

TWO-PORT NETWORKS

Introduction

A pair of terminals at which an electrical signal may enter or leave a network is called a port. The terminals or port is required for connecting input excitation to the network. It is also required for connecting some other networks such as load. The terminals are most useful for making measurements. In general, the minimum number of terminals required is two.

A network having only one pair of terminals or one port is called one port network. The Fig. 7.1 (a) shows a one port network.

A network consisting two pairs of terminals is called two port network as shown in the Fig. 7.1 (b). The terminals are generally named as 1-1' and 2-2' In general, a port designated 1-1', is connected to the driving energy source while the other port designated 2-2' is connected to the load. A port at which energy source is connected is called driving point of the network or input port. A port at which load is connected is called output port.

Fig. 7.1 (c) represents a network with n-port called n-port network. In such networks, generally one port is connected to energy source, one port is connected to load and other ports may be connected to the different networks.

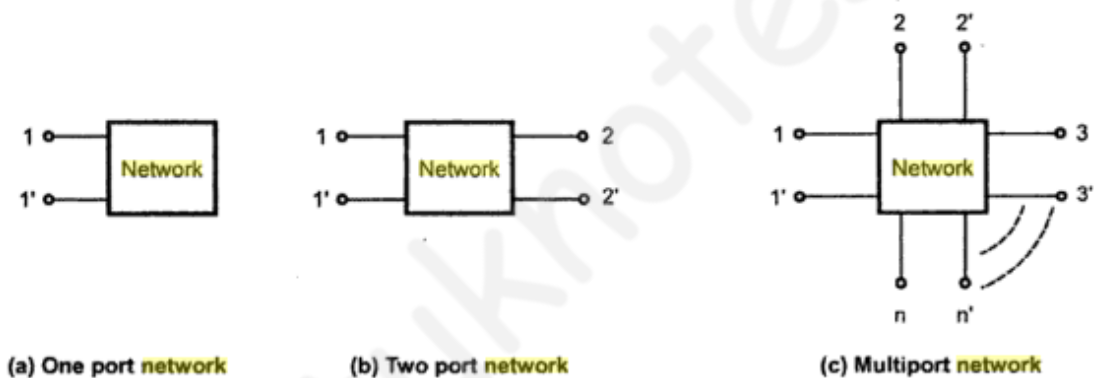


Fig. 7.1

Two Port Network Parameters

Consider a two port network as shown in the Fig. 7.2. In all there are four variables; two voltages and two currents. In general, any two port network has one pair of voltage and current at each port.

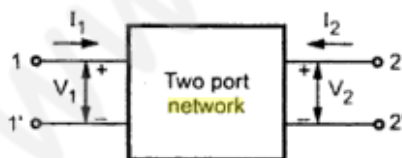


Fig. 7.2 Two port network

Assuming the variables at input and output ports as transformed quantities, the voltage and current at input terminals are V_1 and I_1 while at output terminals are V_2 and I_2 , as shown in the Fig. 7.2. The directions of both the currents I_1 and I_2 are assumed to be flowing into the network. Such currents entering the network are assumed to be positive. Also the voltage have the positive. Also the voltages have the positive reference polarities.

Before starting the discussion on the two port network parameters, let us make certain assumptions necessary in the analysis. The assumptions are as follows.

1. The voltages and currents in the network present inside a box are not available for the measurements.
2. The network should consists only linear elements along with dependent sources if any. But independent or active sources should not be present in the network inside box.

3. If the **network** inside box consists energy storing elements such as inductor and capacitor, then **initial conditions** of such elements should be zero.

In order to describe the relationship between port voltages and currents, one requires the linear equations equal to the number of ports. So in two port **network analysis**, we will require two linear equations interms of four above mentioned variables. We can obtain these equations by considering two variables as dependent variables while other as independent variables. As the **network** consists only linear elements, the linear relationship can be obtained by writing two variables interms of other two variables. There are six possible ways of selecting two independent variables out of four variables. Thus there are six different pairs of equations defining their own sets of parameters such as impedance (z), admittance (y), hybrid (h), inverse hybrid (g), transmission and inverse transmission parameters. In next sections, we will discuss z-parameters, y-parameters, hybrid parameters and transmission parameters in detail.

z-Parameters

These are also called **impedance parameters**. These are obtained by expressing voltages at two ports in terms of currents at two ports. Thus, **currents I_1 and I_2 are independent variables**; while **V_1 and V_2 are dependent variables**.

$$V_1 = f_1(I_1, I_2)$$

$$V_2 = f_2(I_1, I_2)$$

In equation form, above relations can be written as,

$$V_1 = z_{11} I_1 + z_{12} I_2 \quad \dots (1)$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \quad \dots (2)$$

In matrix form, above equations can be written as,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

or $[V] = [z] [I]$

The individual z-parameters can be obtained by assigning values of independent variables to be zero. The z-parameters can be defined as follows.

[A] Let $I_2 = 0$; port - 2 is open circuited.

From equation (1),

$$z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2 = 0} \quad \Omega$$

The parameter z_{11} is called **open circuit driving point input impedance**.

From equation (2),

$$z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2 = 0} \quad \Omega$$

The parameters z_{21} is called **open circuit forward transfer impedance**.

[B] Let $I_1 = 0$; i.e. **port - 1 is open circuited**.

Form equation (1),

$$z_{12} = \frac{V_1}{I_2} \Big|_{I_1 = 0} \Omega$$

The parameter z_{12} is called **open circuit reverse transfer impedance**.

From equation (2),

$$z_{22} = \frac{V_2}{I_2} \Big|_{I_1 = 0} \Omega$$

The parameter z_{22} is called **open circuit driving point output impedance**.

these parameters are defined only when the current in one of the ports is zero. This corresponds to the conditions that the one of the ports is open circuited. Hence z-parameters are named as **open circuit impedance parameters**.

►► **Example 7.1 :** Find the z-parameters for the **network** shown in following Fig. 7.4.

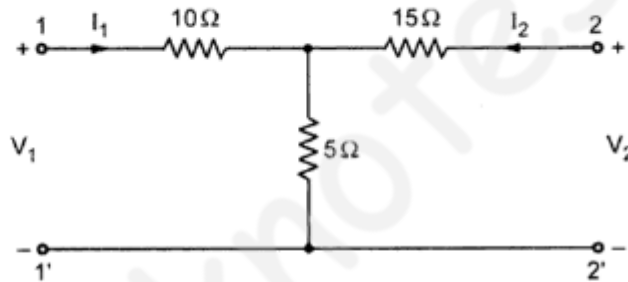


Fig. 7.4

Solution : By definition z-parameters are given as,

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

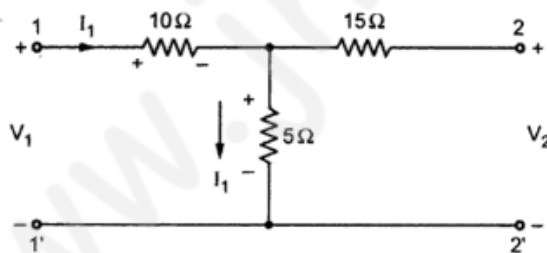


Fig. 7.4 (a)

(A) Let $I_2 = 0$, i.e. **port 2 is open circuited** as shown in the Fig. 7.4 (a).

As port 2 is open circuited, the current flowing through 5Ω is also I_1 . Note that no current will flow through 15Ω as it is connected to open terminals.

Applying KVL at input side, we get,

$$- 10 I_1 - 15 I_1 + V_1 = 0$$

$$\therefore V_1 = 15 I_1$$

$$\therefore z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0} = 15 \Omega$$

From the Fig. 7.4 (a), we can write,

$$V_2 = 5 I_1$$

$$\therefore z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0} = 5 \Omega$$

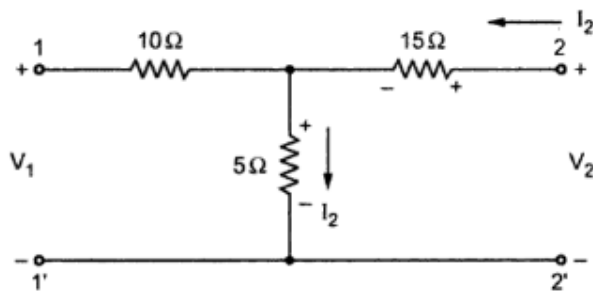


Fig. 7.4 (b)

$$\therefore V_2 = 20 I_2$$

$$\begin{aligned} \therefore z_{22} &= \left. \frac{V_2}{I_2} \right|_{I_1=0} \\ &= 20 \Omega \end{aligned}$$

From the Fig. 7.4 (b), we can write,

$$V_1 = 5 I_2$$

$$\therefore z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = 5 \Omega$$

Hence z-parameters of the given network are

$$[z] = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ 5 & 20 \end{bmatrix} \Omega$$

y-Parameters

These are also called **admittance parameters**. These are obtained by expressing currents at two ports in terms of voltages at two ports. Thus, voltages V_1 and V_2 are independent variables, while I_1 and I_2 are dependent variables. Thus, we have

$$I_1 = f_1(V_1, V_2)$$

$$I_2 = f_2(V_1, V_2)$$

In equation form, above relations can be written as,

$$I_1 = y_{11} V_1 + y_{12} V_2 \quad \dots (1)$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \quad \dots (2)$$

In matrix form, above equations can be written as,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

or $[I] = [y] [V]$

The individual y-parameters are defined as follows,

[A] Let $V_2 = 0$ i.e. port-2 is short circuited.

From equation (1)

$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} \quad \text{U}$$

The parameter y_{11} is called **short circuit driving point input admittance**.

(B) Let $I_1 = 0$, i.e. port 1 is open circuited as shown in the Fig. 7.4 (b).

As port 1 is open circuited, the current flowing through 5Ω is also I_2 . Note that no current will flow through 10Ω as it is connected to open terminals.

Applying KVL at output side, we get,

$$-15 I_2 - 5 I_2 + V_2 = 0$$

From equation (2)

$$y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0}$$

The parameter y_{21} is called **short circuit forward transfer admittance**.

[B] Let $V_1 = 0$ i.e. **port-1 is short circuited**.

From equation (1),

$$y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0}$$

The parameter y_{12} is called **short circuit reverse transfer admittance**.

From equation (2),

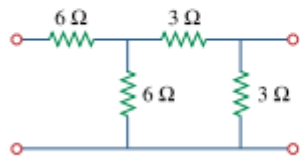
$$y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0}$$

The parameter y_{22} is called **short circuit driving point output admittance**.

These parameters are defined individually only when the voltage in any one of the ports is zero. This corresponds to the condition that one of the ports is short circuited. Hence y-parameters are also called **short circuit admittance parameters**.

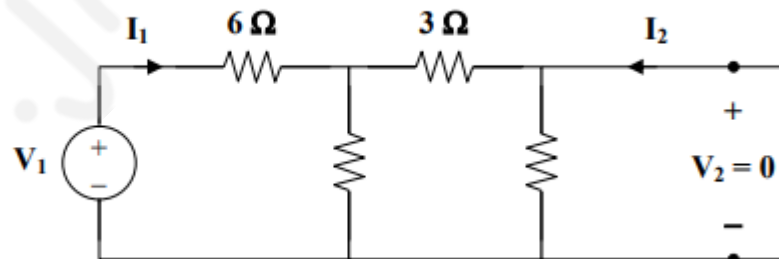
Problem

Calculate the y parameters for the two-port in Fig.



Figure

To get y_{11} and y_{21} , consider the circuit in Fig.(a).



(a)

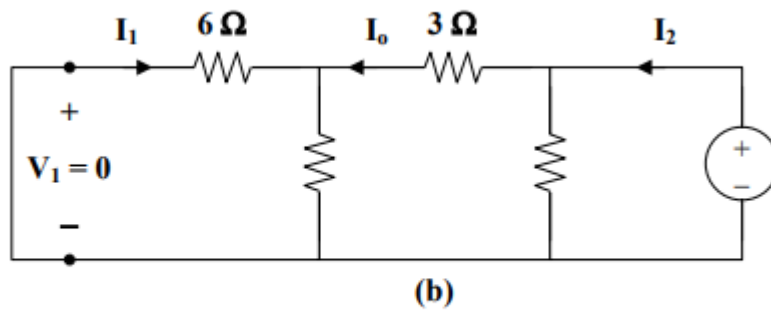
$$V_1 = (6 + 6 \parallel 3) I_1 = 8 I_1$$

$$y_{11} = \frac{I_1}{V_1} = \frac{1}{8}$$

$$I_2 = \frac{-6}{6+3} I_1 = \frac{-2}{3} \frac{V_1}{8} = \frac{-V_1}{12}$$

$$y_{21} = \frac{I_2}{V_1} = \frac{-1}{12}$$

To get y_{22} and y_{12} , consider the circuit in Fig.(b).



$$y_{22} = \frac{I_2}{V_2} = \frac{1}{3 \parallel (3 + 6 \parallel 6)} = \frac{1}{3 \parallel 6} = \frac{1}{2}$$

$$I_1 = \frac{-I_0}{2}, \quad I_0 = \frac{3}{3+6} I_2 = \frac{1}{3} I_2$$

$$I_1 = \frac{-I_2}{6} = \left(\frac{-1}{6} \right) \left(\frac{1}{2} V_2 \right) = \frac{-V_2}{12}$$

$$y_{12} = \frac{I_1}{V_2} = \frac{-1}{12} = y_{21}$$

Thus,

$$[y] = \begin{bmatrix} \frac{1}{8} & \frac{-1}{12} \\ \frac{-1}{12} & \frac{1}{2} \end{bmatrix} \text{S}$$

h-Parameters

These are also called **hybrid parameters**. These parameters are very useful in constructing models for transistors. The transistor parameters cannot be calculated using by either short circuit admittance parameter or open circuit impedance parameter measurement. These parameters are obtained by expressing voltage at input port and the current at output port in terms of the current at the input port and the voltage at the output port. Thus, the current I_1 and voltage V_2 are independent variables; while current I_2 and voltage V_1 are the dependent variables. Thus, we have,

$$V_1 = f_1(I_1, V_2)$$

$$I_2 = f_2(I_1, V_2)$$

In equation form, above relations can be written as,

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \dots (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \dots (2)$$

In matrix form, the above equations can be written as,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

The individual h-parameters can be defined as follows

[A] Let $V_2 = 0$; i.e. port - 2 is short circuited.

From equation (1),

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2 = 0} \quad \Omega$$

The parameter h_{11} is called **short circuit input impedance**.

From equation (2),

$$h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2 = 0}$$

The parameter h_{21} is called **short circuit forward current gain**. It is unitless.

[B] Let $I_1 = 0$; i.e. **port - 1 is circuited**.

From equation (1),

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1 = 0}$$

The parameter h_{12} is called **open circuit reverse voltage gain**. It is also unitless.

From equations (2),

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1 = 0} \quad \text{O}$$

The parameter h_{22} is called **open circuit output admittance**.

All above parameters are having different units such as ohm for short circuit impedance and mho for open circuit output admittance, the name of the parameter is **hybrid parameter**.

Problems-

Determine the h-parameter with the following data:

- i. with the output terminals short circuited, $V_1 = 25 \text{ V}$, $I_1 = 1 \text{ A}$, $I_2 = 2 \text{ A}$
- ii. with the input terminals open circuited, $V_1 = 10 \text{ V}$, $V_2 = 50 \text{ V}$, $I_2 = 2 \text{ A}$

Solution

The h-parameter equations are,

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

- a. With output short-circuited, $V_2 = 0$, given: $V_1 = 25 \text{ V}$, $I_1 = 1 \text{ A}$ and $I_2 = 2 \text{ A}$.

$$\begin{aligned} \therefore & \quad 25 = h_{11} \times 1 \\ \text{and} & \quad 2 = h_{21} \times 1 \end{aligned} \quad \Rightarrow \quad h_{11} = 25 \, \Omega, \text{ and } h_{21} = 2$$

- b. With input open-circuited, $I_1 = 0$, given: $V_1 = 10 \text{ V}$, $V_2 = 50 \text{ V}$ and $I_2 = 2 \text{ A}$.

$$\begin{aligned} \therefore & \quad 10 = h_{12} \times 50 \\ \text{and} & \quad 2 = h_{22} \times 50 \end{aligned} \quad \Rightarrow \quad h_{12} = \frac{1}{5} = 0.2 \text{ and } h_{22} = \frac{2}{50} = 0.04 \, \text{O} = 0.04 \, \text{O}$$

Thus, the h-parameters are:

$$[h] = \begin{bmatrix} 25 \, \Omega & 0.2 \\ 2 & 0.04 \, \Omega^{-1} \end{bmatrix}$$

ABCD Parameters or Transmission Parameters or Chain Parameters

These parameters are known as transmission parameters. These are generally used in the **analysis** of power transmission in which the input port is referred as the sending end while the output port is referred as receiving end. These are obtained by expressing voltage V_1 and current I_1 at input port in terms of voltage V_2 and current I_2 at output port. Thus, **voltage V_2 and current I_2 are independent variables** while **voltage V_1 and current I_1 are dependent variables**. Thus, we have,

$$V_1 = f_1(V_2, -I_2)$$

$$I_1 = f_2(V_2, -I_2)$$

Generally we have considered the currents in both the ports are entering the port and both are positive. The negative sign with I_2 indicates that, for the ABCD parameters the current I_2 is leaving the port-2.

In equation form, above relations can be written as,

$$V_1 = A V_2 + B (-I_2) \quad \dots (1)$$

$$I_1 = C V_2 + D (-I_2) \quad \dots (2)$$

In matrix form, above equations can be written as,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

The individual ABCD parameters can be defined as follows.

[A] Let $-I_2 = 0$, i.e. **port - 2 is circuited**.

From equation (1),

$$A = \left. \frac{V_1}{V_2} \right|_{-I_2 = 0}$$

The parameter A is called **open circuit reverse voltage gain**. It is unitless.

From equation (2),

$$C = \left. \frac{I_1}{V_2} \right|_{-I_2 = 0} \quad \text{U}$$

The parameter C is called **open circuit reverse transfer admittance**.

[B] Let $V_2 = 0$, i.e. **port - 2 is short circuited**.

From equation (1),

$$B = \left. \frac{V_1}{-I_2} \right|_{V_2 = 0} \quad \Omega$$

The parameter B is called **short circuit reverse transfer impedance**.

From equation (2),

$$D = \left. \frac{I_1}{-I_2} \right|_{V_2 = 0}$$

The parameter D is called **short circuit reverse current gain**. It is also unitless.

ABCD or Transmission parameters are also called **chain parameters**.

These parameter are effectively used for the **analysis** of power transmission line, so commonly known as **transmission parameters**.

►►► **Example** : Find the transmission or general parameters for the circuit shown in Fig. 4.10.

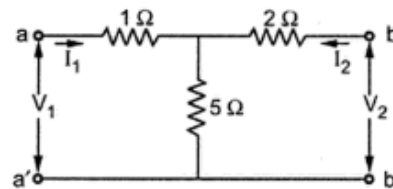


Fig. 4.10

Solution : By definition, transmission parameters are given by,

$$V_1 = A V_2 + B(-I_2)$$

$$I_1 = C V_2 + D(-I_2)$$

A) Let $-I_2 = 0$ i.e. open circuit terminals $b - b'$ (i.e. port-2) as shown in the Fig. 4.10 (a).

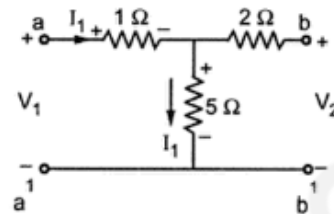


Fig. 4.10 (a)

From circuit drawn above, we can write,

$$V_2 = 5 I_1 \quad \dots (i)$$

$$\therefore C = \left. \frac{I_1}{V_2} \right|_{-I_2=0} = \frac{1}{5} \text{ U}$$

Applying KVL at the input side, we get,

$$-I_1 - 5 I_1 + V_1 = 0$$

$$\therefore 6 I_1 = V_1 \quad \dots (ii)$$

From equation (i),

$$I_1 = \frac{V_2}{5} \quad \dots (iii)$$

Substituting value of I_1 in equation (ii), we get,

$$\therefore 6 \left(\frac{V_2}{5} \right) = V_1$$

$$\therefore V_2 = \frac{5}{6} V_1$$

$$\therefore A = \left. \frac{V_1}{V_2} \right|_{-I_2=0} = \frac{6}{5}$$

B) Let $V_2 = 0$ i.e. short circuit port-2 (terminals $b - b'$) as shown in the Fig. 4.10 (b).

Applying current divider rule, we get,

$$I_2 = -I_1 \left[\frac{5}{2+5} \right]$$

$$\therefore -I_2 = I_1 \left(\frac{5}{7} \right)$$

$$\therefore D = \left. \frac{I_1}{-I_2} \right|_{V_2=0} = \frac{7}{5}$$

Applying KVL at the input side, we get,

$$-I_1 - 5(I_1 + I_2) + V_1 = 0$$

$$\therefore -6 I_1 - 5 I_2 = -V_1$$

$$\therefore 6 I_1 + 5 I_2 = V_1$$

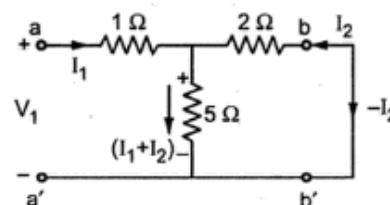


Fig. 4.10 (b)

$$\therefore 6\left(\frac{-7}{5}I_2\right) + 5I_2 = V_1$$

$$\therefore \left(\frac{-42}{5} + 5\right)I_2 = V_1$$

$$\therefore \frac{-17}{5}I_2 = V_1$$

$$\therefore \left(\frac{17}{5}\right)(-I_2) = V_1$$

$$\therefore B = \frac{V_1}{-I_2}\bigg|_{V_2=0} = +\frac{17}{5} \Omega$$

Hence transmission parameters matrix is given by,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{17}{5} \\ \frac{1}{5} & \frac{7}{5} \end{bmatrix}$$

INVERSE HYBRID (OR g) PARAMETERS

If V_1 and I_2 are chosen as independent variables, the two-port network equations may be written as

$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

In matrix form, these equations are written as

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

The constants g_{11} , g_{12} , g_{21} , and g_{22} are known as inverse hybrid parameters or g-parameters. The g-parameters are defined as follows by using Equations

If $I_2 = 0$ the output port is open circuit.

$$g_{11} = \frac{I_1}{V_1} \bigg|_{I_2=0} \text{ open circuit input admittance.}$$

$$g_{21} = \frac{V_2}{V_1} \bigg|_{I_2=0} \text{ open circuit forward voltage gain.}$$

If

$V_1 = 0$ the input port is short circuit.

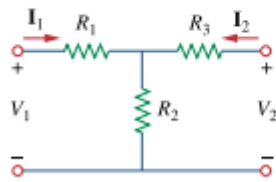
$$g_{12} = \frac{I_1}{I_2} \bigg|_{V_1=0} \text{ short circuit reverse current gain.}$$

$$g_{22} = \frac{V_2}{I_2} \bigg|_{V_1=0} \text{ short circuit output impedance.}$$

From the definitions of the g-parameters, it is seen that g_{11} has the dimensions of admittance, g_{21} and g_{12} are dimensionless, and g_{22} has the dimensions of impedance.

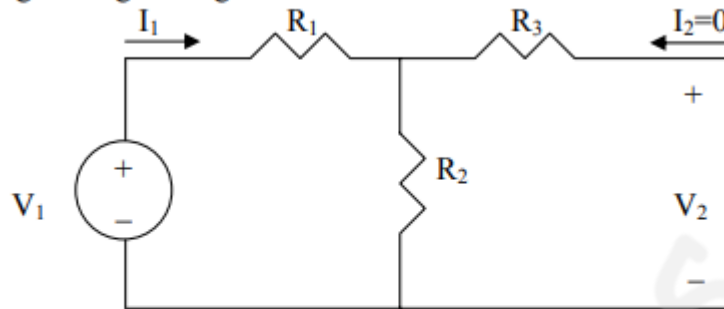
Problem

Obtain the g parameters for the circuit of Fig.



Figure

We obtain g_{11} and g_{21} using the circuit below.

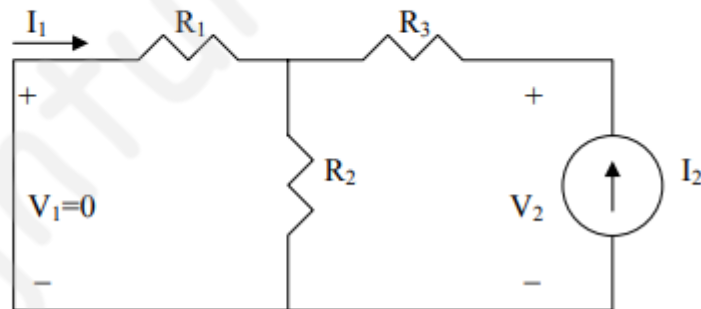


$$I_1 = \frac{V_1}{R_1 + R_2} \longrightarrow g_{11} = \frac{I_1}{V_1} = \frac{1}{R_1 + R_2}$$

By voltage division,

$$V_2 = \frac{R_2}{R_1 + R_2} V_1 \longrightarrow g_{21} = \frac{V_2}{V_1} = \frac{R_2}{R_1 + R_2}$$

We obtain g_{12} and g_{22} using the circuit below.



By current division,

$$I_1 = -\frac{R_2}{R_1 + R_2} I_2 \longrightarrow g_{12} = \frac{I_1}{I_2} = -\frac{R_2}{R_1 + R_2}$$

Also,

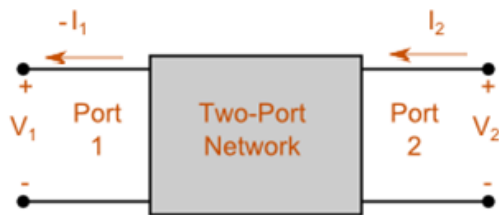
$$V_2 = I_2 (R_3 + R_1 \parallel R_2) = I_2 \left(R_3 + \frac{R_1 R_2}{R_1 + R_2} \right) \quad g_{22} = \frac{V_2}{I_2} = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

$$g_{11} = \frac{1}{R_1 + R_2}, g_{12} = -\frac{R_2}{R_1 + R_2}$$

$$g_{21} = \frac{R_2}{R_1 + R_2}, g_{22} = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

The transmission parameter and the inverse transmission parameter are duals of each other.

If, instead of quantities V_1 and I_1 , quantities V_2 and I_2 are expressed in terms of V_1 and I_1 , the resulting parameter (A', B', C', D') are called inverse transmission parameter.



The inverse transmission parameters of the two port network in figure having direction of voltages and current as shown, are given by

$$V_2 = A'V_1 + B'(-I_1)$$

$$I_2 = C'V_1 + D'(-I_1)$$

in matrix form,

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

The inverse transmission parameters can be defined as

$A' = \frac{V_2}{V_1}; I_1 = 0$ forward voltage ratio with sending end open circuited.

$C' = \frac{I_2}{V_1}; I_1 = 0$ transfer admittance with sending end open circuited.

$B' = \frac{V_2}{-I_1}; V_1 = 0$ transfer impedance with sending end short circuited.

$D' = \frac{I_2}{-I_1}; V_1 = 0$ forward current ratio with sending end short circuited.

Name of the parameters	Port variables		Equations
	Express (Dependent)	Intems of (Independent)	
Open circuit impedance i.e. z-parameters	V_1, V_2	I_1, I_2	$V_1 = z_{11} I_1 + z_{12} I_2$ $V_2 = z_{21} I_1 + z_{22} I_2$
Short circuit admittance i.e. y-parameters	I_1, I_2	V_1, V_2	$I_1 = y_{11} V_1 + y_{12} V_2$ $I_2 = y_{21} V_1 + y_{22} V_2$
h-parameters	V_1, I_2	I_1, V_2	$V_1 = h_{11} I_1 + h_{12} V_2$ $I_2 = h_{21} I_1 + h_{22} V_2$
ABCD or transmission parameters	V_1, I_1	$V_2, -I_2$	$V_1 = AV_2 + B(-I_2)$ $I_1 = CV_2 + D(-I_2)$
Inverse hybrid or g-parameters	I_1, V_2	V_1, I_2	$I_1 = g_{11} V_1 + g_{12} I_2$ $V_2 = g_{21} V_1 + g_{22} I_2$
Inverse transmission parameters	V_2, I_2	$V_1, -I_1$	$V_2 = A' V_1 + B'(-I_1)$ $I_2 = C' V_1 + D'(-I_1)$

Table 4.1 Summary of two port network parameters

Interrelationships between the Parameters

During **analysis** of a two port **network** it is often required to obtain more than one set of parameters for a given two port **network**. Then it becomes time consuming and tedious to obtain different sets of parameter by using their respective basic definitions. Hence to make the **analysis** easier, any set of parameters is expressed interms of all the remaining sets of parameters. In other words, parameters are derived from each other. In this section, we will derive such interrelationships between the parameters.

z-Parameters Interm of other Parameters

The equations for z-parameters are as follows,

$$V_1 = z_{11} I_1 + z_{12} I_2 \quad \dots (A)$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \quad \dots (B)$$

[A] Interm of y-Parameters

The equations for y-parameters are as follows,

$$I_1 = y_{11} V_1 + y_{12} V_2 \quad \dots (1)$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \quad \dots (2)$$

Writing equations in matrix form, we have,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Solving above equations, using Cramer's rule for V_1 and V_2 . We can write,

$$\begin{aligned} V_1 &= \frac{\begin{vmatrix} I_1 & y_{12} \\ I_2 & y_{22} \end{vmatrix}}{\begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}} = \frac{y_{22} I_1 - y_{12} I_2}{y_{11} y_{22} - y_{12} y_{21}} \\ &= \frac{y_{22}}{y_{11} y_{22} - y_{12} y_{21}} I_1 - \frac{y_{12}}{y_{11} y_{22} - y_{12} y_{21}} I_2 \quad \dots (3) \end{aligned}$$

Similarly,

$$\begin{aligned} V_2 &= \frac{\begin{vmatrix} y_{11} & I_1 \\ y_{21} & I_2 \end{vmatrix}}{\begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}} = \frac{y_{11} I_2 - y_{21} I_1}{y_{11} y_{22} - y_{12} y_{21}} \\ &= \frac{-y_{21}}{y_{11} y_{22} - y_{12} y_{21}} I_1 + \frac{y_{11}}{y_{11} y_{22} - y_{12} y_{21}} I_2 \quad \dots (4) \end{aligned}$$

Let $y_{11} y_{22} - y_{12} y_{21} = \Delta y$

Rewriting equations (3) and (4), we have,

$$V_1 = \left[\frac{y_{22}}{\Delta y} \right] I_1 + \left[\frac{-y_{12}}{\Delta y} \right] I_2 \quad \dots (5)$$

and $V_2 = \left[\frac{-y_{21}}{\Delta y} \right] I_1 + \left[\frac{y_{11}}{\Delta y} \right] I_2 \quad \dots (6)$

Comparing equations (5) and (6) to equations (A) and (B), we have,

$$z_{11} = \frac{y_{22}}{\Delta y} \quad z_{12} = \frac{-y_{12}}{\Delta y}$$

$$z_{21} = \frac{-y_{21}}{\Delta y} \quad z_{22} = \frac{y_{11}}{\Delta y}$$

[B] Inters of h-Parameters

The equations for h-parameters are as follows,

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \dots (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \dots (2)$$

From equation (2) we can write,

$$h_{22} V_2 = -h_{21} I_1 + I_2$$

$$\therefore V_2 = \left[\frac{-h_{21}}{h_{22}} \right] I_1 + \left[\frac{1}{h_{22}} \right] I_2 \quad \dots (3)$$

Substituting value of V_2 in equation (1) we have,

$$V_1 = h_{11} I_1 + h_{12} \left[\frac{-h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \right]$$

$$\therefore V_1 = \left[h_{11} - \frac{h_{12} h_{21}}{h_{22}} \right] I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$\therefore V_1 = \left[\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} \right] I_1 + \left[\frac{h_{12}}{h_{22}} \right] I_2 \quad \dots (4)$$

Comparing equations (4) and (3) with equations (A) and (B) respectively, we have,

$z_{11} = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}}$	$z_{12} = \frac{h_{12}}{h_{22}}$
$z_{21} = \frac{-h_{21}}{h_{22}}$	$z_{22} = \frac{1}{h_{22}}$

[C] Inters of Transmission (ABCD) Parameters

The equations for transmission parameters are as follows,

$$V_1 = A V_2 + B (-I_2) \quad \dots (1)$$

$$I_1 = C V_2 + D (-I_2) \quad \dots (2)$$

We can rewrite equation (2) as follows,

$$C V_2 = I_1 + D I_2$$

$$\therefore V_2 = \left[\frac{1}{C} \right] I_1 + \left[\frac{D}{C} \right] I_2 \quad \dots (3)$$

Substituting value of V_2 in equation (1), we have,

$$V_1 = A \left[\left(\frac{1}{C} \right) I_1 + \left(\frac{D}{C} \right) I_2 \right] - B \cdot I_2$$

$$\therefore V_1 = \left[\frac{A}{C} \right] I_1 + \left[\frac{AD}{C} - B \right] I_2$$

$$\therefore V_1 = \left[\frac{A}{C} \right] I_1 + \left[\frac{AD - BC}{C} \right] I_2 \quad \dots (4)$$

Comparing equations (4) and (3) with the equations (A) and (B) respectively, we have,

$z_{11} = \frac{A}{C}$	$z_{12} = \frac{AD - BC}{C}$
$z_{21} = \frac{1}{C}$	$z_{22} = \frac{D}{C}$

y-Parameters Interm of other Parameters

The equations for y-parameters are as follows,

$$I_1 = y_{11} V_1 + y_{12} V_2 \quad \dots (A)$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \quad \dots (B)$$

[A] Interm of z-Parameters

The equations for z-parameters are as follows,

$$V_1 = z_{11} I_1 + z_{12} I_2 \quad \dots (1)$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \quad \dots (2)$$

Writing equations in matrix form,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Solving above equations using Cramer's rule for I_1 and I_2 , we can write,

$$I_1 = \frac{\begin{vmatrix} V_1 & z_{12} \\ V_2 & z_{22} \end{vmatrix}}{\begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{vmatrix}} = \frac{z_{22} \cdot V_1 - z_{12} \cdot V_2}{z_{11} \cdot z_{22} - z_{12} \cdot z_{21}}$$
$$= \frac{z_{22}}{z_{11} z_{22} - z_{12} z_{21}} \cdot V_1 + \frac{-z_{12}}{z_{11} z_{22} - z_{12} z_{21}} \cdot V_2 \quad \dots (3)$$

Similarly,

$$I_2 = \frac{\begin{vmatrix} z_{11} & V_1 \\ z_{21} & V_2 \end{vmatrix}}{\begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{vmatrix}} = \frac{z_{11} \cdot V_2 - z_{21} \cdot V_1}{z_{11} \cdot z_{22} - z_{12} \cdot z_{21}}$$
$$= \frac{-z_{21}}{z_{11} \cdot z_{22} - z_{12} z_{21}} \cdot V_1 + \frac{z_{11}}{z_{11} z_{22} - z_{12} z_{21}} \cdot V_2 \quad \dots (4)$$

Let $\Delta z = z_{11} \cdot z_{22} - z_{12} \cdot z_{21}$

Rewriting equations (3) and (4),

$$I_1 = \left[\frac{z_{22}}{\Delta z} \right] V_1 + \left[\frac{-z_{12}}{\Delta z} \right] V_2 \quad \dots (5)$$

and

$$I_2 = \left[\frac{-z_{21}}{\Delta z} \right] V_1 + \left[\frac{z_{11}}{\Delta z} \right] V_2 \quad \dots (6)$$

Comparing equations (5) and (6) with the equations (A) and (B) respectively, we have,

$y_{11} = \frac{z_{22}}{\Delta z}$	$y_{12} = \frac{-z_{12}}{\Delta z}$
$y_{21} = \frac{-z_{21}}{\Delta z}$	$y_{22} = \frac{z_{11}}{\Delta z}$

[B] Interm of h-Parameters

The equations for h-parameters are as follows,

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \dots (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \dots (2)$$

We can rewrite equation (1) as follows,

$$h_{11} I_1 = V_1 - h_{12} V_2$$

$$\therefore I_1 = \left[\frac{1}{h_{11}} \right] V_1 + \left[\frac{-h_{12}}{h_{11}} \right] V_2 \quad \dots (3)$$

Substituting equation (3) in equation (2), we have,

$$I_2 = h_{21} \left[\frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2 \right] + h_{22} V_2$$

$$\therefore I_2 = \left[\frac{h_{21}}{h_{11}} \right] V_1 + \left[h_{22} - \frac{h_{12} h_{21}}{h_{11}} \right] V_2$$

$$\therefore I_2 = \left[\frac{h_{21}}{h_{11}} \right] V_1 + \left[\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{11}} \right] V_2 \quad \dots (4)$$

Comparing equations (3) and (4) with the equations (A) and (B) respectively, we have,

$y_{11} = \frac{1}{h_{11}}$	$y_{12} = \frac{-h_{12}}{h_{11}}$
$y_{21} = \frac{h_{21}}{h_{11}}$	$y_{22} = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{11}}$

[C] Inters of Transmission (ABCD) Parameters

The equations for transmission parameters are as follows,

$$V_1 = A V_2 + B (-I_2) \quad \dots (1)$$

$$I_1 = C V_2 + D (-I_2) \quad \dots (2)$$

We can rewrite equation (1) as follows,

$$-B \cdot I_2 = V_1 - A V_2$$

$$\therefore I_2 = \left[-\frac{1}{B} \right] V_1 + \left[\frac{A}{B} \right] V_2 \quad \dots (3)$$

Substituting value of I_2 in equation (2), we have,

$$I_1 = C \cdot V_2 + D \left[\frac{+1}{B} V_1 - \frac{A}{B} \right] V_2$$

$$\therefore I_1 = \left[\frac{D}{B} \right] V_1 + \left[C - \frac{AD}{B} \right] V_2$$

$$\therefore I_1 = \left[\frac{D}{B} \right] V_1 + \left[\frac{BC - AD}{B} \right] V_2 \quad \dots (4)$$

Comparing equations (4) and (3) with the equations (A) and (B) respectively, we have,

$y_{11} = \frac{D}{B}$	$y_{12} = \frac{BC - AD}{B}$
$y_{21} = \frac{-1}{B}$	$y_{22} = \frac{A}{B}$

h-Parameters inters of other Parameters

The equations for h-parameters are as follows,

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \dots (A)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \dots (B)$$

[A] Inters of z-Parameters

The equations for z-parameter are as follows,

$$V_1 = z_{11} I_1 + z_{12} I_2 \quad \dots (1)$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \quad \dots (2)$$

We can rewrite equations (2) as follows,

$$z_{22} I_2 = -z_{21} I_1 + V_2$$

$$I_2 = \left[\frac{-z_{21}}{z_{22}} \right] I_1 + \left[\frac{+1}{z_{22}} \right] V_2 \quad \dots (3)$$

Substituting value of I_2 in equation (1), we have,

$$V_1 = z_{11} I_1 + z_{12} \left(\left[\frac{-z_{21}}{z_{22}} \right] I_1 + \left[\frac{+1}{z_{22}} \right] V_2 \right)$$

$$\therefore V_1 = \left[\frac{z_{11} z_{22} - z_{12} z_{21}}{z_{22}} \right] I_1 + \left[\frac{z_{12}}{z_{22}} \right] V_2 \quad \dots (4)$$

Comparing equations (4) and (3) with equations (A) and (B) respectively, we have,

$h_{11} = \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{22}}$	$h_{12} = \frac{z_{12}}{z_{22}}$
$h_{21} = \frac{-z_{21}}{z_{22}}$	$h_{22} = \frac{1}{z_{22}}$

[B] Inters of y-Parameters

The equations of y-parameters are as follows,

$$I_1 = y_{11} V_1 + y_{12} V_2 \quad \dots (1)$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \quad \dots (2)$$

We can rewrite equation (1) as follows,

$$y_{11} V_1 = I_1 - y_{12} V_2$$

$$\therefore V_1 = \left[\frac{1}{y_{11}} \right] I_1 + \left[\frac{-y_{12}}{y_{11}} \right] V_2 \quad \dots (3)$$

Substituting value of V_1 in equation (2), we have,

$$I_2 = y_{21} \left(\left[\frac{1}{y_{11}} \right] I_1 + \left[\frac{-y_{12}}{y_{11}} \right] V_2 \right) + y_{22} V_2$$

$$\therefore I_2 = \left[\frac{y_{21}}{y_{11}} \right] I_1 + \left[y_{22} - \frac{y_{12} y_{21}}{y_{11}} \right] V_2$$

$$\therefore I_2 = \left[\frac{y_{21}}{y_{11}} \right] I_1 + \left[\frac{y_{11} y_{22} - y_{12} y_{21}}{y_{11}} \right] V_2 \quad \dots (4)$$

Comparing equations (3) and (4) with equation (A) and (B) respectively, we have,

$h_{11} = \frac{1}{y_{11}}$	$h_{12} = \frac{-y_{12}}{y_{11}}$
$h_{21} = \frac{y_{21}}{y_{11}}$	$h_{22} = \frac{y_{11} y_{22} - y_{12} y_{21}}{y_{11}}$

[C] Inters of Tramission Parameters

The equations for transmission parameters are as follows,

$$V_1 = A V_2 + B (-I_2) \quad \dots (1)$$

$$I_2 = C V_2 + D (-I_2) \quad \dots (2)$$

We can rewrite equations (2) as follows,

$$D (I_2) = -I_2 + C V_2$$

$$\therefore I_2 = \left[\frac{-1}{D} \right] I_2 + \left[\frac{C}{D} \right] V_2 \quad \dots (3)$$

Substituting value of I_2 in equation (1), we have,

$$V_1 = A V_2 - B \left(\left[\frac{-1}{D} \right] I_1 + \left[\frac{C}{D} \right] V_2 \right)$$

$$\therefore V_1 = \left[A - \frac{BC}{D} \right] V_2 + \left[\frac{B}{D} \right] I_1$$

$$\therefore V_1 = \left[\frac{B}{D} \right] I_1 + \left[\frac{AD - BC}{D} \right] V_2 \quad \dots (4)$$

Comparing equations (4) and (3) with equations (A) and (B) respectively we have,

$h_{11} = \frac{B}{D}$	$h_{12} = \frac{AD - BC}{D}$
$h_{21} = \frac{-1}{D}$	$h_{22} = \frac{C}{D}$

T-parameters in terms of other parameters

(i) T-parameters in terms of Z-parameters:

Step-I: We know that T-parameter equations;

$$V_1 = AV_2 - BI_2 \quad \dots(1)$$

$$I_1 = CV_2 - DI_2 \quad \dots(2)$$

and Z-parameters equations;

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \dots(3)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \dots(4)$$

Step-II: In order to obtain as equation (1), eliminated I_1 from equations (3) and (4).

From equation (3), we have

$$I_1 = \frac{V_1 - Z_{12} I_2}{Z_{11}}$$

Put this value in equation (4), then we get

$$V_2 = Z_{21} \left(\frac{V_1 - Z_{12} I_2}{Z_{11}} \right) + Z_{22} I_2$$

or
$$V_2 = \frac{Z_{21}}{Z_{11}} V_1 + I_2 \left(Z_{22} - \frac{Z_{12} Z_{21}}{Z_{11}} \right)$$

or
$$\frac{Z_{21}}{Z_{11}} V_1 = V_2 - I_2 \left(\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{11}} \right)$$

or
$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \left(\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}} \right) I_2 \quad \dots(5)$$

For obtaining as equations (2), rewrite equation (4), then we get

$$Z_{21} I_1 = V_2 - Z_{22} I_2$$

or
$$I_1 = \frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2 \quad \dots(6)$$

Step-III: Comparing equations (1) and (5), then we get

$A = \frac{Z_{11}}{Z_{21}}$ and $B = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}} = \frac{\Delta Z}{Z_{21}}$

Similarly, comparing equation (2) and (6), then we get

$C = \frac{1}{Z_{21}}$ and $D = \frac{Z_{22}}{Z_{21}}$

(ii) T-parameters in terms of Y-parameters:

Step-I: We know that T-parameter equations:

$$V_1 = AV_2 - BI_2 \quad \dots(1)$$

$$I_1 = CV_2 - DI_2 \quad \dots(2)$$

and Y-parameters equations:

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \dots(3)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \dots(4)$$

Handwritten derivation showing the elimination of V_1 from equations (3) and (4) to find I_1 in terms of I_2 and V_2 .

From equation (4): $V_1 = \frac{I_2 - Y_{22}V_2}{Y_{21}}$

Substituting into equation (3): $I_1 = Y_{11} \left[\frac{I_2 - Y_{22}V_2}{Y_{21}} \right] + Y_{12}V_2$

Simplifying: $I_1 = \frac{Y_{11}}{Y_{21}} I_2 - \frac{Y_{11}Y_{22}}{Y_{21}} V_2 + Y_{12}V_2$

Further simplification: $I_1 = \frac{Y_{11}}{Y_{21}} I_2 - \left[\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{21}} \right] V_2$

Comparing equation (6) with equation (2):

$A = -\frac{Y_{22}}{Y_{21}}$; $B = -\frac{1}{Y_{21}}$

$D = -\frac{Y_{11}}{Y_{21}}$; $C = \frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{21}}$

(iv) T-parameters in terms of h-parameters:

Step-I: We know that T-parameter equations:

$$V_1 = AV_2 - BI_2 \quad \dots(1)$$

$$I_1 = CV_2 - DI_2 \quad \dots(2)$$

and h-parameters equations:

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \dots(3)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \dots(4)$$

Step-II: In order to obtain as equation (1), eliminated I_1 from equations (3) and (4).

From equation (3), we have

$$I_1 = \frac{V_1 - h_{12} V_2}{h_{11}}$$

Put this value in equation (4), then we get

$$I_2 = h_{21} \left(\frac{V_1 - h_{12} V_2}{h_{11}} \right) + h_{22} V_2$$

or

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + V_2 \left(h_{22} - \frac{h_{12} h_{21}}{h_{11}} \right)$$

or

$$\frac{h_{21}}{h_{11}} V_1 = \left(\frac{h_{12} h_{21} - h_{11} h_{22}}{h_{11}} \right) V_2 + I_2$$

or

$$V_1 = \left(\frac{h_{12} h_{21} - h_{11} h_{22}}{h_{21}} \right) V_2 + \frac{h_{11}}{h_{21}} I_2 \quad \dots(5)$$

For obtaining as equation (2), rewrite equation (4), then we get

$$h_{21} I_1 = -h_{22} V_2 + I_2$$

or

$$I_1 = -\frac{h_{22}}{h_{21}} V_2 + \frac{1}{h_{21}} I_2 \quad \dots(6)$$

Step-III: Comparing equations (1) and (5), then we get

$$A = \frac{h_{12} h_{21} - h_{11} h_{22}}{h_{21}} = \frac{-\Delta h}{h_{21}} \text{ and } B = \frac{-h_{11}}{h_{21}}$$

Similarly, comparing equations (2) and (6), then we get

$$C = \frac{-h_{22}}{h_{21}} \text{ and } D = -\frac{1}{h_{21}}$$

INTERCONNECTIONS OF TWO-PORT NETWORKS

Two-port networks may be interconnected in various configurations, such as series, parallel, cascade, series-parallel, and parallel-series connections. For each configuration a certain set of parameters may be more useful than others to describe the network.

Series Connection

Figure 10.19 shows a series connection of two two-port networks N_a and N_b .

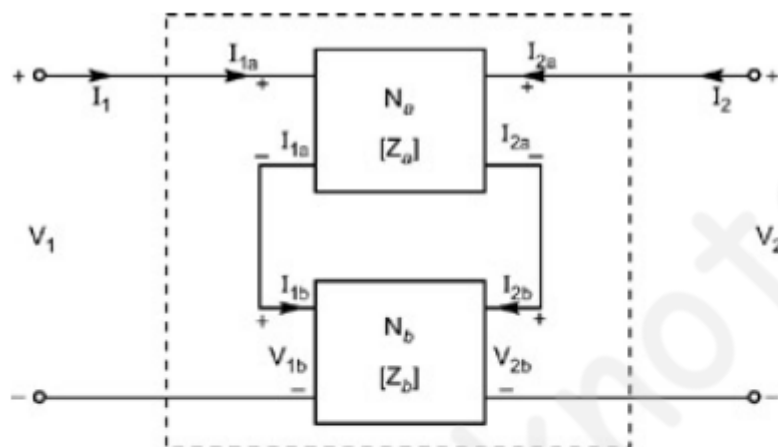


Figure 10.19: Series connection of two two-port networks
For network N_a ,

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}$$

$$(10.62) V_{1a} = Z_{11a} I_{1a} + Z_{12a} I_{2a}$$

$$(10.63) V_{2a} = Z_{21a} I_{1a} + Z_{22a} I_{2a}$$

For network N_b ,

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22b} \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

$$(10.64) V_{1b} = Z_{11b} I_{1b} + Z_{12b} I_{2b}$$

$$(10.65) V_{2b} = Z_{21b} I_{1b} + Z_{22b} I_{2b}$$

The condition for series connection is

$$I_{1a} = I_{1b} = I_1, \text{ and } I_2 = I_{2b} = I_2 \text{ (current same)}$$

$$(10.66) V_1 = V_{1a} + V_{1b}$$

$$(10.67) V_2 = V_{2a} + V_{2b}$$

Putting the values of V_{1a} and V_{1b} from Equation (10.62) and Equation (10.64),

$$\begin{aligned} V_1 &= Z_{11a}I_{1a} + Z_{12a}I_{2a} + Z_{11b}I_{1b} + Z_{12b}I_{2b} \\ (10.68) \quad &= Z_{11a}I_1 + Z_{12a}I_2 + Z_{11b}I_1 + Z_{12b}I_2 \quad [I_{1a} = I_{1b} = I_1, I_{2a} = I_{2b} = I_2] \\ V_1 &= (Z_{11a} + Z_{11b})I_1 + (Z_{12a} + Z_{12b})I_2. \end{aligned}$$

Putting the values of V_{2a} and V_{2b} from Equation (10.63) and Equation (10.65) into Equation (10.67), we get

$$(10.69) V_2 = (Z_{21a} + Z_{21b})I_1 + (Z_{22a} + Z_{22b})I_2$$

The Z-parameters of the series-connected combined network can be written as

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2,$$

where

$$Z_{11} = Z_{11a} + Z_{11b}$$

$$Z_{12} = Z_{12a} + Z_{12b}$$

$$Z_{21} = Z_{21a} + Z_{21b}$$

$$Z_{22} = Z_{22a} + Z_{22b}$$

or in the matrix form,

$$[Z] = [Z_a] + [Z_b].$$

The overall Z-parameter matrix for series connected two-port networks is simply

the sum of Z-parameter matrices of each individual two-port network connected in series.

Parallel Connection

Figure 10.20 shows a parallel connection of two two-port networks N_a and N_b .

The resultant of two admittances connected in parallel is $Y_1 + Y_2$. So in parallel connection, the parameters are Y-parameters.

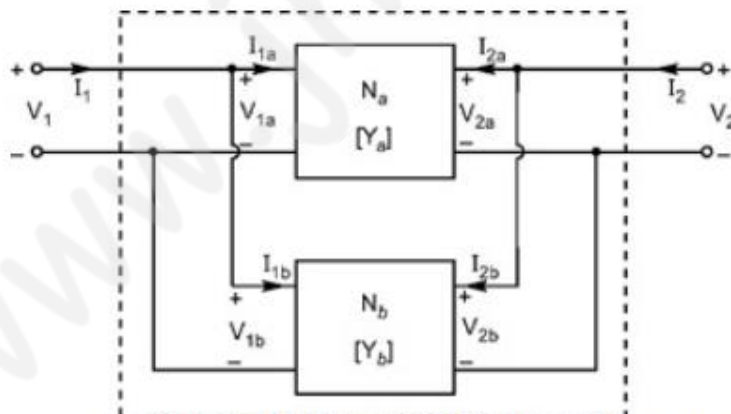


Figure 10.20: Parallel connections for two two-port networks
For network N_a ,

$$\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} Y_{11a} & Y_{12a} \\ Y_{21a} & Y_{22a} \end{bmatrix} \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix}$$

or

$$(10.70) I_{1a} = Y_{11a}V_{1a} + Y_{12a}V_{2a}$$

$$(10.71) I_{2a} = Y_{21a}V_{1a} + Y_{22a}V_{2a}.$$

For network N_b,

$$\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} Y_{11b} & Y_{12b} \\ Y_{21b} & Y_{22b} \end{bmatrix} \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

$$(10.72) I_{1b} = Y_{11b}V_{1b} + Y_{12b}V_{2b}$$

$$(10.73) I_{2b} = Y_{21b}V_{1b} + Y_{22b}V_{2b}$$

Now the condition for parallel,

$$V_{1a} = V_{1b} = V_1, \quad V_{2a} = V_{2b} = V_2 \quad [\text{Same voltage}]$$

and

$$(10.74) I_1 = I_{1a} + I_{1b}$$

$$(10.75) I_2 = I_{2a} + I_{2b}$$

$$\begin{aligned} I_1 &= Y_{11a}V_{1a} + Y_{12a}V_{2a} + Y_{11b}V_{1b} + Y_{12b}V_{2b} \\ &= Y_{11a}V_1 + Y_{12a}V_2 + Y_{11b}V_1 + Y_{12b}V_2 \end{aligned}$$

$$(10.76) I_1 = (Y_{11a} + Y_{11b})V_1 + (Y_{12a} + Y_{12b})V_2$$

Similarly,

$$(10.77) I_2 = (Y_{21a} + Y_{21b})V_1 + (Y_{22a} + Y_{22b})V_2$$

The Y-parameters of the parallel connected combined network can be written as

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$Y_{11} = Y_{11a} + Y_{11b}$$

$$Y_{12} = Y_{12a} + Y_{12b}$$

$$Y_{21} = Y_{21a} + Y_{21b}$$

$$Y_{22} = Y_{22a} + Y_{22b}$$

