


Fig. 4.28 Impedance triangles

In both the cases, $R = Z\cos\phi$ and $X = Z\sin\phi$

Power and Power Triangle

The average power consumed by the circuit is,

$$P_{av} = \text{Average power consumed by R+ Average power consumed by L}$$
$$+ \text{ Average power consumed by C}$$

But, pure L and C never consume any power.

$\therefore \quad P_{av} = \text{Power taken by R} = I^2 R = I(IR) = IV_R$

But, $\quad V_R = V\cos\phi$ in both the cases

$\therefore \quad P = VI\cos\phi$ W

Thus, for any condition, $X_L > X_C$ or $X_L < X_C$, the power can be expressed as,

P = Voltage × Component of current in phase with voltage

Key Point : *The power triangle can be obtained by multiplying each side of impedance triangle by* I^2.

The power triangles are shown in the Fig. 4.29.

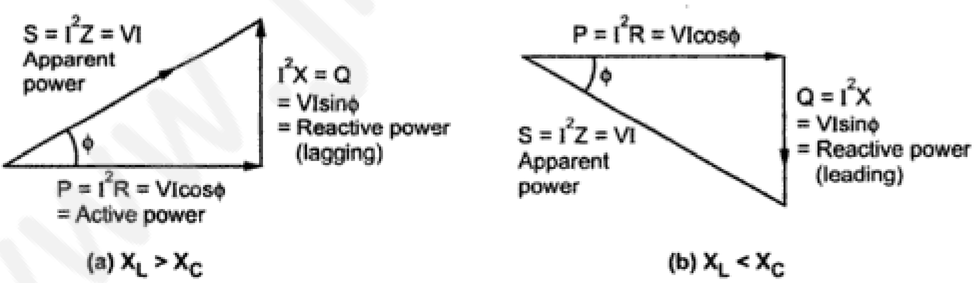


Fig. 4.29

Summary of R, L and C circuits

No.	Circuit	Impedance (Z)		ϕ	p.f. cos ϕ	Remark
		Polar	Rectangular			
1.	Pure R	$R \angle 0° \ \Omega$	$R + j0 \ \Omega$	0°	1	Unity p.f.
2.	Pure L	$X_L \angle 90° \ \Omega$	$0 + j X_L \ \Omega$	90°	0	Zero lagging
3.	Pure C	$X_C \angle -90° \ \Omega$	$0 - j X_C \ \Omega$	– 90°	0	Zero leading
4.	Series RL	$\lvert Z\rvert \angle +\phi° \ \Omega$	$R + j X_L \ \Omega$	$0° \angle \phi \angle 90°$	cos ϕ	Lagging
5.	Series RC	$\lvert Z\rvert \angle -\phi° \ \Omega$	$R - j X_C \ \Omega$	$-90° \angle \phi \angle 0°$	cos ϕ	Leading
6.	Series RLC	$\lvert Z\rvert \angle \pm \phi° \ \Omega$	$R + j X \ \Omega$ $X = X_L - X_C$	ϕ	cos ϕ	$X_L > X_C$ Lagging $X_L < X_C$ Leading $X_L = X_C$ Unity

Example 4.7 : *A series circuit consisting of 25 Ω resistor, 64 mH inductor and 80 μF capacitor, is connected to a 110 V, 50 Hz, single phase supply as shown in Fig. 4.30. Calculate the current, voltage across individual element and the overall p.f. of the circuit. Draw a neat phasor diagram showing* $\bar{I}$, $\bar{V}_R$, $\bar{V}_L$, $\bar{V}_C$ *and* $\bar{V}$.

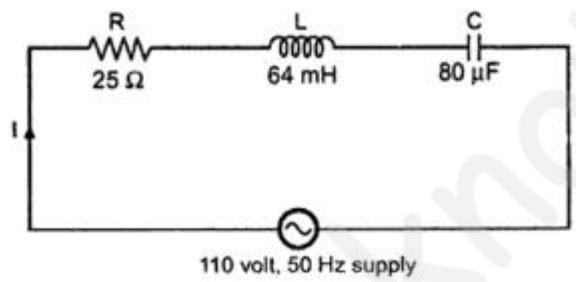


Fig. 4.30

Solution : From Fig. 4.30,

$$R = 25 \ \Omega$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 64 \times 10^{-3} = 20.10 \ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 80 \times 10^{-6}} = 39.78 \ \Omega$$

$$Z = R + jX_L - jX_C = 25 + j\,20.10 - j\,39.78$$

$$Z = (25 - j19.68) \ \Omega$$

$$I = \frac{V}{Z} = \frac{110 \angle 0°}{25 - j\,19.68} = \frac{110 \angle 0°}{31.81 \angle -38.20°}$$

$$I = \mathbf{3.4580 \angle 38.20° \ A}$$

$$\therefore \quad I = 3.4580 \ A$$

$$V_R = IR = (3.4580 \angle 38.20°)(25) = 86.45 \angle 38.20° \ \text{volts}$$

$$V_L = I(jX_L) = (3.4580 \angle 38.20°)\ (j20.10)$$

$$= (3.4580 \angle 38.20°)\ (20.10 \angle 90°) = \mathbf{69.50 \angle 128.2°} \ \text{volts}$$

$$V_C = I(-jX_C) = (3.4580 \angle 38.20°)\ (-j\,39.78)$$

$$= (3.4580 \angle 38.20°)\ (38.78 \angle -90°) = \mathbf{134.10 \angle -51.9°} \ \textbf{volts}$$

$$V = 110 \angle 0° \ \text{volts}$$

Overall p.f., $\cos\phi = \cos 38.20° = \mathbf{0.7858}$ **leading.**

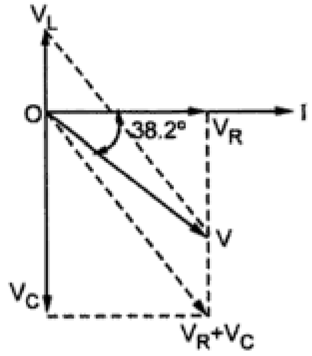


Fig. 4.30 (a)

A. C. Parallel Circuit

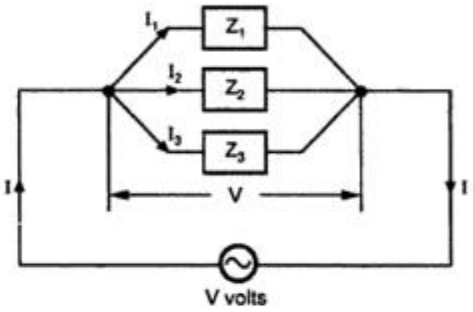


Fig. 4.33 A.C. parallel circuit

A parallel circuit is one in which two or more impedances are connected in parallel across the supply voltage. Each impedance may be a separate series circuit. Each impedance is called branch of the parallel circuit.

The Fig. 4.33 shows a parallel circuit consisting of three impedances connected in parallel across an a.c. supply of V volts.

Key Point : *The voltage across all the impedances is same as supply voltage of V volts.*

The current taken by each impedance is different.

Applying Kirchhoff's law, $\bar{I} = \bar{I_1} + \bar{I_2} + \bar{I_3}$ ... (phasor addition)

$$\therefore \quad \frac{\bar{V}}{\bar{Z}} = \frac{\bar{V}}{\bar{Z_1}} + \frac{\bar{V}}{\bar{Z_2}} + \frac{\bar{V}}{\bar{Z_3}}$$

$$\therefore \quad \frac{1}{\bar{Z}} = \frac{1}{\bar{Z_1}} + \frac{1}{\bar{Z_2}} + \frac{1}{\bar{Z_3}}$$

where Z is called **equivalent impedance.** This result is applicable for 'n' such impedances connected in parallel.

Following are the steps to solve parallel a.c. circuit :

1) The currents in the individual branches are to be calculated by using the relation

$$\bar{I_1} = \frac{\bar{V}}{\bar{Z_1}}, \quad \bar{I_2} = \frac{\bar{V}}{\bar{Z_2}}, \ldots\ldots, \quad \overline{I_n} = \frac{\bar{V}}{\bar{Z_n}}$$

while the individual phase angles can be calculated by the relation,

$$\tan\phi_1 = \frac{X_1}{R_1}, \ \tan\phi_2 = \frac{X_2}{R_2}, \ \ldots, \ \tan\phi_n = \frac{X_n}{R_n}$$

2) Voltage must be taken as reference phasor as it is common to all branches.
3) Represent all the currents on the phasor diagram and add them graphically or mathematically by expressing them in **rectangular form**. This is the resultant current drawn from the supply.
4) The phase angle of resultant current I is power factor angle. Cosine of this angle is the power factor of the circuit..

Concept of Admittance

Admittance is defined as the reciprocal of the impedance. It is denoted by Y and is measured in unit **siemens** or **mho.**

Now, current equation for the circuit shown in the Fig. 4.34 is,

$$\bar{I} = \bar{I_1} + \bar{I_2} + \bar{I_3}$$

$$\bar{I} = \bar{V} \times \left(\frac{1}{Z_1}\right) + \bar{V} \times \left(\frac{1}{Z_2}\right) + \bar{V} \times \left(\frac{1}{Z_3}\right)$$

$$\overline{VY} = \overline{VY_1} + \overline{VY_2} + \overline{VY_3}$$

$$\therefore \quad \bar{Y} = \bar{Y_1} + \bar{Y_2} + \bar{Y_3}$$

where Y is the **admittance of the total circuit. The three impedances connected in parallel can be replaced by an equivalent circuit, where three admittances are connected in series,** as shown in the Fig. 4.34.

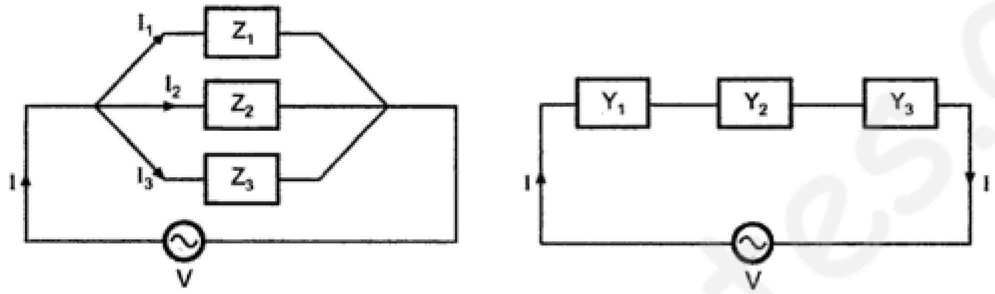


Fig. 4.34 Equivalent parallel circuit using admittances

Components of Admittance

Consider an impedance given as,

$$Z = R \pm j X$$

Positive sign for inductive and negative for capacitive circuit.

Admittance $\quad Y = \frac{1}{Z} = \frac{1}{R \pm j X}$

Rationalising the above expression,

$$Y = \frac{R \mp jX}{(R \pm jX)(R \mp jX)} = \frac{R \mp jX}{R^2 + X^2}$$

$$= \left(\frac{R}{R^2+X^2}\right) \mp j\left(\frac{X}{R^2+X^2}\right) = \frac{R}{Z^2} \mp j\frac{X}{Z^2}$$

$\therefore$ $\quad Y = G \mp j B$

In the above expression, $\quad G = \text{Conductance} = \frac{R}{Z^2}$

and $\quad B = \text{Susceptance} = \frac{X}{Z^2}$

Conductance (G)

It is defined as the ratio of the resistance to the square of the impedance. It is measured in the unit **siemens.**

Susceptance (B)

It is defined as the ratio of the reactance to the square of the impedance. It is measured in the unit **siemens.**

Example 4.9 : *Two impedance $Z_1 = 5 - j13.1\ \Omega$ and $Z_1 = 8.57 + j6.42\ \Omega$ are connected in parallel across a voltage of (100 + j200) volts.*

Estimate :-

i) branch currents in complex form ii) total power consumed,

Draw a neat phasor diagram showing voltage, branch currents and all phase angles.

Solution : The circuit is shown in the Fig. 4.36.

$$V = 100 + j\,200 = 223.607 \angle 63.43^\circ\ V$$

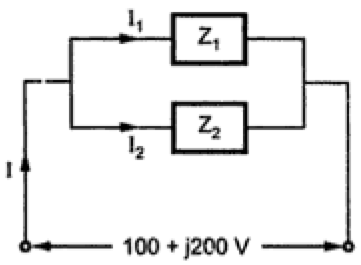


Fig. 4.36

$$Z_1 = 5 - j\,13.1 = 14.021 \angle -69.109^\circ\ \Omega$$

$$Z_2 = 8.57 + j\,6.42 = 10.71 \angle +36.83^\circ\ \Omega$$

i) $$I_1 = \frac{V}{Z_1} = \frac{223.607 \angle 63.43^\circ}{14.021 \angle 69.109^\circ}$$

$$= 15.948 \angle 132.539^\circ\ A$$

$$= \mathbf{-10.782 + j\,11.75\ A}$$

$$I_2 = \frac{V}{Z_2} = \frac{223.607 \angle 63.43^\circ}{10.71 \angle +36.83^\circ}$$

$$= 20.878 \angle 26.6^\circ\ A = \mathbf{18.668 + j\,9.3483\ A}$$

$\therefore$ $$I_T = \bar{I}_1 + \bar{I}_2 = -10.782 + j\,11.75 + 18.668 + j\,9.3483$$

$$= 7.886 + j\,21.0983\ A = \mathbf{22.5239} \angle 69.5^\circ\ A$$

$\therefore$ ϕ_T = Angle between V and I_T

$$= 69.5 - 63.43 = \mathbf{6.075^\circ}\ \text{leading}$$

$\therefore$ $$P_T = V\,I_T \cos\phi_T = 223.607 \times 22.5239 \times \cos(6.075)$$

$$= \mathbf{5008.212\ W}$$

The phasor diagram is shown in the Fig. 4.37.

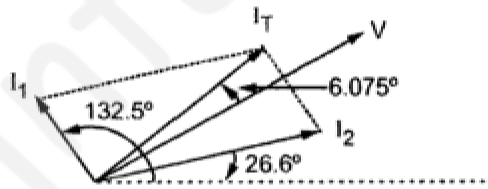


Fig. 4.37

Example 4.11 : *Find the current through 4 Ω resistor by using loop current method.*

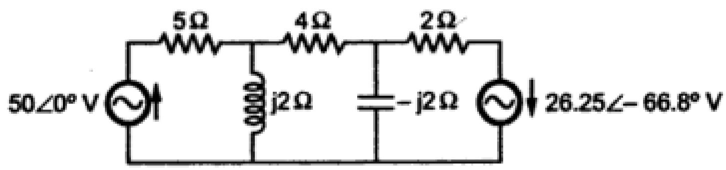


Fig. 4.39

Solution : The various loop currents are shown in the Fig. 4.39 (a),

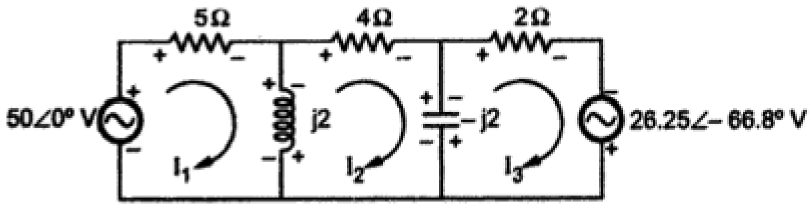


Fig. 4.39 (a)

Loop 1, $-5I_1 - j2I_1 + j2I_2 + 50\angle 0^\circ = 0$

$\therefore$ $I_1(5+j2) - I_2\, j2 = 50\angle 0^\circ$ ... (1)

Loop 2, $-4I_2 - I_2(-j2) + I_3(-j2) - j2I_2 + j2I_1 = 0$

$\therefore$ $I_1(j2) + I_2(-4) - I_3(j2) = 0$ ... (2)

Loop 3, $-2I_3 + 26.25\angle -66.8^\circ - I_3(-j2) + I_2(-j2) = 0$

$\therefore$ $I_2(j2) + I_3(2-j2) = 26.25\angle -66.8^\circ$ ... (3)

$$\therefore \quad D = \begin{vmatrix} 5+j2 & -j2 & 0 \\ j2 & -4 & -j2 \\ 0 & j2 & 2-j2 \end{vmatrix}$$

$$= -4(2-j2)(5+j2) - 4(2-j2) - 4(5+j2)$$

$$= -84 + j24$$

and
$$D_2 = \begin{vmatrix} 5+j2 & 50\angle 0^\circ & 0 \\ j2 & 0 & -j2 \\ 0 & 26.25\angle -66.8^\circ & 2-j2 \end{vmatrix}$$

$$= -j2\,(2-j2)\,50\angle 0^\circ + j2\,(5+j2)\,(26.25\angle -66.8^\circ)$$

$$= [2\angle -90^\circ \times 2.828\angle -45^\circ \times 50\angle 0^\circ] + [2\angle 90^\circ \times 5.385$$
$$\angle 21.8^\circ \times 26.25\angle -66.8^\circ]$$

$$= -199.969 - j199.969 + 199.9 + j199.9 = 0$$

$$\therefore \quad I_2 = \frac{D_2}{D} = 0 \text{ A} = \text{current through } 4\ \Omega$$

Example 4.12 : *Use the nodal analysis to find the value of V_x in the circuit shown in the Fig. 4.40 such that the current through (2 + j3) Ω impedance is zero.*

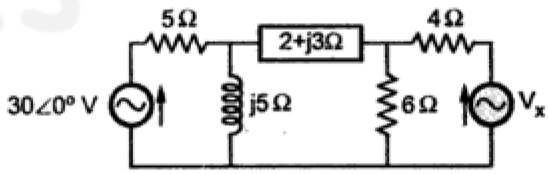


Fig. 4.40

Solution : The various node voltages and currents are shown in the Fig. 4.40 (a).

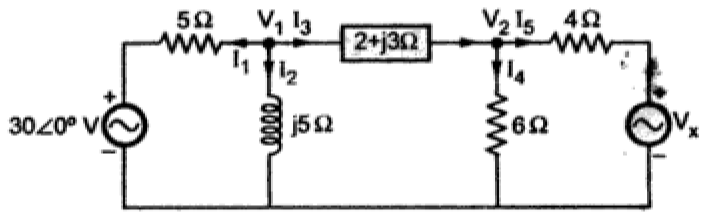


Fig. 4.40 (a)

At node 1, $-I_1 - I_2 - I_3 = 0$

$$\therefore \quad -\left[\frac{V_1 - 30\angle 0^\circ}{5}\right] - \left[\frac{V_1}{j5}\right] - \left[\frac{V_1 - V_2}{2+j3}\right] = 0$$

But current through (2 + j3) Ω i.e. $I_3 = 0$

$$\therefore \quad -\left[\frac{V_1 - 30\angle 0^\circ}{5}\right] - \frac{V_1}{j5} = 0$$

$$\therefore \quad \frac{-V_1}{j5} + \frac{30\angle 0^\circ}{5} - \frac{V_1}{5\angle 90^\circ} = 0$$

$$\therefore \quad V_1\,[0.2 + 0.2\angle -90^\circ] = 6\angle -0^\circ$$

$$\therefore \quad V_1 = \frac{6\angle 0^\circ}{0.2 - j0.2} = \frac{6\angle 0^\circ}{0.2828\angle -45^\circ}$$

$$= 21.216\angle +45^\circ \text{ V}$$

At node 2, $I_3 - I_4 - I_5 = 0$

i.e. $I_4 + I_5 = 0$ ... as $I_3 = 0$

$$\therefore \quad \frac{V_2}{6} + \frac{V_2 - V_x}{4} = 0$$

$$\therefore \quad V_2\left[\frac{1}{6} + \frac{1}{4}\right] - \frac{V_x}{4} = 0$$

$$\therefore \quad 0.4166\,V_2 - 0.25\,V_x = 0$$

$$\therefore \quad V_x = 1.667\,V_2 \qquad \text{... (1)}$$

➡ **Example 4.13 :** *Use the node voltage technique to obtain the current I in the network shown in the Fig. 4.41.*

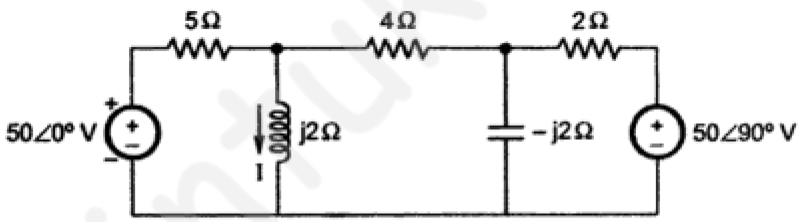


Fig. 4.41

As $I_3 = 0$, V_1 and V_2 must be equal.

$$\therefore \quad V_2 = V_1 = \mathbf{21.2\angle 45^\circ\ V} \qquad \text{...(2)}$$

$$\therefore \quad V_x = 1.667 \times 21.2\angle +45^\circ = \mathbf{35.33\angle 45^\circ\ V}$$

Solution : The various currents and node voltages are shown in the Fig. 4.41 (a).

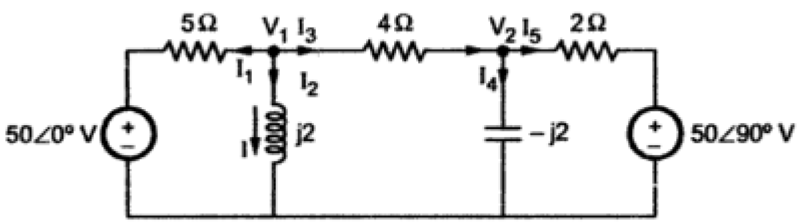


Fig. 4.41 (a)

At node 1, $-I_1 - I_2 - I_3 = 0$ i.e. $I_1 + I_2 + I_3 = 0$

$$\therefore \quad \frac{V_1 - 50\angle 0^\circ}{5} + \frac{V_1}{j2} + \frac{V_1 - V_2}{4} = 0$$

$\therefore \quad V_1\left[\frac{1}{5}-j\frac{1}{2}+\frac{1}{4}\right]-V_2\left[\frac{1}{4}\right] = 10 \quad \ldots \frac{1}{j}=-j$

$\therefore \quad V_1\,[0.45 - j0.5] - V_2\,(0.25) = 10 \quad \ldots (1)$

At node 2, $\quad I_3 - I_4 - I_5 = 0$

$\therefore \quad \frac{V_1 - V_2}{4} - \frac{V_2}{-j2} - \frac{V_2 - 50\angle 90^\circ}{2} = 0$

$\therefore \quad V_1\left[\frac{1}{4}\right]+V_2\left[-\frac{1}{4}-j\frac{1}{2}-\frac{1}{2}\right] = -25\angle 90^\circ \quad \ldots \frac{1}{j}=-j$

$\therefore \quad 0.25\,V_1 + V_2\,[-0.75 - j\,0.5] = -j25 \quad \ldots (2)$

For calculating $I = I_2$, only V_1 is required.

$$\therefore \quad D = \begin{vmatrix} 0.45-j0.5 & -0.25 \\ 0.25 & -0.75-j0.5 \end{vmatrix}$$

$$= -0.3375 + j\,0.375 - j\,0.225 - 0.25 + 0.0625$$

$$= -0.525 + j0.15 = 0.546\angle 164.05^\circ$$

$$D_1 = \begin{vmatrix} 10 & -0.25 \\ -j25 & -0.75-j0.5 \end{vmatrix} = -7.5 - j5 - j6.25$$

$$= -7.5 - j\,11.25 = 13.52\angle -123.69^\circ$$

$$\therefore \quad V_1 = \frac{D_1}{D} = \frac{13.52\angle -123.69^\circ}{0.546\angle 164.05^\circ} = \mathbf{24.762\angle 40.36^\circ\ V}$$

$$\therefore \quad I = \frac{V_1}{j2} = \frac{24.762\angle 40.36^\circ}{2\angle 90^\circ} = \mathbf{12.381\angle -49.64^\circ\ A}$$