Unit 2

Mobile Radio Propagation

UNIT – II

Mobile Radio Propagation: Large Scale Fading: Free space propagation model, The Three basic propagation mechanisms: Reflection, Ground Reflection (Two-Ray) Model, Diffraction, scattering, Practical Link budget design using path loss models.

Small Scale Fading: Multipath Propagation, Parameters of mobile multipath channel, Types of small scale fading: Fading effects due to multipath time delay spread and Doppler spread.

Source: Theodore S. Rappaport, Wireless Communications Principles and Practice, 2nd Edition, Pearson Education, 2003.

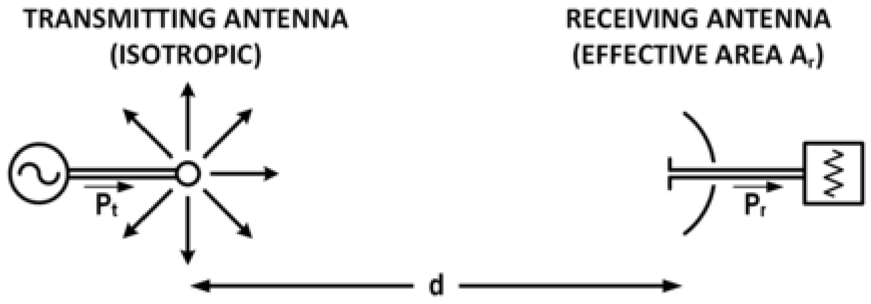
I. Large Scale Fading

Mobile Radio Propagation: Large Scale Fading: Free space propagation model, The Three basic propagation mechanisms: Reflection, Ground Reflection (Two-Ray) Model, Diffraction, scattering, Practical Link budget design using path loss models.

1. Free Space Propagation Model

- The free space propagation model is used to predict received signal strength when the transmitter and receiver have a clear, unobstructed line-of-sight path between them.
- Example: Satellite communication systems and microwave line-of-sight radio links.
- The free space model predicts that received power decays as a function of the T-R separation distance raised to some power (i.e. a power law function).
- The free space power received by a receiver antenna which is separated from a radiating transmitter antenna by a distance *d*, is given by the ***Friis free space equation***,

$$P_r(d) = P_d A_e = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}$$



- The Friis free space model is only a valid predictor ***P_r*** for for values of ***d*** which are in the far-field of the 'transmitting antenna.
- The far-field, or Fraunhofer region, of a transmitting antenna is defined as the region beyond the far-field distance ***d_f***, which is related to the largest linear dimension of the transmitter antenna aperture and the carrier wavelength.
- The Fraunhofer distance is given by

$$d_f = \frac{2D^2}{\lambda}$$

where ***D*** is the largest physical linear dimension of the antenna.

- Additionally, to be in the far-field region, ***d_f*** must satisfy

$$d_f \gg D \qquad\qquad d_f \gg \lambda$$

$$P_r(d) = P_r(d_0)\left(\frac{d_0}{d}\right)^2 \qquad\qquad d \geq d_0 \geq d_f$$

Example 3.1

Find the far-field distance for an antenna with maximum dimension of 1 m and operating frequency of 900 MHz.

Solution to Example 3.1

Given:

Largest dimension of antenna, $D = 1$ m

Operating frequency $f = 900$ MHz, $\lambda = c/f = \dfrac{3 \times 10^8 \text{ m/s}}{900 \times 10^6 \text{ Hz}}$ m

Using equation (3.7.a), far-field distance is obtained as

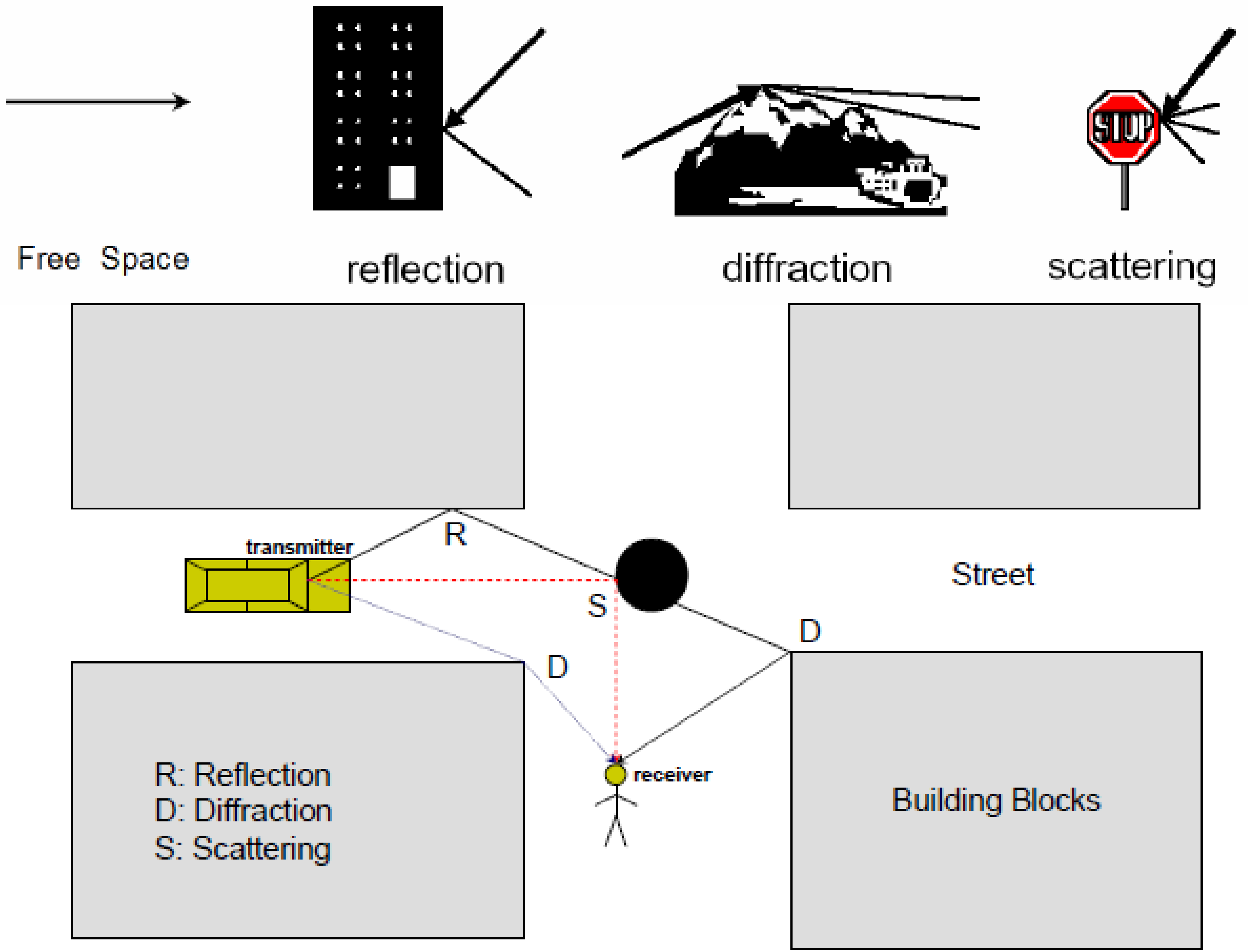
$$d_f = \frac{2(1)^2}{0.33} = 6 \text{ m}$$

2. The Three Basic Propagation Mechanisms

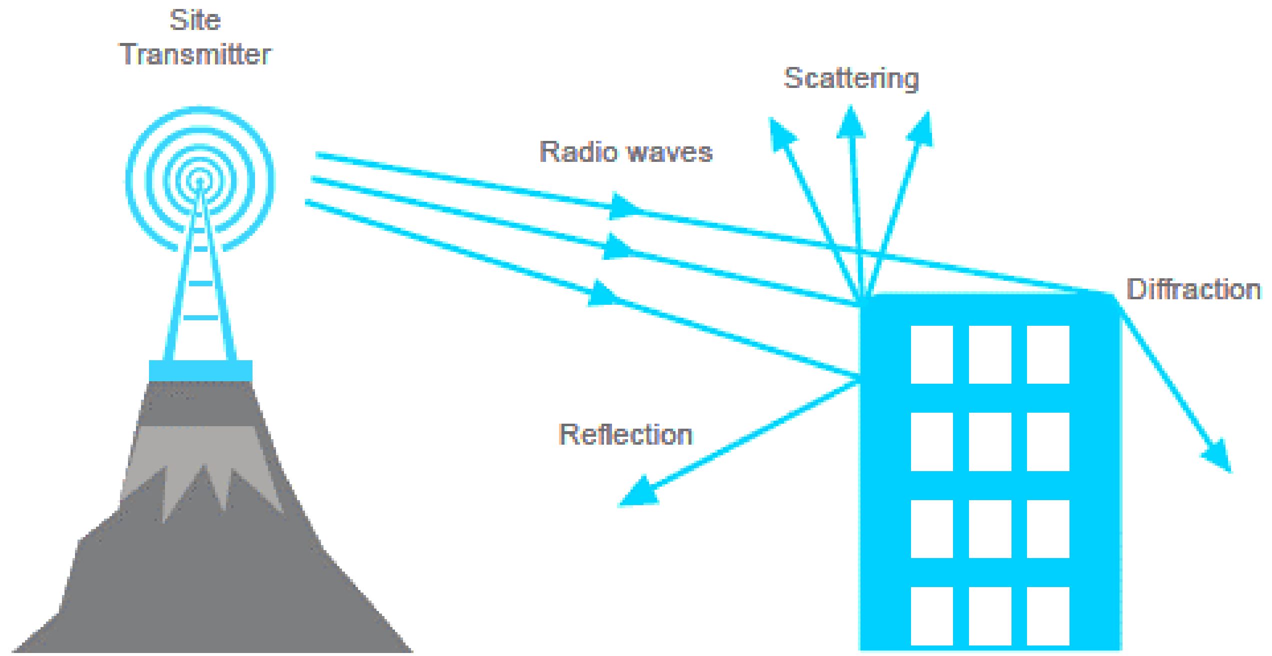
Three basic propagation mechanisms which impact the propagation in mobile radio communication system are: Reflection, Diffraction, and Scattering.

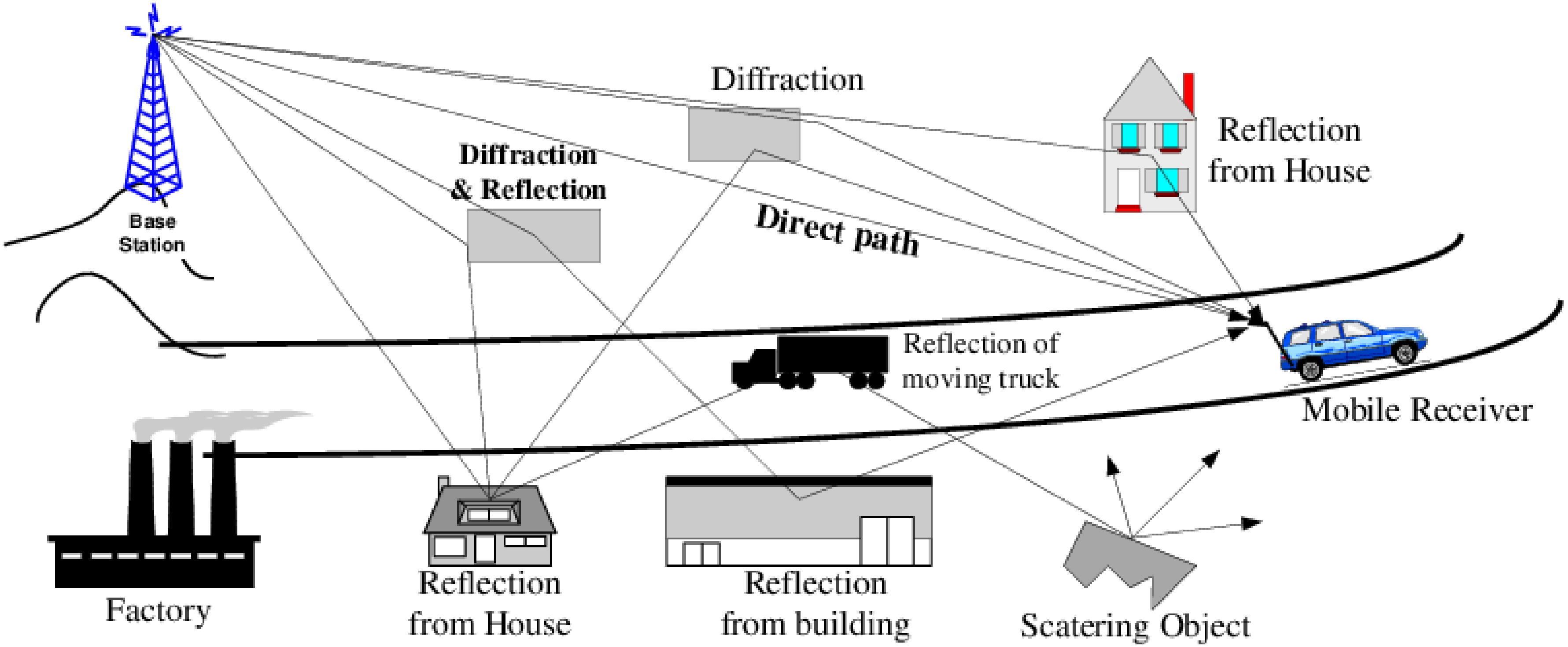
- Reflection: It occurs when a propagating electromagnetic wave impinges upon an object which has very large dimensions when compared to the wavelength of the propagating wave. Reflection occurs from the surface of earth, buildings and walls.

- Diffraction: It occurs when the radio transmission path between the transmitter and receiver is obstructed by a surface that has sharp irregularities (edges).
  Secondary waves resulting from the obstructing surface are present throughtout the space and even behind the obstacle.
  The secondary waves causes bending of waves around the obstacle, even when a LOS path does not exist between the transmitter and receiver.
  Bendingof electromagnetic waves around sharp edges such as, sharp towers or peaks.
- Scattering: It occurs when medium has objects that are smaller or comparable to the wavelength.
  Scattered waves are produced by rough surfaces, small objects, or by irregularities in the channel, foliage, street signs etc.



Free Space
reflection
diffraction
scattering
STOP
transmitter
R
S
D
D
Street
receiver
R: Reflection
D: Diffraction
S: Scattering
Building Blocks


Site
Transmitter
Radio waves
Scattering
Diffraction
Reflection


Diffraction
Reflection
from House
Diffraction
& Reflection
Direct path
Base
Station
Reflection of
moving truck
Mobile Receiver
Factory
Reflection
from House
Reflection
from building
Scatering Object

3. Reflection

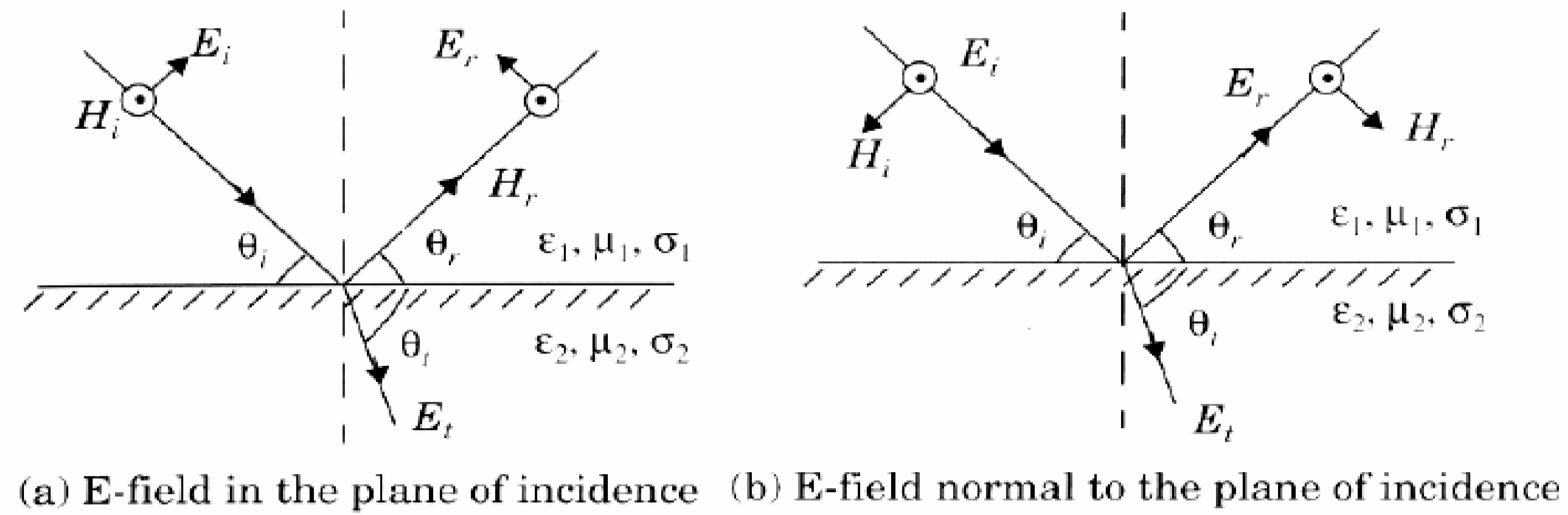
- when a radio wave propagating in one medium impinges upon another medium having different electrical properties, the wave is partially reflected and partially transmitted.
- If the plane radio wave is incident on a perfect dielectric,
  - Part of energy is transmitted into the second medium and
  - Part of energy is reflected back into the first medium, and there is no loss of energy in absorption.

  If the second medium is a perfect conductor, then all incident energy is reflected back into the first medium without loss of energy.
- The electric field intensity of the reflected and transmitted waves may be related to the incident wave amplitude by Fresnel reflection coefficients.

  The reflection coefficient is a function of the material properties, and generally depend on the wave polarization, angle of incidence, and the frequency of propagating wave.

3.1. Reflection from Dielectrics

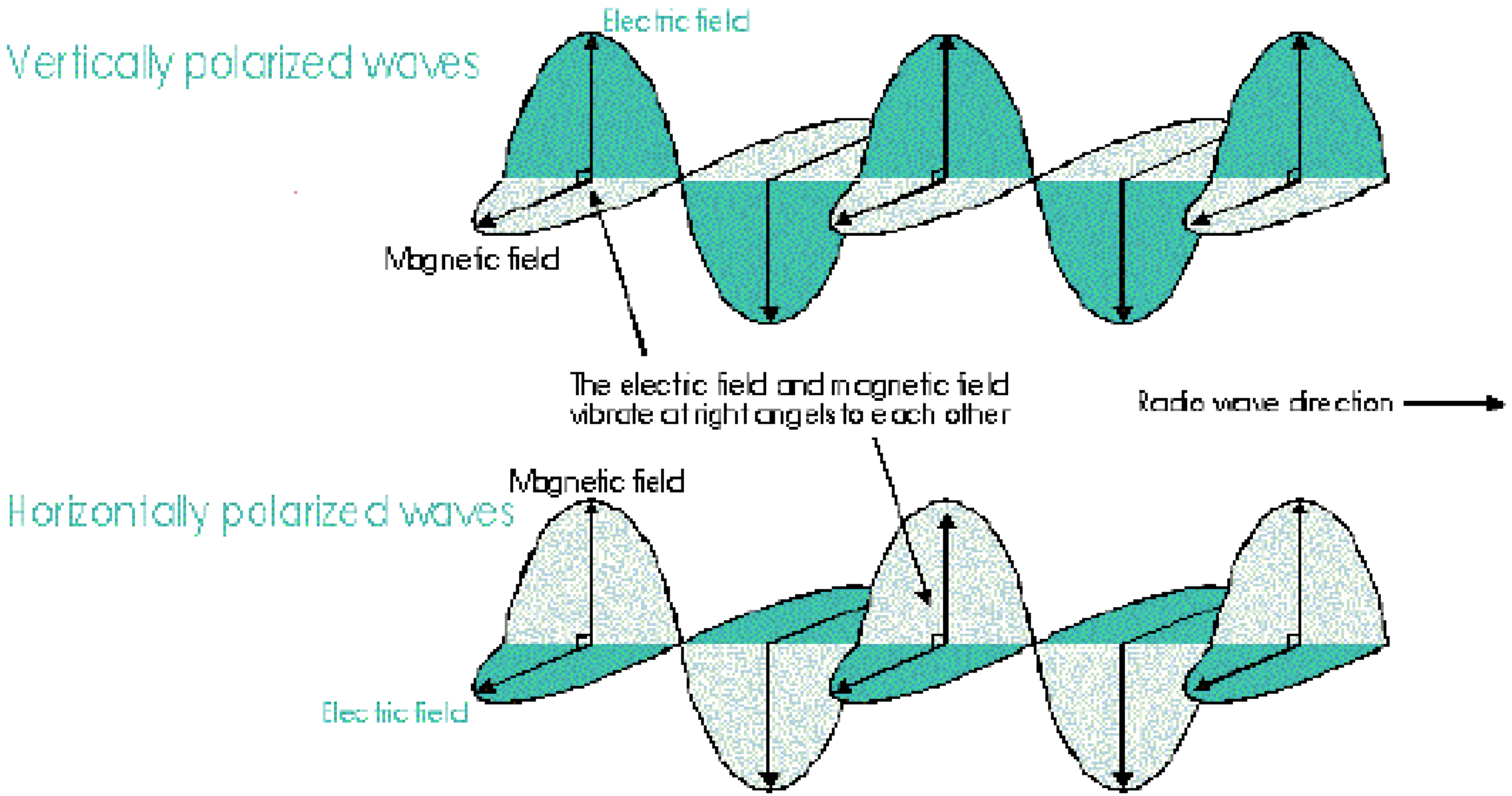
Figure shows an EM wave incidents at an angle (θ_i) with the plane of boundary between two dielectrics.
The part of the energy is reflected back to the first media at an angel (θ_r), and part of the energy is transmitted (refracted) into the second media at an angle (θ_t).



(vertical polarization) (horizontal polarization)

Fig. 1. Geometry for calculating the reflection coefficients between two dielectrics.

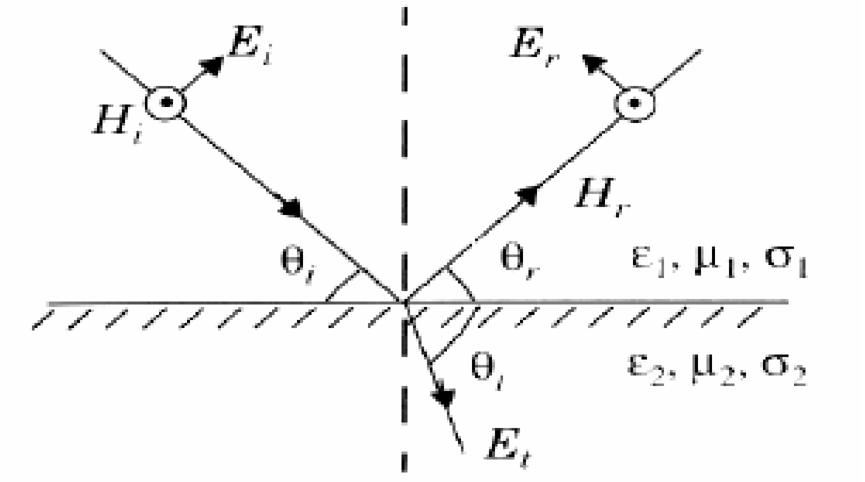
The nature of reflectio varies with the direction of propagation of E-field.
In general, EM waves are polarized means they have instantaneous electric field components in orthogonal directions in space.



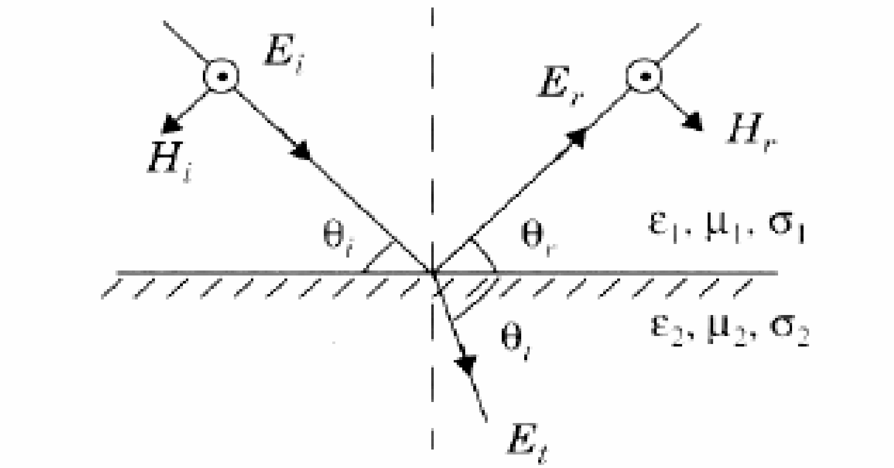
The plane of incidence is defind as the plane containing the incident, reflected, and transmitted rays.

In Fig. 1(a), the E-field polarization is parallel with the plane of incidence (i.e. E-field has a vertical polarization, or normal component, with respect to the reflecting surface).

And in Fig. 1(b), the E-field polarization is perpendicular to the plane of incidence (i.e. incident E-field is pointing out of the page towards the reader, and is perpendicular to the page and parallel to the reflecting surface).



(a) E-field in the plane of incidence



(b) E-field normal to the plane of incidence

For lossless dielectrics, the permittivity is real and can be expressed as $\varepsilon = \varepsilon_0 \varepsilon_r$, where $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m is the permittivity of free space, and ε_r is relative permittivity (dielectric constant)

For lossy dielectric materials, $\varepsilon = \varepsilon_0 \varepsilon_r - j \varepsilon'$, where

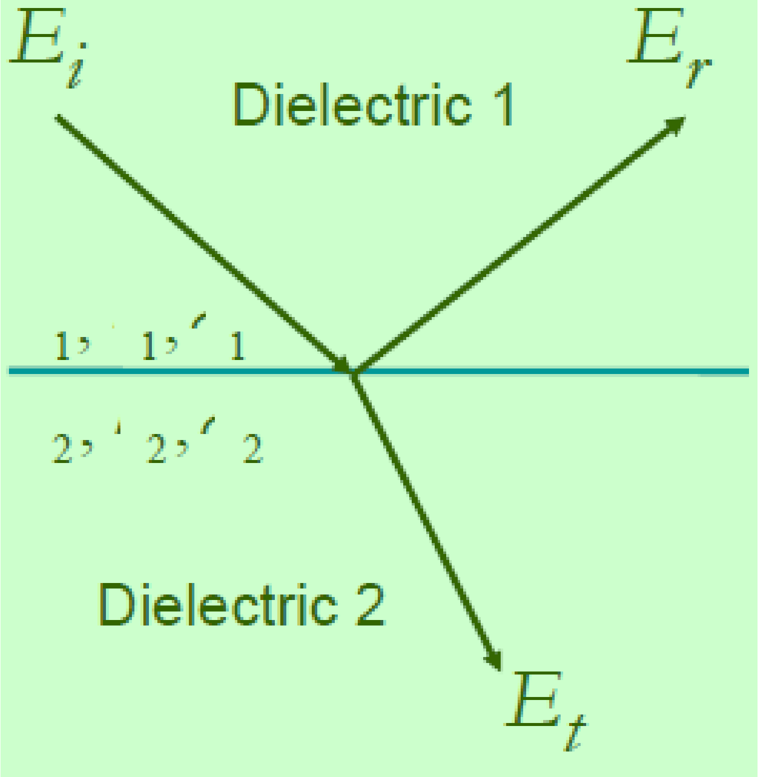
$$\varepsilon' = \frac{\sigma}{2\pi f}$$

Speed of propagation $\quad v_i = \frac{1}{\sqrt{\mu_i \varepsilon_i}}$

(at free space 3×10^8 m/s)

Intrinsic impedance $\quad \eta_i = \sqrt{\frac{\mu_i}{\varepsilon_i}}$

(at free space $120\pi\ \Omega$)



Reflection coefficient $\frac{E_r}{E_i} = \Gamma$

Transmission coefficient $\frac{E_t}{E_i} = T = 1 + \Gamma$

The value of Γ depends on polarization.

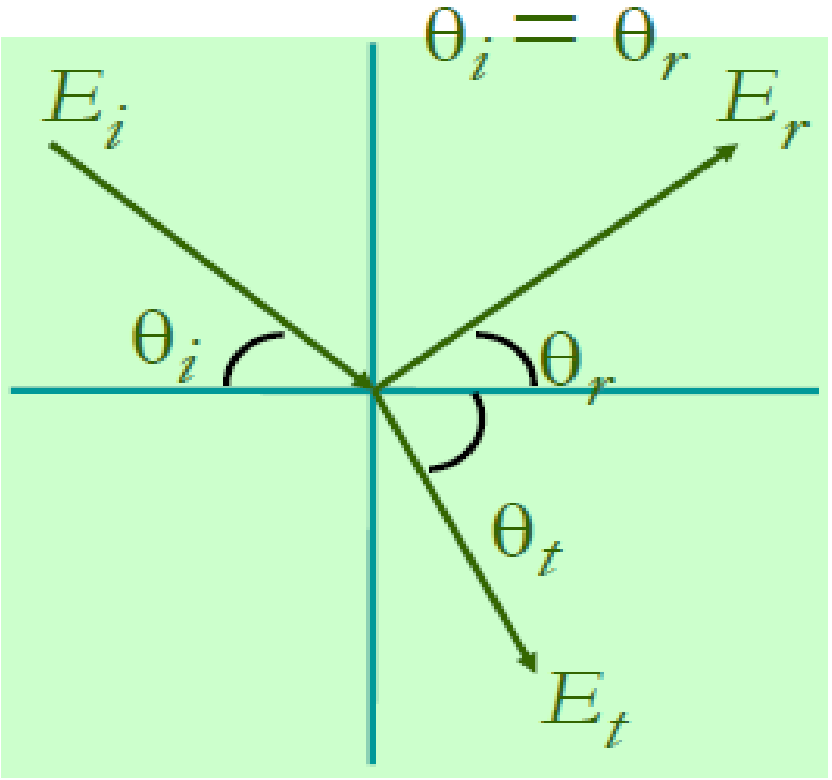
The reflection coefficient for the two cases of parallel and perpendicular E-field polarization at the boundary of the two dielectrics is given by:

E-field in the plane of incident

$$\Gamma_{\parallel} = \frac{E_r}{E_i} = \frac{\eta_2 \sin\theta_t - \eta_1 \sin\theta_i}{\eta_2 \sin\theta_t + \eta_1 \sin\theta_i}$$

E-field normal to the plane of incident

$$\Gamma_{\perp} = \frac{E_r}{E_i} = \frac{\eta_2 \sin\theta_i - \eta_1 \sin\theta_t}{\eta_2 \sin\theta_i + \eta_1 \sin\theta_t}$$



Relationship between angles

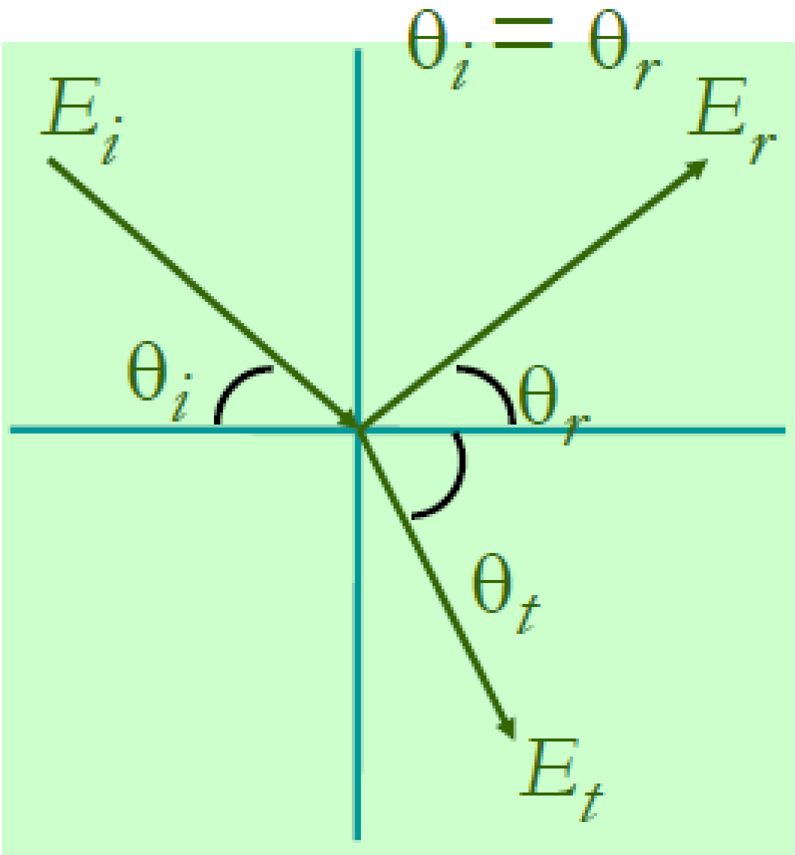
$$\theta_i = \theta_r$$

Snell's Law

$$\sqrt{\mu_1\varepsilon_1}\sin(90-\theta_i) = \sqrt{\mu_2\varepsilon_2}\sin(90-\theta_t)$$

When $\mu_1 = \mu_2$

$$\sqrt{\varepsilon_1}\sin(90-\theta_i) = \sqrt{\varepsilon_2}\sin(90-\theta_t)$$



When the first medium is free space, and $\mu_1=\mu_2$

$$\sin(90-\theta_i)=\sqrt{\varepsilon_r}\sin(90-\theta_t) \qquad \sin\theta_t=\sqrt{1-\cos^2\theta_t}=\sqrt{1-\frac{\cos^2\theta_i}{\varepsilon_r}}$$

$$\Gamma_{\parallel}=\frac{E_r}{E_i}=\frac{\eta_2\sin\theta_t-\eta_1\sin\theta_i}{\eta_2\sin\theta_t+\eta_1\sin\theta_i}=\frac{\dfrac{\sin\theta_t}{\sqrt{\mu_0\varepsilon_r\varepsilon_0}}-\dfrac{\sin\theta_i}{\sqrt{\mu_0\varepsilon_0}}}{\dfrac{\sin\theta_t}{\sqrt{\mu_0\varepsilon_r\varepsilon_0}}+\dfrac{\sin\theta_i}{\sqrt{\mu_0\varepsilon_0}}}=\frac{\sin\theta_t-\sqrt{\varepsilon_r}\sin\theta_i}{\sin\theta_t+\sqrt{\varepsilon_r}\sin\theta_i}$$

$$=\frac{\sqrt{1-\dfrac{\cos^2\theta_i}{\varepsilon_r}}-\sqrt{\varepsilon_r}\sin\theta_i}{\sqrt{1-\dfrac{\cos^2\theta_i}{\varepsilon_r}}+\sqrt{\varepsilon_r}\sin\theta_i}=\frac{-\varepsilon_r\sin\theta_i+\sqrt{\varepsilon_r-\cos^2\theta_i}}{\varepsilon_r\sin\theta_i+\sqrt{\varepsilon_r-\cos^2\theta_i}}$$

$$\Gamma_{\perp}=\frac{\sin\theta_i-\sqrt{\varepsilon_r-\cos^2\theta_i}}{\sin\theta_i+\sqrt{\varepsilon_r-\cos^2\theta_i}}$$

Example 3.4

Demonstrate that if medium 1 is free space and medium 2 is a dielectric, both $|\Gamma_{\parallel}|$ and $|\Gamma_{\perp}|$ approach 1 as θ_i approaches $0°$ regardless of ε_r.

Solution to Example 3.4

Substituting $\theta_i = 0°$ in equation (3.24)

$$\Gamma_{\parallel} = \frac{-\varepsilon_r \sin 0 + \sqrt{\varepsilon_r - \cos^2 0}}{\varepsilon_r \sin 0 + \sqrt{\varepsilon_r - \cos^2 0}}$$

$$\Gamma_{\parallel} = \frac{\sqrt{\varepsilon_r - 1}}{\sqrt{\varepsilon_r - 1}}$$

$$= 1$$

Substituting $\theta_i = 0^o$ in equation (3.25)

$$\Gamma_{\perp} = \frac{\sin 0 - \sqrt{\varepsilon_r - \cos^2 0}}{\sin 0 + \sqrt{\varepsilon_r - \cos^2 0}}$$

$$\Gamma_{\perp} = \frac{-\sqrt{\varepsilon_r - 1}}{\sqrt{\varepsilon_r - 1}}$$

$$= -1.$$

- This example illustrates that ground may be modeled as a perfect reflector with a reflection coefficient of unit magnitude when an incident wave grazes the earth, regardless of polarization or ground dielectric properties

3.5.2 Brewster Angle

The *Brewster angle* is the angle at which no reflection occurs in the medium of origin. It occurs when the incident angle θ_B is such that the reflection coefficient $\Gamma_{\|}$ is equal to zero (see Figure 3.6). The Brewster angle is given by the value of θ_B which satisfies

$$\sin(\theta_B) = \sqrt{\frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2}} \tag{3.27}$$

For the case when the first medium is free space and the second medium has a relative permittivity ε_r, equation (3.27) can be expressed as

$$\sin(\theta_B) = \frac{\sqrt{\varepsilon_r - 1}}{\sqrt{\varepsilon_r^2 - 1}} \tag{3.28}$$

Note that the Brewster angle occurs only for vertical (i.e. parallel) polarization.

Example 3.5

Calculate the Brewster angle for a wave impinging on ground having a permittivity of $\varepsilon_r = 4$.

Solution to Example 3.5

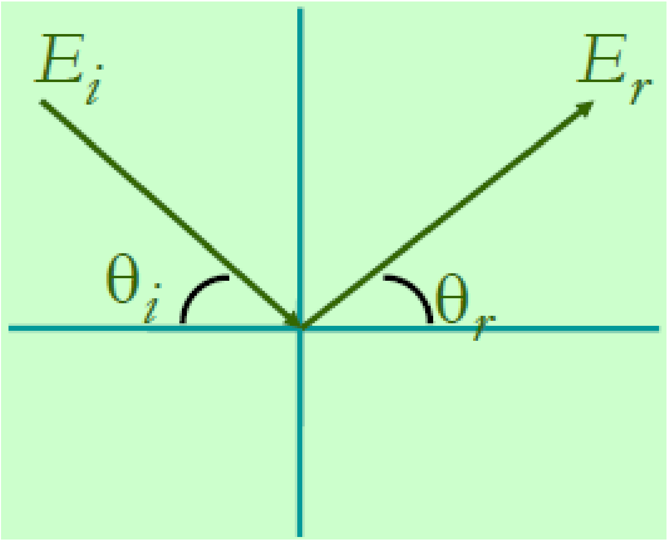
The Brewster angle can be found by substituting the values for ε_r in equation (3.28).

$$\sin(\theta_i) = \frac{\sqrt{(4) - 1}}{\sqrt{(4)^2 - 1}} = \sqrt{\frac{3}{15}} = \sqrt{\frac{1}{5}}$$

$$\theta_i = \sin^{-1}\sqrt{\frac{1}{5}} = 26.56^o$$

Thus Brewster angle for $\varepsilon_r = 4$ is equal to $26.56°$.

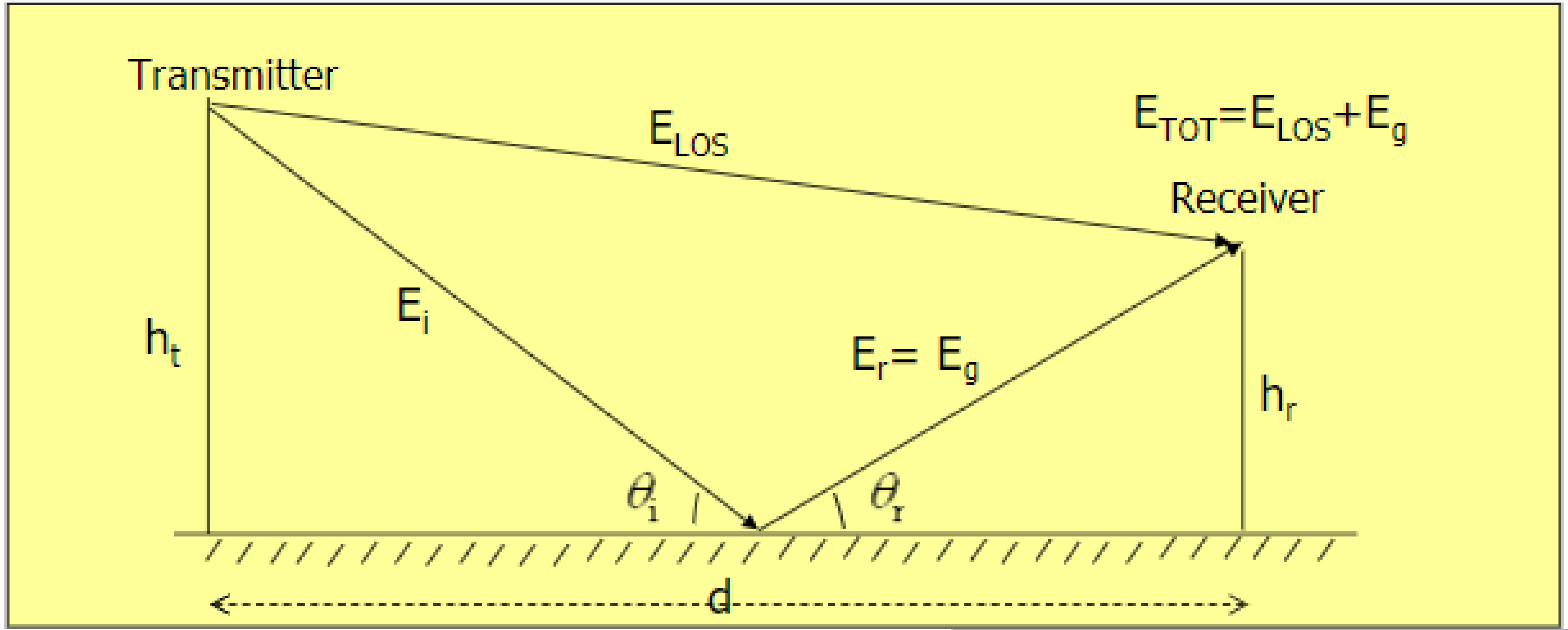
3.2. Reflection from Conductors



$\theta_i = \theta_r$

Parallel / vertical polarization	Perpendicular / horizontal polarization
$\theta_i = \theta_r$	$\theta_i = \theta_r$
$E_i = E_r$	$E_i = -E_r$

4. Ground Reflection (2-ray) Model



- A single direct path between the base station and a mobile is seldom the only means for propagation
- Friis equation is most likely inaccurate
- *Two ray ground reflection model* has been found to be reasonably accurate for predicting large scale strength over distances of several Kms for mobile radio systems that use tall towers

Power Flux Density

$$\Phi_R = \frac{|E_{TOT}|^2}{\eta} = \frac{|E_{TOT}|^2}{120\pi\Omega} \ \mathrm{W/m^2}$$

$$E(d,t) = \frac{E_0 d_0}{d} \cos\left(\omega_c\left(t - \frac{d}{c}\right)\right), d > d_0$$

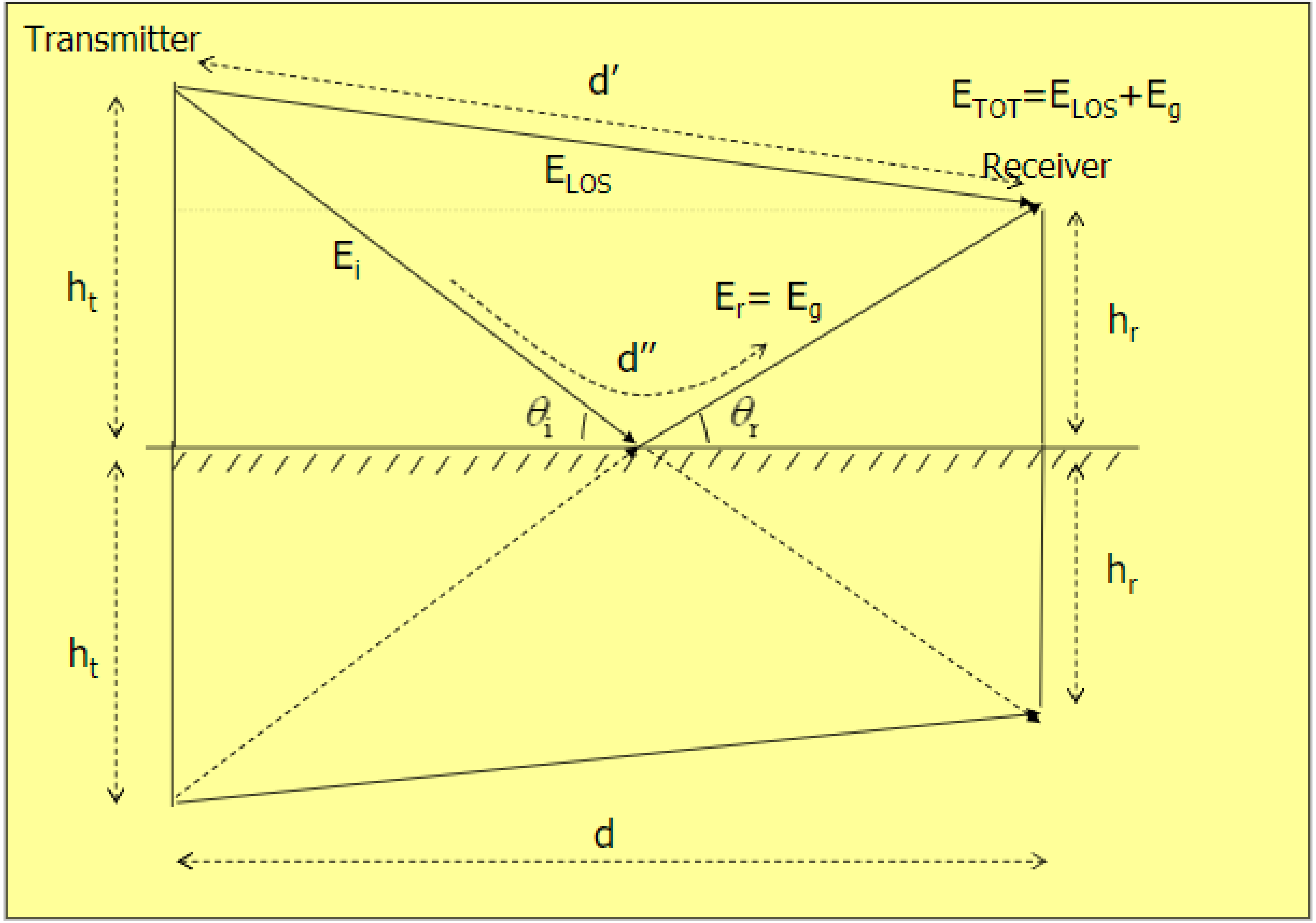
$$E_{LOS}(d',t) = \frac{E_0 d_0}{d'} \cos\left(\omega_c\left(t - \frac{d'}{c}\right)\right)$$

$$E_g(d'',t) = \Gamma \frac{E_0 d_0}{d''} \cos\left(\omega_c\left(t - \frac{d''}{c}\right)\right)$$

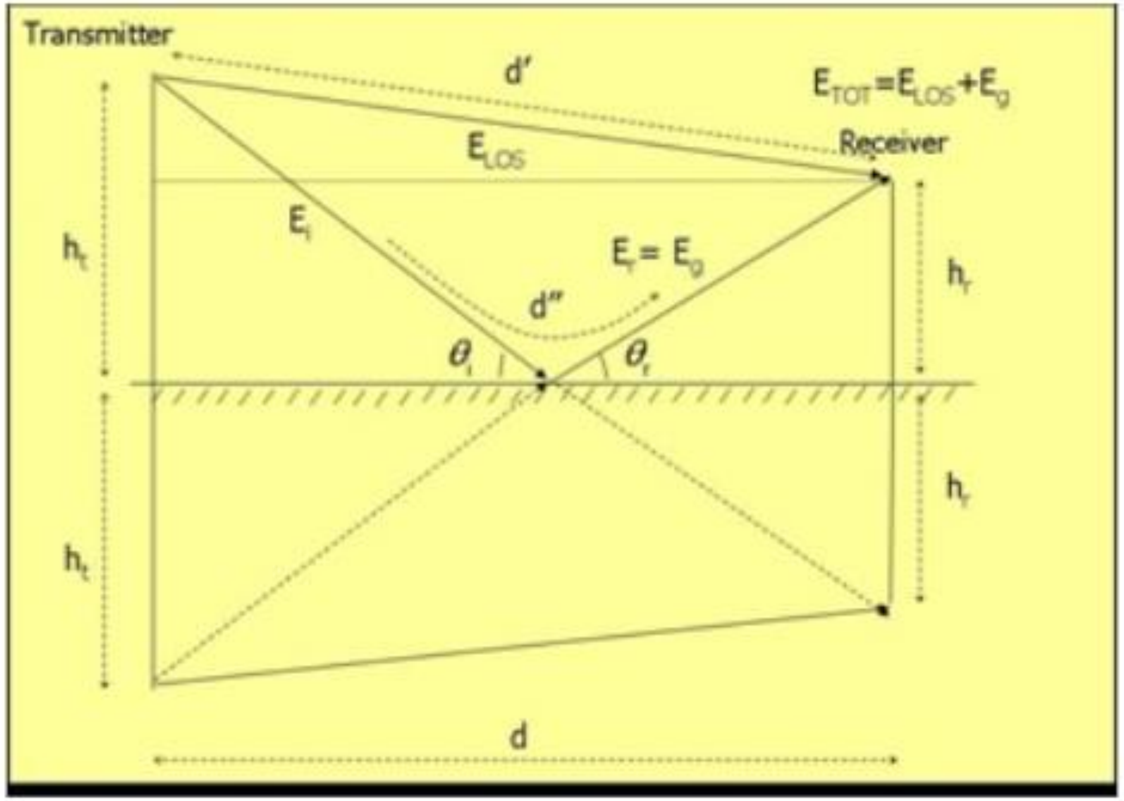
Assuming grazing incidence **Γ=-1 for normal incidence**

$$|E_{TOT}| = |E_{LOS} + E_g|$$

$$E_{TOT}(d,t) = \frac{E_0 d_0}{d'} \cos\left(\omega_c\left(t - \frac{d'}{c}\right)\right) + (-1)\frac{E_0 d_0}{d''} \cos\left(\omega_c\left(t - \frac{d''}{c}\right)\right)$$



Transmitter
d′
$E_{TOT}=E_{LOS}+E_g$
Receiver
E_{LOS}
E_i
h_t
$E_r= E_g$
h_r
d″
θ_i
θ_r
h_r
h_t
d



$$\Delta = d'' - d' = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$$

$$\Delta = d\sqrt{1 + \left(\frac{h_t + h_r}{d}\right)^2} - d\sqrt{1 + \left(\frac{h_t - h_r}{d}\right)^2}$$

$$\Delta \approx d\left\{\left(1 + \frac{1}{2}\left(\frac{h_t + h_r}{d}\right)^2\right) - \left(1 + \frac{1}{2}\left(\frac{h_t - h_r}{d}\right)^2\right)\right\}$$

$$\Delta \approx \frac{4\,h_t\,h_r}{2d^2}$$

Path Difference (Δ) $\approx \frac{2.h_t h_r}{d}$

$$d'' = \sqrt{(h_t + h_r)^2 + d^2} = d\sqrt{1 + \frac{(h_t + h_r)^2}{d^2}} \approx d\left(1 + \frac{(h_t + h_r)^2}{2d^2}\right), d \ll (h_t + h_r)$$

$$d' = \sqrt{(h_t - h_r)^2 + d^2} = d\sqrt{1 + \frac{(h_t - h_r)^2}{d^2}} \approx d\left(1 + \frac{(h_t - h_r)^2}{2d^2}\right), d \ll (h_t + h_r)$$

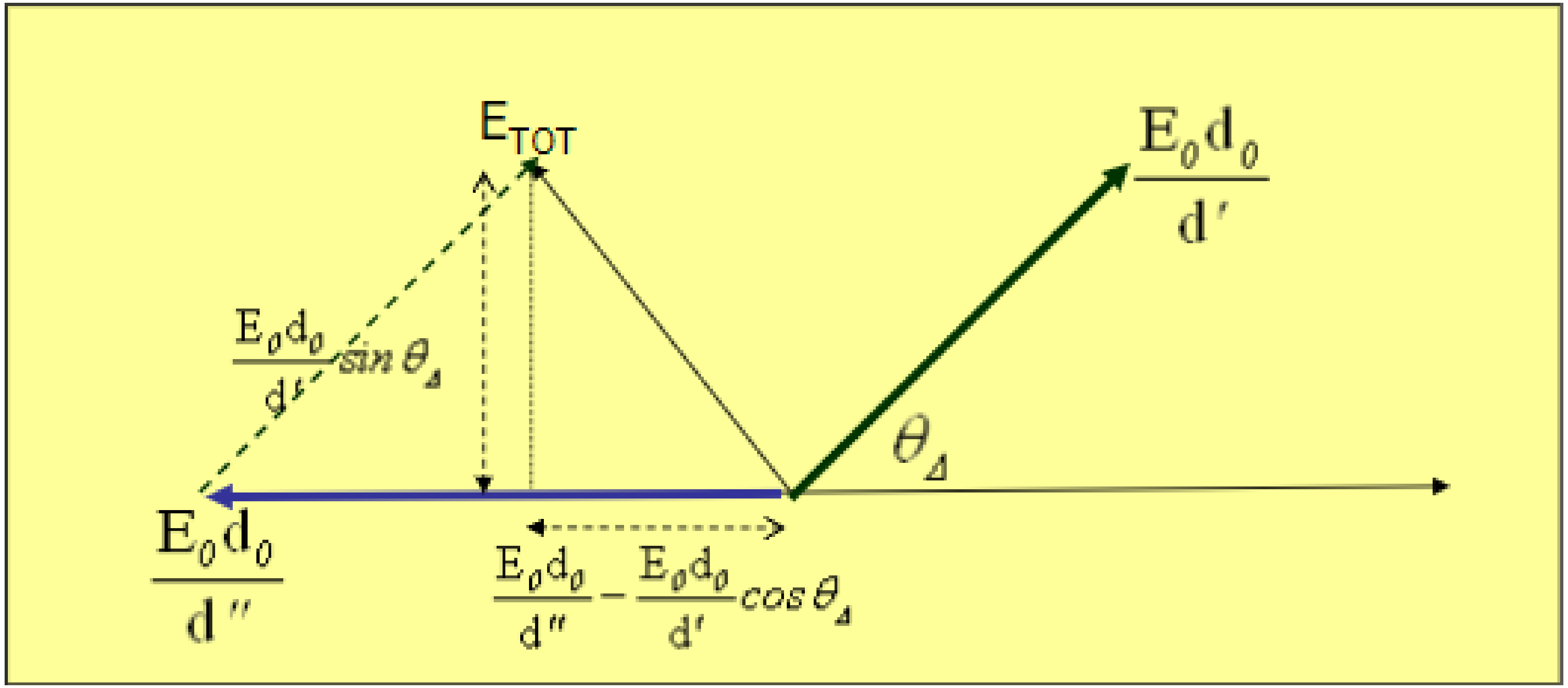
where $\sqrt{1 + x} \approx 1 + \frac{x}{2}$ for $x \ll 1$

$$\Delta = d'' - d' \approx 2\frac{h_t h_r}{d}, \theta_\Delta = \frac{\Delta\omega_c}{c}$$

$$E_{TOT}(d,t) = \frac{E_0 d_0}{d'} \cos\left(\omega_c \left(t - \frac{d'}{c}\right)\right) + (-1) \frac{E_0 d_0}{d''} \cos\left(\omega_c \left(t - \frac{d''}{c}\right)\right)$$

Let evaluate E $_{TOT}$ at t=d"/c

$$E_{TOT}\left(d, t = \frac{d''}{c}\right) = \frac{E_0 d_0}{d'} \cos\left(\omega_c \left(\frac{d''}{c} - \frac{d'}{c}\right)\right) + (-1) \frac{E_0 d_0}{d''} \cos(0)$$



Assume for large d $\left|\frac{E_0 d_0}{d'}\right| \approx \left|\frac{E_0 d_0}{d''}\right| \approx \left|\frac{E_0 d_0}{d}\right|$

$$|E_{TOT}(d)| = \sqrt{\left(\frac{E_0 d_0}{d}\right)^2 (\cos\theta_\Delta - 1)^2 + \left(\frac{E_0 d_0}{d}\right)^2 \sin^2\theta_\Delta}$$

$$|E_{TOT}(d)| = \left(\frac{E_0 d_0}{d}\right)\sqrt{2 - 2\cos\theta_\Delta}$$

$$|E_{TOT}(d)| = 2\left(\frac{E_0 d_0}{d}\right)\sin\frac{\theta_\Delta}{2}$$

Note that rather than using phasors, we could also use $2\sin\left(\frac{\theta_\Delta}{2}\right)\sin\left(\omega_c t + \frac{\theta_\Delta}{2}\right) = \cos(\omega_c t) - \cos(\omega_c t + \theta_\Delta)$

Important Note:

The significance of the path difference Δ appears in the phase difference θ_Δ between the two waves (LOS and Reflected Wave)

Under the assumption

$$\frac{\theta_\Delta}{2} < 0.3 \text{ rads} \Rightarrow \sin\frac{\theta_\Delta}{2} \approx \frac{\theta_\Delta}{2}$$

$$\frac{\theta_\Delta}{2} = \frac{\pi}{\lambda}\Delta = \frac{2\pi h_t h_r}{\lambda d} < 0.3\text{rad}$$

$$d > \frac{20\pi h_t h_r}{3\lambda} \approx \frac{20 h_t h_r}{\lambda}$$

$$|E_{TOT}(d)| = 2\left(\frac{E_0 d_0}{d}\right)\sin\frac{\theta_\Delta}{2}$$

$$|E_{TOT}(d)| \approx 2\left(\frac{E_0 d_0}{d}\right)\frac{2\pi h_t h_r}{\lambda d}$$

$$\Phi_R = \frac{|E_{TOT}|^2}{\eta} = \frac{\left(2\left(\frac{E_0 d_0}{d}\right)\frac{2\pi h_t h_r}{\lambda d}\right)^2}{120\pi\Omega} = \frac{\left(\frac{E_0 d_0}{d}\right)^2}{120\pi\Omega}\left(\frac{4\pi h_t h_r}{\lambda d}\right)^2$$

$$\Phi_R = \frac{|E_{TOT}|^2}{\eta} = \frac{\left(2\left(\frac{E_0 d_0}{d}\right)\frac{2\pi h_t h_r}{\lambda d}\right)^2}{120\pi\Omega} = \frac{\left(\frac{E_0 d_0}{d}\right)^2}{120\pi\Omega}\left(\frac{4\pi h_t h_r}{\lambda d}\right)^2$$

But from LOS analysis

$$\frac{\left(\frac{E_0 d_0}{d}\right)^2}{120\pi\Omega} = \frac{P_T G_T}{4\pi d^2}$$

$$\Phi_R = \frac{|E_{TOT}|^2}{\eta} = \frac{P_T G_T}{4\pi d^2}\left(\frac{4\pi h_t h_r}{\lambda d}\right)^2$$

$$P_R(d) = \Phi_R (A_e)_{dir} = \Phi_R G_R (A_e)_{iso}$$

$$P_R(d) = \frac{P_T G_T}{4\pi d^2}\left(\frac{4\pi h_t h_r}{\lambda d}\right)^2 G_R \frac{\lambda^2}{4\pi} = \boxed{\frac{P_T G_T G_R (h_t h_r)^2}{d^4}}$$

- The received power falls off with distance raised to the fourth power, 40 dB/decade
- Much rapid than free space
- The received power and path-loss are no longer dependent on frequency
  - $PL(dB)=40\log d-(10\log G_T+10\log G_R+20\log h_t+20\log h_r)$

Example 3.6

A mobile is located 5 km away from a base station and uses a vertical $\lambda/4$ monopole antenna with a gain of 2.55 dB to receive cellular radio signals. The E-field at 1 km from the transmitter is measured to be 10^{-3} V/m. The carrier frequency used for this system is 900 MHz.

(a) Find the length and the gain of the receiving antenna.

(b) Find the received power at the mobile using the 2-ray ground reflection model assuming the height of the transmitting antenna is 50 m and the receiving antenna is 1.5 m above ground.

Solution to Example 3.6

Given:

T-R separation distance = 5 km
E-field at a distance of 1 km = 10^{-3} V/m
Frequency of operation, f = 900 MHz

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{900\times10^6} = 0.333 \text{ m}.$$

Length of the antenna, $L = \lambda/4 = 0.333/4 = 0.0833$ m = 8.33 cm.
Gain of $\lambda/4$ monopole antenna can be obtained using equation (3.2).
Gain of antenna =1.8 = 2.55 dB.

(b) Since $d \gg \sqrt{h_t h_r}$, the electric field is given by

$$E_R(d) \approx \frac{2E_0 d_0}{d} \frac{2\pi h_t h_r}{\lambda d} \approx \frac{k}{d^2} \text{ V/m}$$

$$= \frac{2 \times 10^{-3} \times 1 \times 10^3}{5 \times 10^3} \left[\frac{2\pi (50)(1.5)}{0.333 \left(5 \times 10^3 \right)} \right]$$

$$= 113.1 \times 10^{-6} \text{ V/m.}$$

The received power at a distance d can be obtained using equation (3.15)

$$P_r(d) = \frac{\left(113.1 \times 10^{-6} \right)^2}{377} \left[\frac{1.8 (0.333)^2}{4\pi} \right]$$

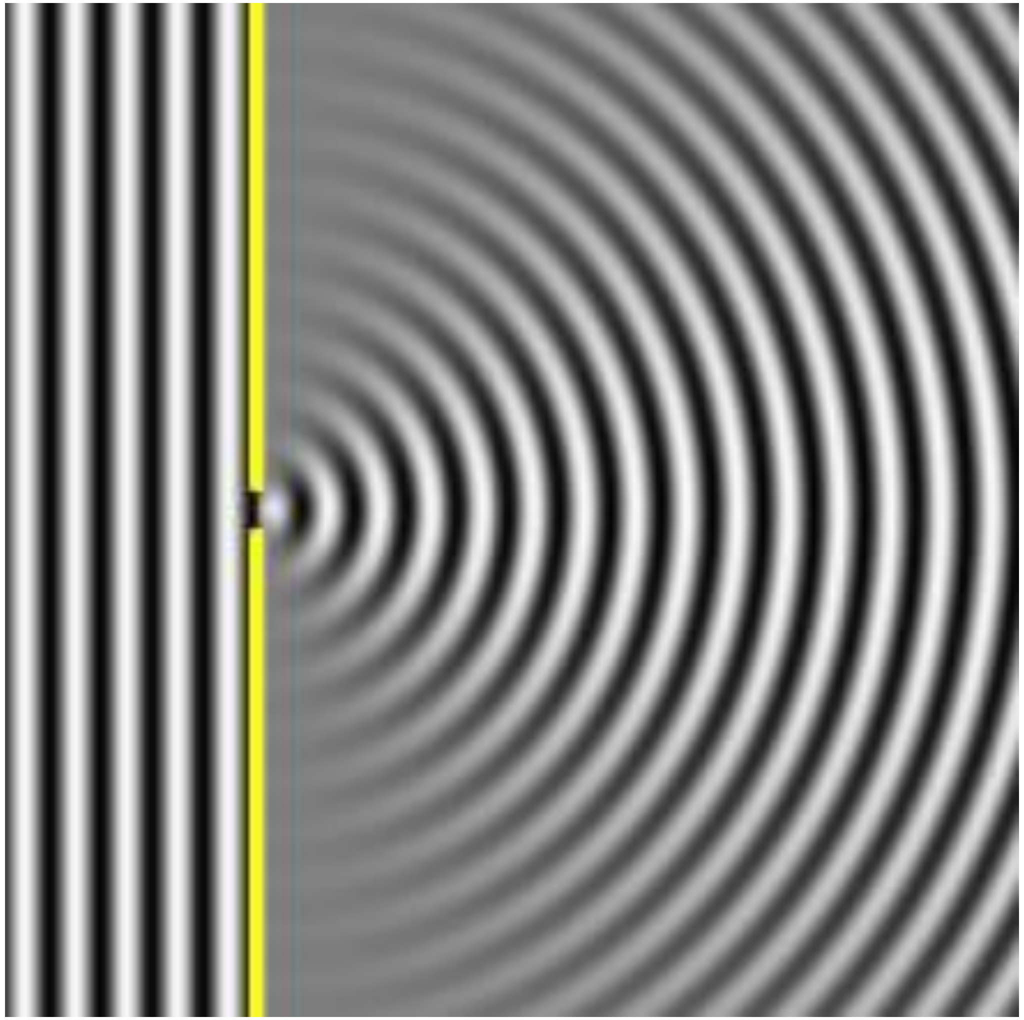
$$P_r(d = 5 \text{ km}) = 5.4 \times 10^{-13} \text{ W} = -122.68 \text{ dBW or } -92.68 \text{ dBm.}$$

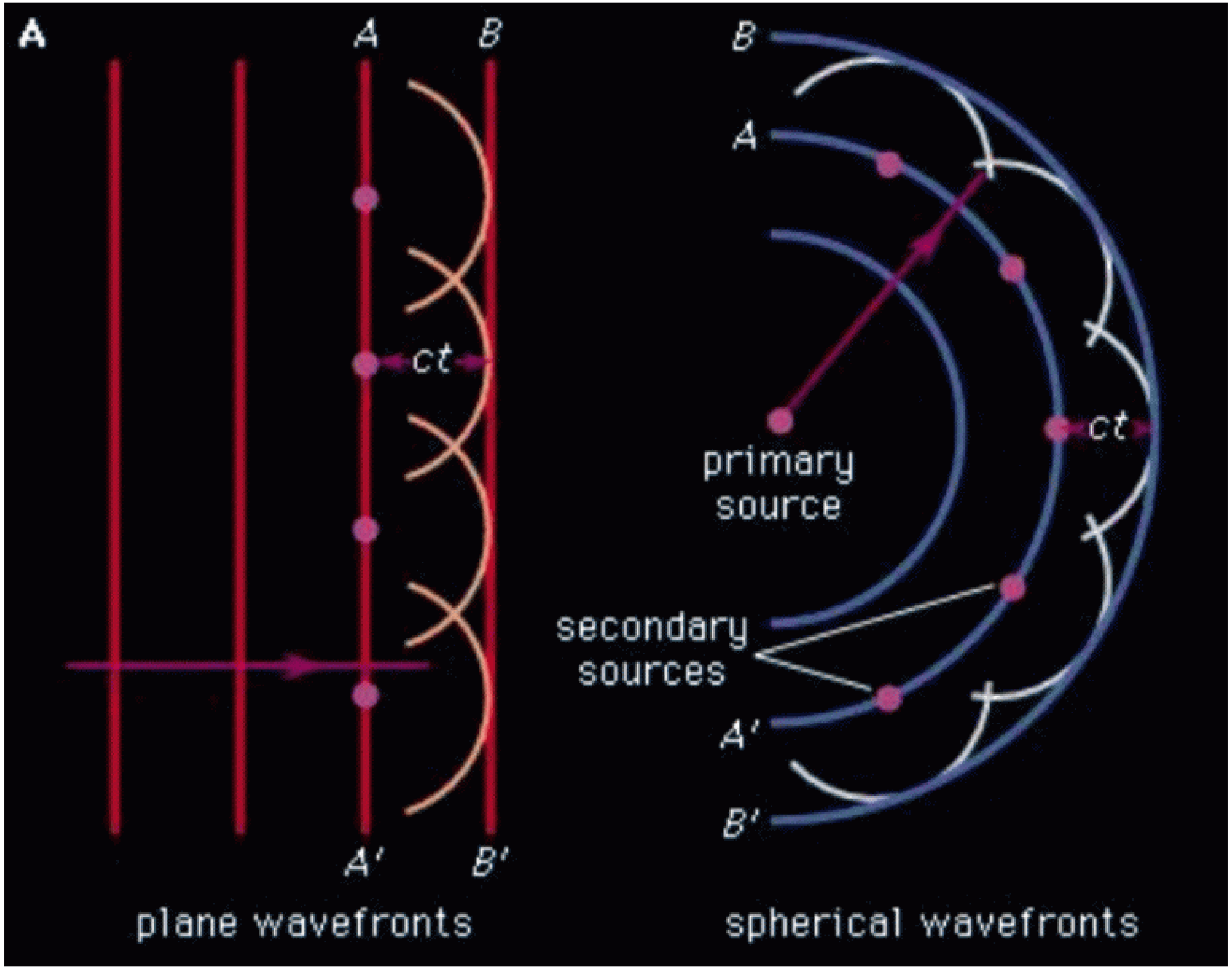
5. Diffraction

- Diffraction is the bending of wave fronts around obstacles.
- Diffraction allows radio signals to propagate behind obstructions.
- Although the received field strength decreases rapidly as a receiver moves deeper into an obstructed (shadowed) region, the diffraction field still exists and often has sufficient signal strength to produce a useful signal.

  And is thus one of the factors why we receive signals at locations where there is no line-of-sight from base stations.

The diffraction phenomenon can be explained by Haygen's principle, which states that all the points on a wavefront can be considered as point sources for production of secondary waves, and these waves combine to produce new waves in the direction of propagation.




A
A
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plane wavefronts
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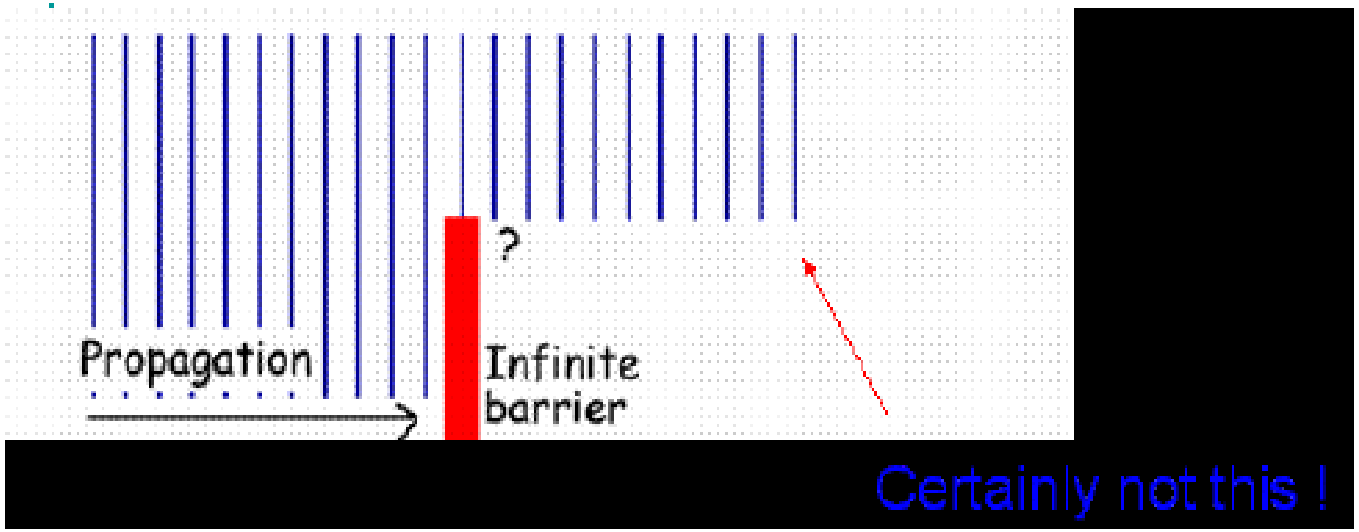
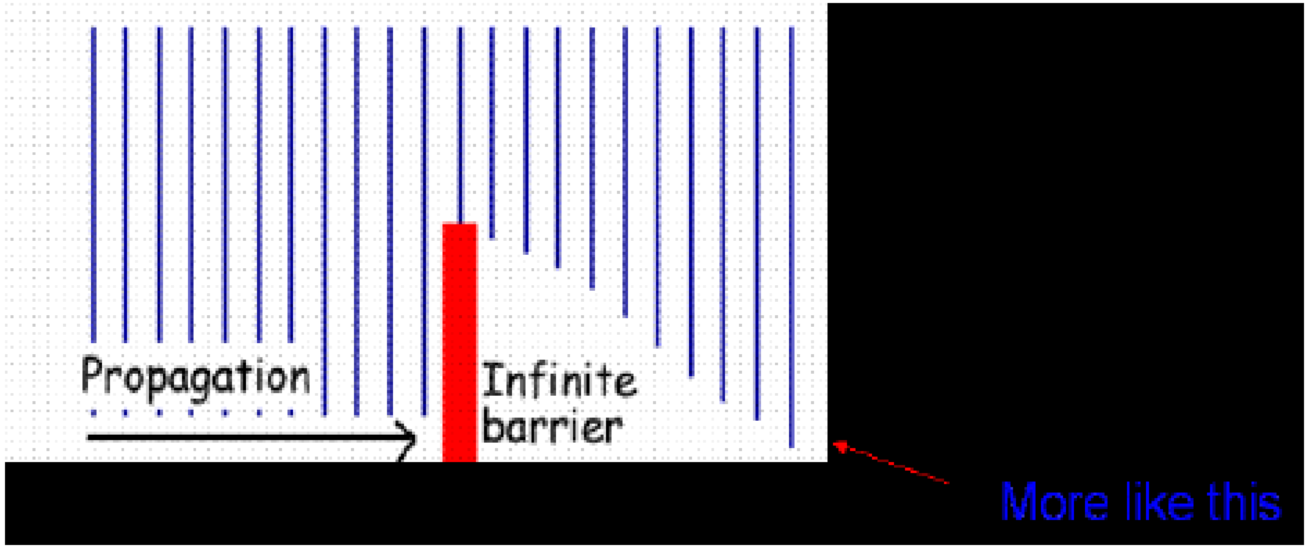
spherical wavefronts



Diffraction is caused by the propagation of secondary waves into a shadowed region.

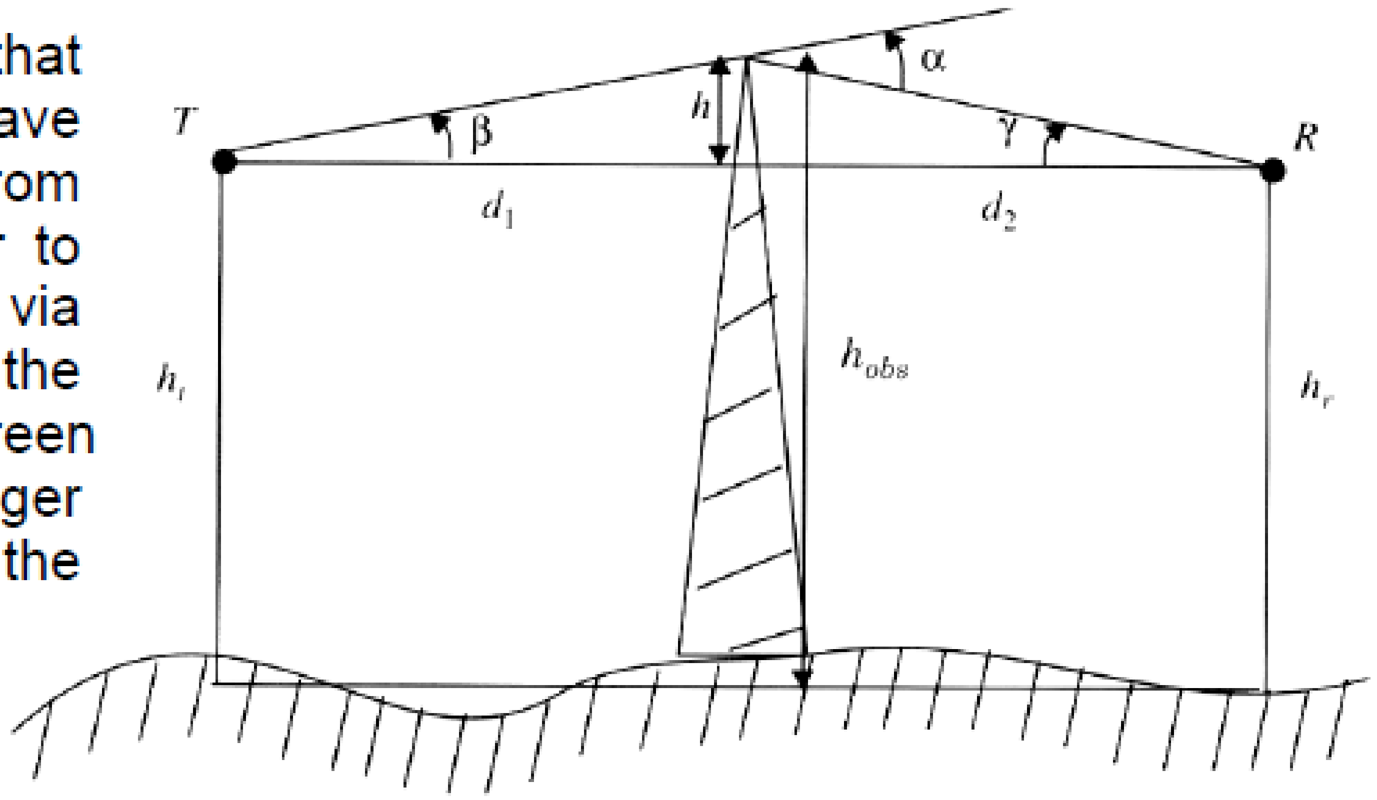


The field strength of a diffracted wave in the in the shadowed region is the vector sum of the electric field components of all the secondary waves in the space around the obstacle.

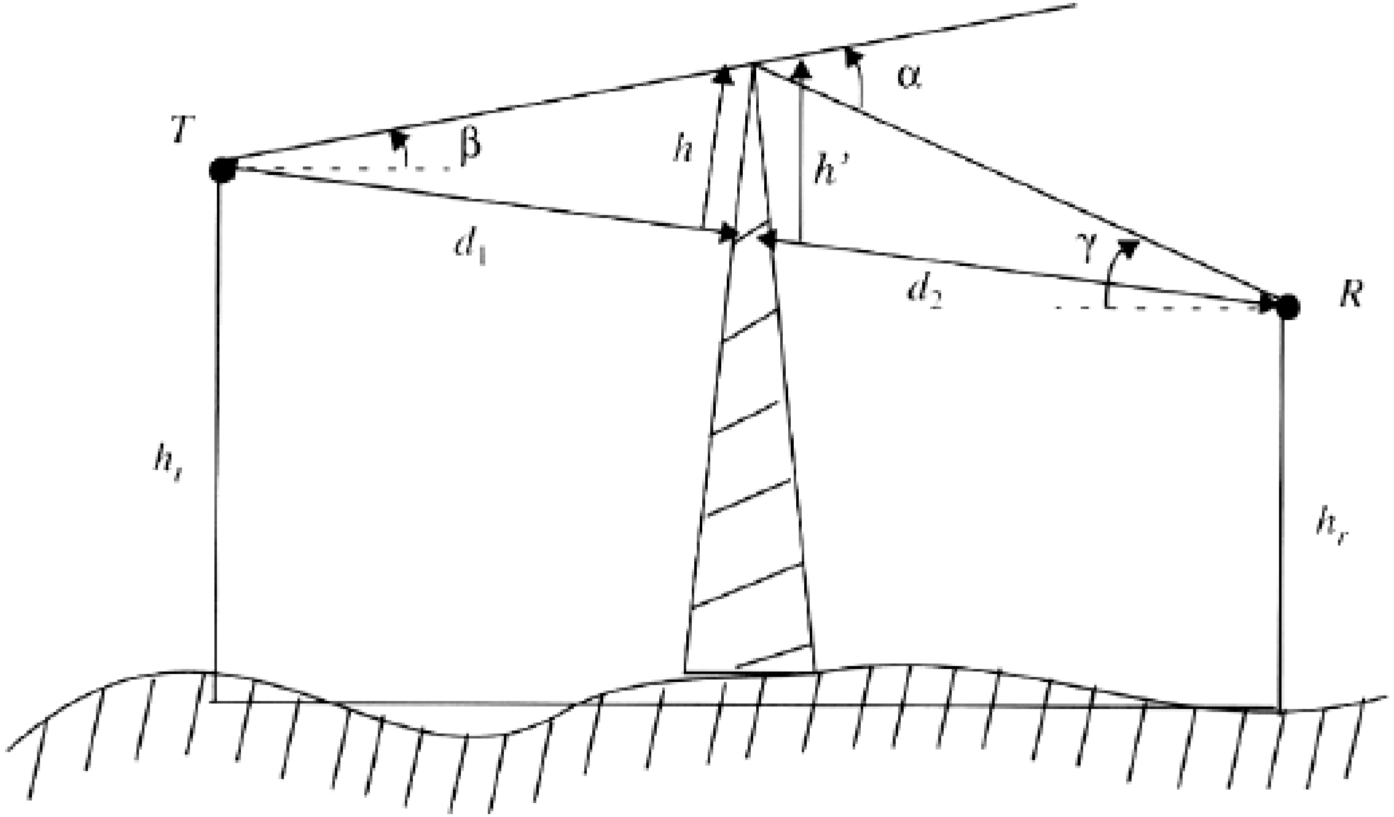
5.1. Fresnel Zone Geometry

Consider a transmitter and receiver separated in free space as shown in below figure. Let an obstructing screen of effective height h with infinite width be placed between them at a distance d1 from the transmitter and d2 from the receiver.

It is apparent that the wave propagating from the transmitter to the receiver via top of the obstructing screen travels longer distance than the LOS path.



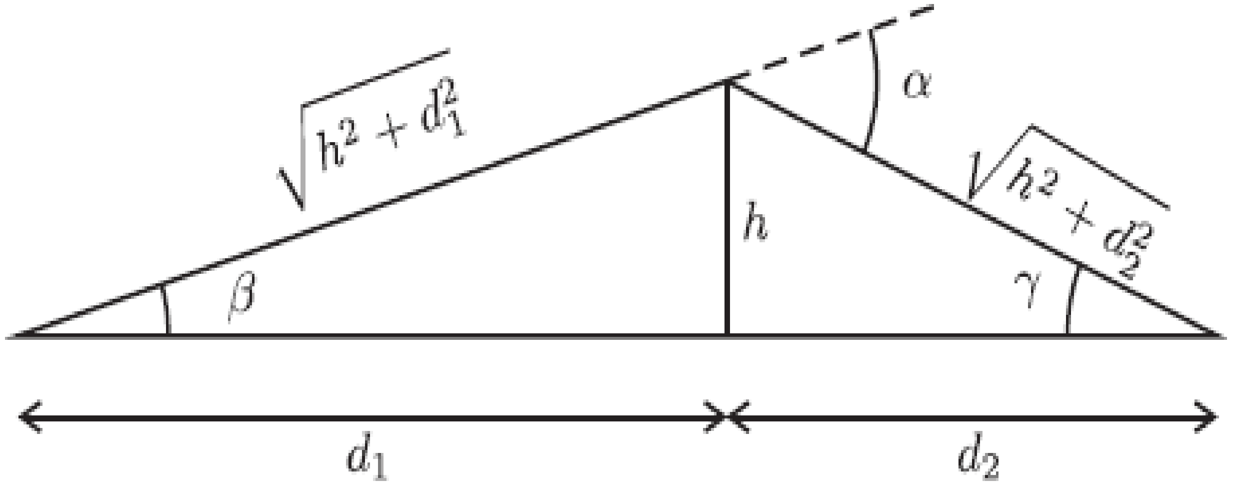
(a) Knife-edge diffraction geometry. The point *T* denotes the transmitter and *R* denotes the receiver, with an infinite knife-edge obstruction blocking the line-of-sight path.



(b) Knife-edge diffraction geometry when the transmitter and receiver are not at the same height. Note that if α and β are small and $h \ll d_1$ and d_2, then h and h' are virtually identical and the geometry may be redrawn as shown in Figure 4.10c.

When d_1 , $d_2 \gg h$, $h \gg \lambda$, the excess path length (difference between the direct path and the diffracted path) is

$$\Delta \approx \frac{h^2}{2}\frac{d_1 + d_2}{d_1 d_2}$$



$$
\begin{aligned}
\Delta &= \sqrt{d_1^2+h^2}+\sqrt{d_2^2+h^2}-(d_1+d_2) \\
&= d_1\sqrt{1+\frac{h^2}{d_1^2}}+d_2\sqrt{1+\frac{h^2}{d_2^2}}-d_1-d_2 \\
&\approx d_1\left(1+\frac{h^2}{2d_1^2}\right)+d_2\left(1+\frac{h^2}{2d_2^2}\right)-d_1-d_2 \\
&= \frac{h^2}{2}\left(\frac{1}{d_1}+\frac{1}{d_2}\right) \\
&= \frac{h^2}{2}\frac{d_1+d_2}{d_1 d_2}
\end{aligned}
$$

The angle $\alpha = \beta + \gamma$. Since $d_1, d_2 \gg h$,

$$\beta = \tan^{-1}\frac{h}{d_1} \approx \frac{h}{d_1}$$

$$\gamma = \tan^{-1}\frac{h}{d_2} \approx \frac{h}{d_2}$$

$$\alpha \approx \frac{h(d_1 + d_2)}{d_1 d_2}$$

The corresponding phase difference is given by

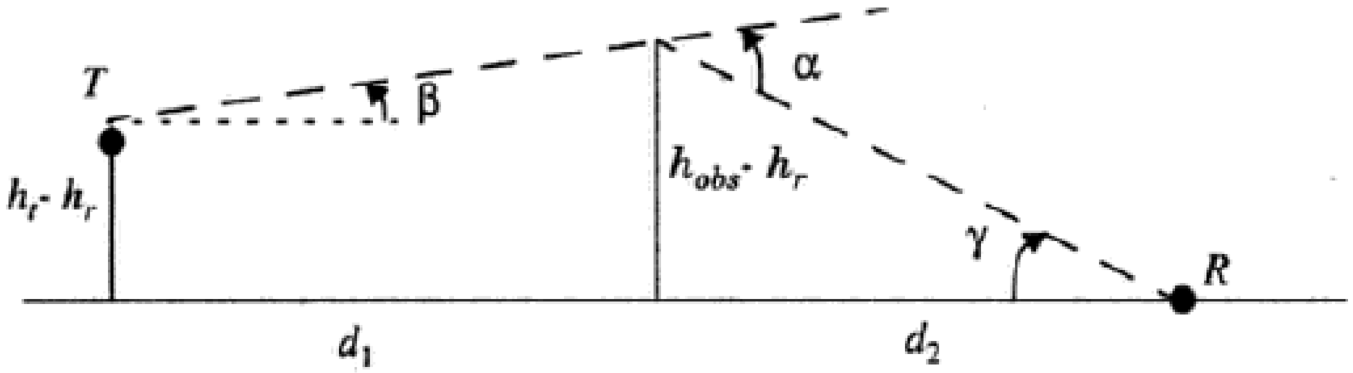
$$\phi = \frac{2\pi\Delta}{\lambda} \approx \frac{2\pi}{\lambda}\frac{h^2}{2}\frac{(d_1 + d_2)}{d_1 d_2} \tag{4.55}$$

and when $\tan x \approx x$, then $\alpha = \beta + \gamma$ from Figure 4.10c and

$$\alpha \approx h\left(\frac{d_1 + d_2}{d_1 d_2}\right)$$

Equation (4.55) is often normalized using the dimensionless *Fresnel–Kirchoff* diffraction parameter v which is given by

$$v = h\sqrt{\frac{2(d_1 + d_2)}{\lambda d_1 d_2}} = \alpha\sqrt{\frac{2 d_1 d_2}{\lambda(d_1 + d_2)}} \tag{4.56}$$



(c) Equivalent knife-edge geometry where the smallest height (in this case h_r) is subtracted from all other heights.

where α has units of radians and is shown in Figure 4.10b and Figure 4.10c. From Equation (4.56), ϕ can be expressed as

$$\phi = \frac{\pi}{2}v^2 \tag{4.57}$$

From the above equations it is clear that the phase difference between a direct line-of-sight path and diffracted path is a function of height and position of the obstruction, as well as the transmitter and receiver location.

Fresnel zones are used to explane the concept of diffraction loss as a function of the path difference around an obstruction.

Fresnel zones represent successive regions where secondary waves have a path length from the transmitter to the receiver which are $n\lambda/2$ greater than the total path length of LOS path.

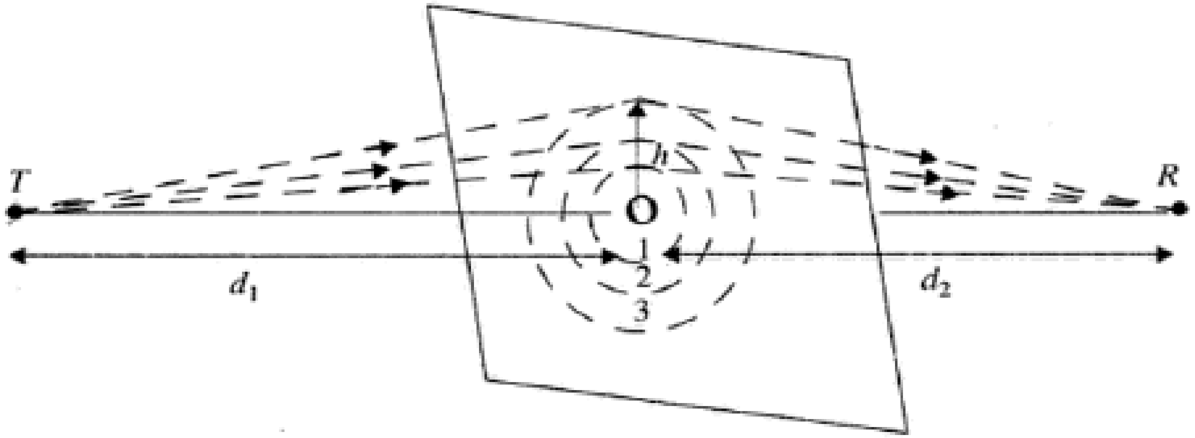


Figure shows a transparent plane is placed between the transmitter and receiver

Figure 4.11 Concentric circles which define the boundaries of successive Fresnel zones.

The concentric circles on the plane represent the loci of the origins of secondary waveswhich propagate to the receiver such that the total path length increses by $\lambda/2$ succesive circles. These circles are called Fresnel zones.

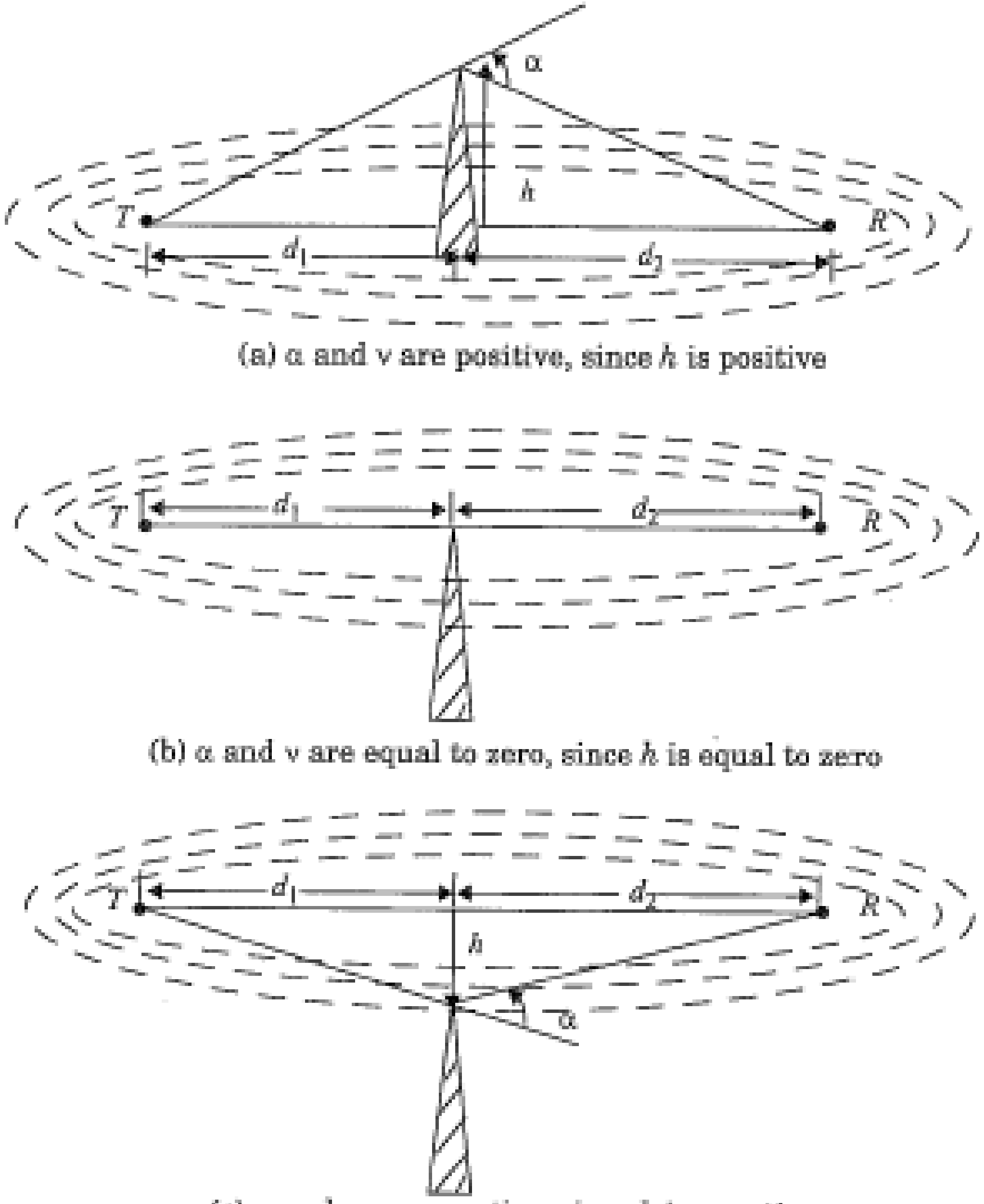
$$r_n = \sqrt{\frac{n\lambda d_1 d_2}{d_1 + d_2}}$$ is the radius corresponding to the nth Fresnel zone,

which has $n\lambda/2$ path difference, or $n\pi$ phase difference to the LOS.

If n is equal to 1, then the excessive path length is $\lambda/2$
If n is equal to 2, then the excessive path length is λ
If n is equal to 3, then the excessive path length is $3\lambda/2$

The radius of the concentric circle depends on the location of the plane. The circle has maximum radius if plane is at the middle and circle has minimum radius when the plane is moved towards either the transmitter or receiver.

In mobile communication systems, the diffraction loss occurs from the blockage of secondary waves such that only portion of the energy is diffracted around an obstacle.
That is, an obstuction causes blockage of energy from some Fresnel zones, thus allowing only some of the transmitted energy to reach the receiver.



(a) α and ν are positive, since h is positive

(b) α and ν are equal to zero, since h is equal to zero

(c) α and ν are negative, since h is negative

Figure 9: Cases of Fresnel zone blockage

5.2. Knife-edge Diffraction Model

- Estimating the signal attenuation caused by diffraction of radio waves over hills and buildings is essential in predicting the field strength in a given service area.
- Generally, it is impossible to make very precise estimate of diffraction losses because of complex and irregular terrain.
- Knife edge diffraction model gives good approximation of magnitude diffraction loss.
- When shadowing is caused by a single object such as a hill or mountain or building, the attenuation caused by diffraction can be estimated by treating the obstruction as a diffracting knife edge
- It is the simplest diffraction model, and the diffraction loss in this model can be estimated using the Fresnel solution.

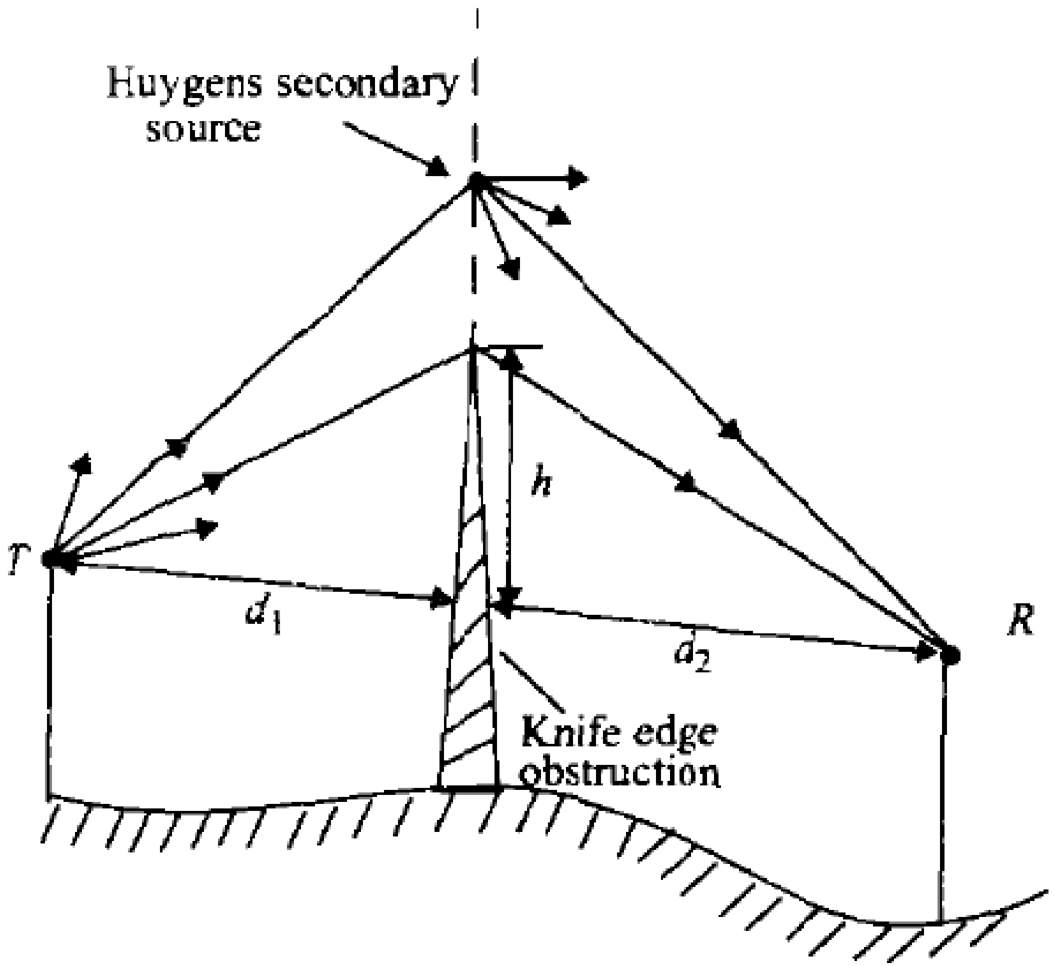


Figure 3.13
Illustration of knife-edge diffraction geometry. The receiver R is located in the shadow region.

- The normalized electric field produced at the receiver, relative to the LOS path (free space field) is,

$$\frac{E_d}{E_{LOS}} = F(v) = \frac{1+j}{2}\int_v^{\infty} \exp\left(-j\frac{\pi}{2}t^2\right) dt,$$

$$F(v) = \frac{1+j}{2}\left[\int_v^{\infty} \cos\left(\frac{\pi}{2}t^2\right) dt - j\int_v^{\infty} \sin\left(\frac{\pi}{2}t^2\right) dt\right]$$

- The diffraction gain due to the presence of a knife edge, as compared to the free space E-field, is given by

$$G_d(\text{dB}) = 20\log|F(v)|$$

- A graphical representation of Gd (dB) as a function of u is given in Figure 3.14.

$G_d(dB) = 0$	$v \le -1$
$G_d(\mathrm{dB}) = 20\log(0.5 - 0.62v)$	$-1 \le v \le 0$
$G_d(\mathrm{dB}) = 20\log(0.5\exp(-0.95v))$	$0 \le v \le 1$
$G_d(\mathrm{dB}) = 20\log(0.4 - \sqrt{0.1184 - (0.38 - 0.1v)^2})$	$1 \le v \le 2.4$
$G_d(\mathrm{dB}) = 20\log\left(\frac{0.225}{v}\right)$	$v > 2.4$

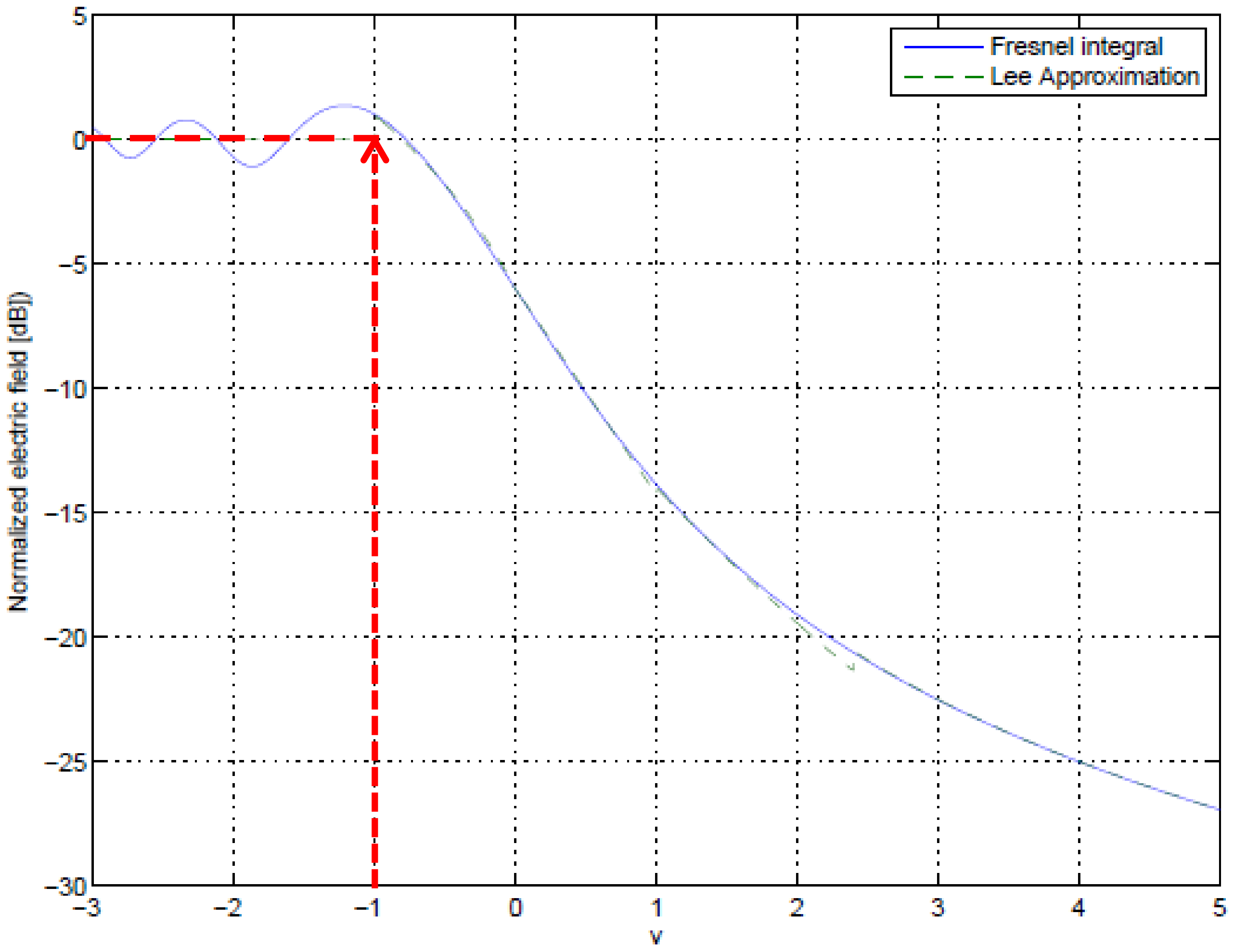


Figure 5: Diffraction gain as a function of v

Example 4.7

Compute the diffraction loss for the three cases shown in Figure 4.12. Assume $\lambda = 1/3$ m, $d_1 = 1$km, $d_2 = 1$ km, and (a) h = 25 m, (b) $h = 0$, (c) h = −25 m. Compare your answers using values from Figure 4.14, as well as the approximate solution given by Equation (4.61.a)–(4.61.e). For each of these cases, identify the Fresnel zone within which the tip of the obstruction lies.

Given:

$\lambda = 1/3$ m

$d_1 = 1$ km

$d_2 = 1$ km

(a)h = 25 m

Using Equation (4.56), the Fresnel diffraction parameter is obtained as

$$v = h\sqrt{\frac{2(d_1 + d_2)}{\lambda d_1 d_2}} = 25\sqrt{\frac{2(1000 + 1000)}{(1/3) \times 1000 \times 1000}} = 2.74.$$

From Figure 4.14, the diffraction loss is obtained as 22 dB.

Using the numerical approximation in Equation (4.61.e), the diffraction loss is equal to 21.7 dB.

The path length difference between the direct and diffracted rays is given by Equation (4.54) as

$$\Delta \approx \frac{h^2}{2}\frac{(d_1 + d_2)}{d_1 d_2} = \frac{25^2}{2}\frac{(1000 + 1000)}{1000 \times 1000} = 0.625 \text{ m}.$$

(b) $h = 0$ m

Therefore, the Fresnel diffraction parameter $v = 0$.

From Figure 4.14, the diffraction loss is obtained as 6 dB.

Using the numerical approximation in Equation (4.61.b), the diffraction loss is equal to 6 dB.

For this case, since $h = 0$, we have $\Delta = 0$, and the tip of the obstruction lies in the middle of the first Fresnel zone.

(c) $h = -25$ m

Using Equation (4.56), the Fresnel diffraction parameter is obtained as –2.74.

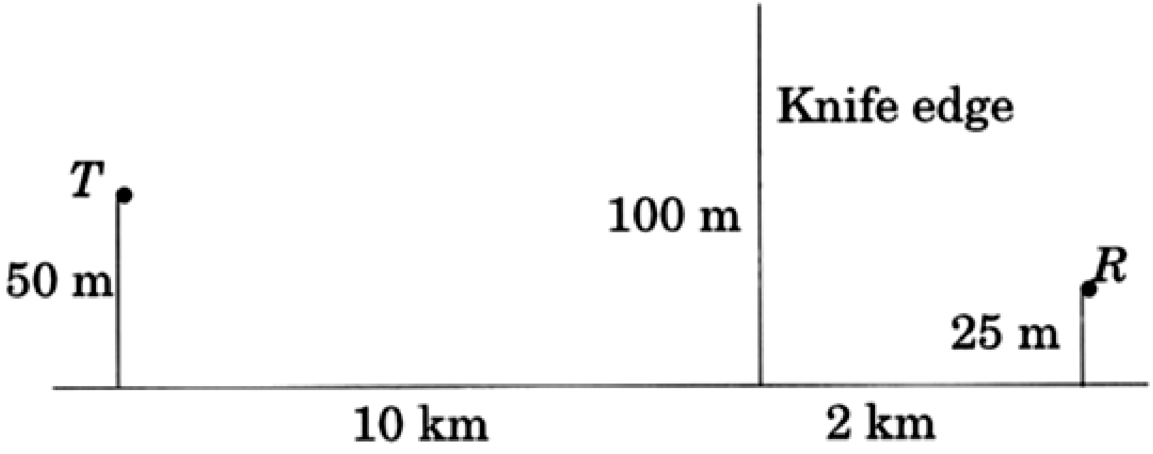
From Figure 4.14, the diffraction loss is approximately equal to 1 dB.

Using the numerical approximation in Equation (4.61.a), the diffraction loss is equal to 0 dB.

Since the absolute value of the height h is the same as part (a), the excess path length Δ and hence n will also be the same. It should be

Example 4.8

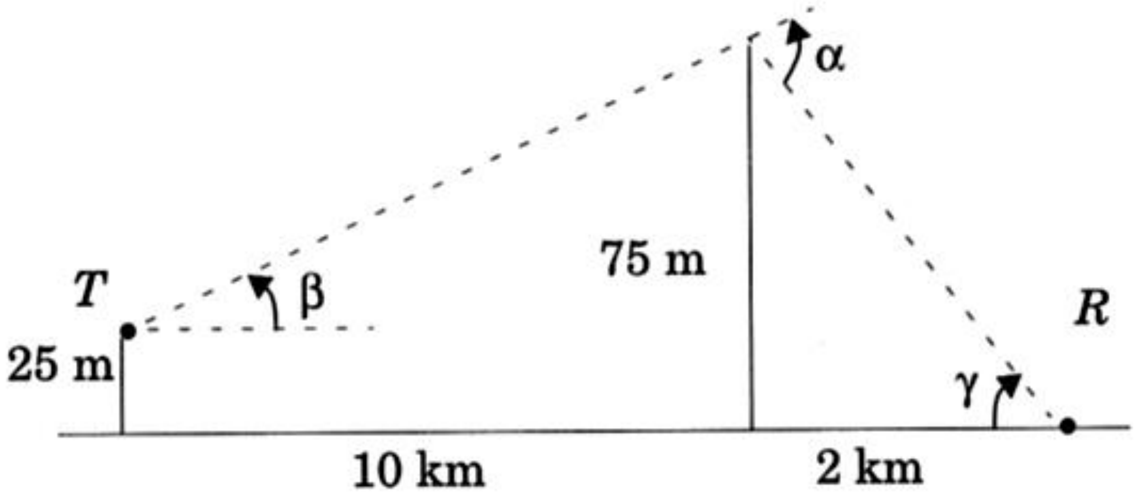
Given the following geometry, determine (a) the loss due to knife-edge diffraction, and (b) the height of the obstacle required to induce 6 dB diffraction loss. Assume $f = 900$ MHz.



Solution

(a) The wavelength $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{900 \times 10^6} = \frac{1}{3}$ m.

Redraw the geometry by subtracting the height of the smallest structure.



$$\beta = \tan^{-1}\left(\frac{75-25}{10000}\right) = 0.2865°$$

$$\gamma = \tan^{-1}\left(\frac{75}{2000}\right) = 2.15°$$

and

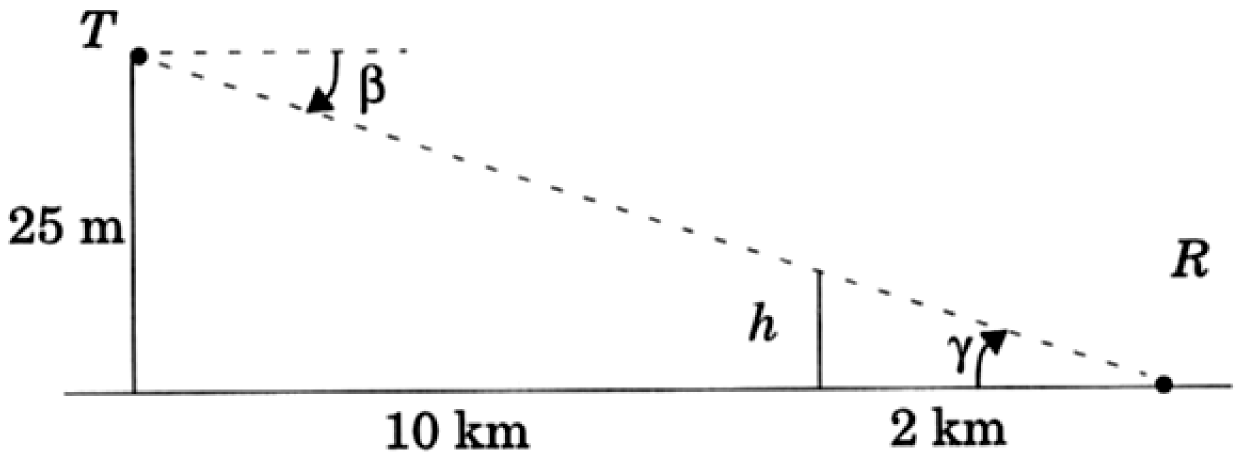
$$\alpha = \beta + \gamma = 2.434° = 0.0424 \text{ rad}$$

Then using Equation (4.56)

$$\nu = 0.0424\sqrt{\frac{2 \times 10000 \times 2000}{(1/3) \times (10000 + 2000)}} = 4.24.$$

From Figure 4.14 or (4.61.e), the diffraction loss is 25.5 dB.

(b) For 6 dB diffraction loss, $\nu = 0$. The obstruction height h may be found using similar triangles ($\beta = \gamma$), as shown below.



It follows that $\frac{h}{2000} = \frac{25}{12000}$, thus $h = 4.16$ m.

5.3.Multiple Knife-edge Diffraction

- In many practical situations, especially in hilly terrain, the propagation path may consist of more than one obstruction, in which case the total diffraction loss due to all of the obstacles must be computed.
- Bullington suggested that the series of obstacles be replaced by a single equivalent obstacle so that the path loss can be obtained using single knife-edge diffraction models.

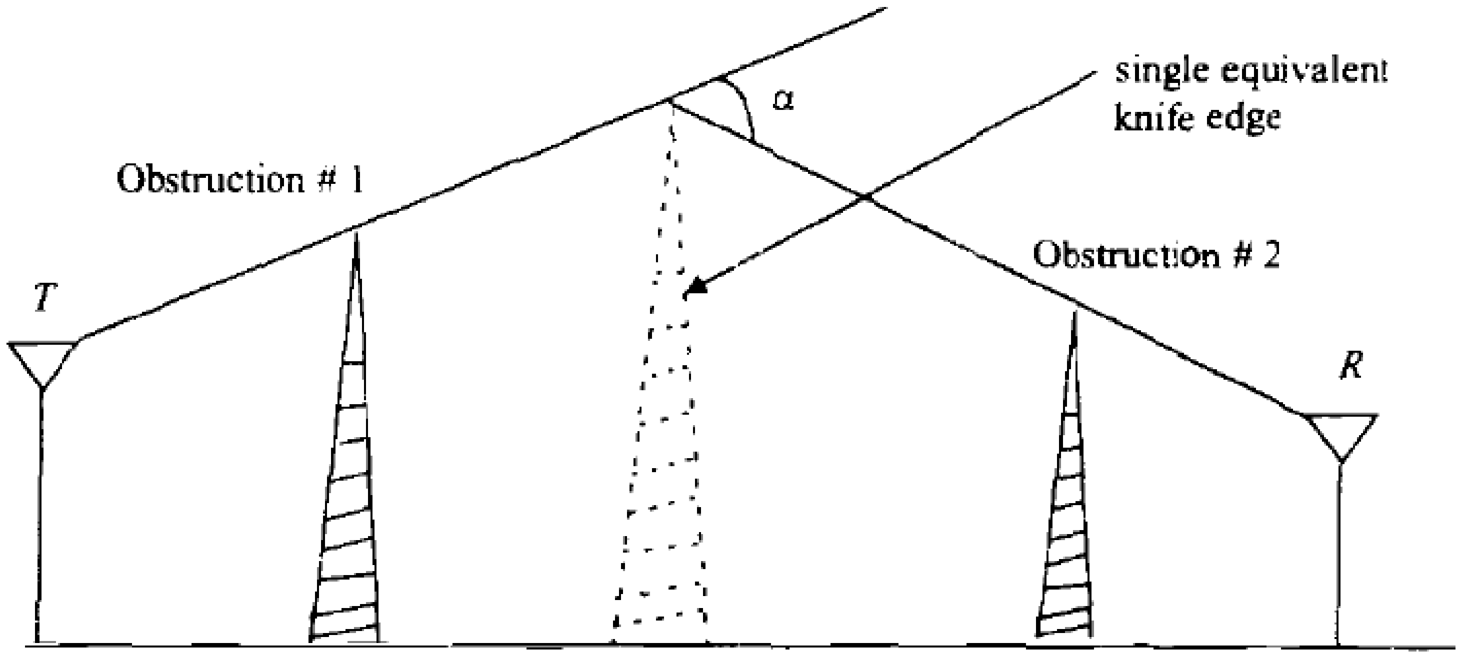
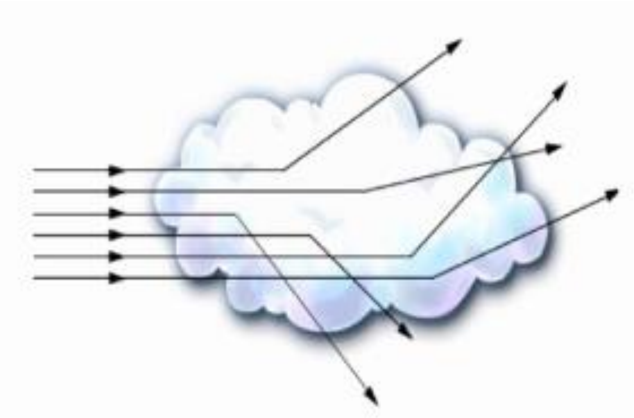
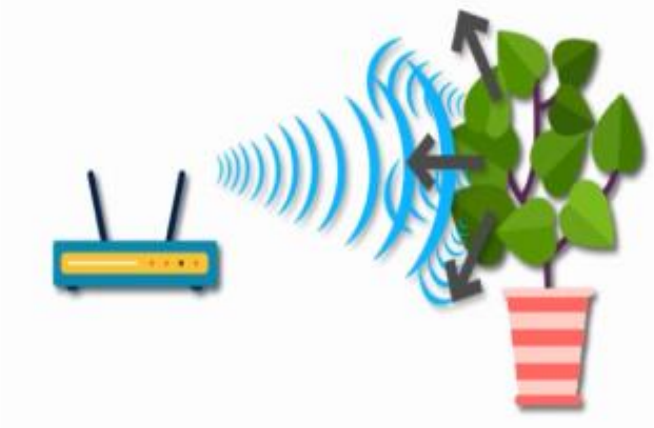


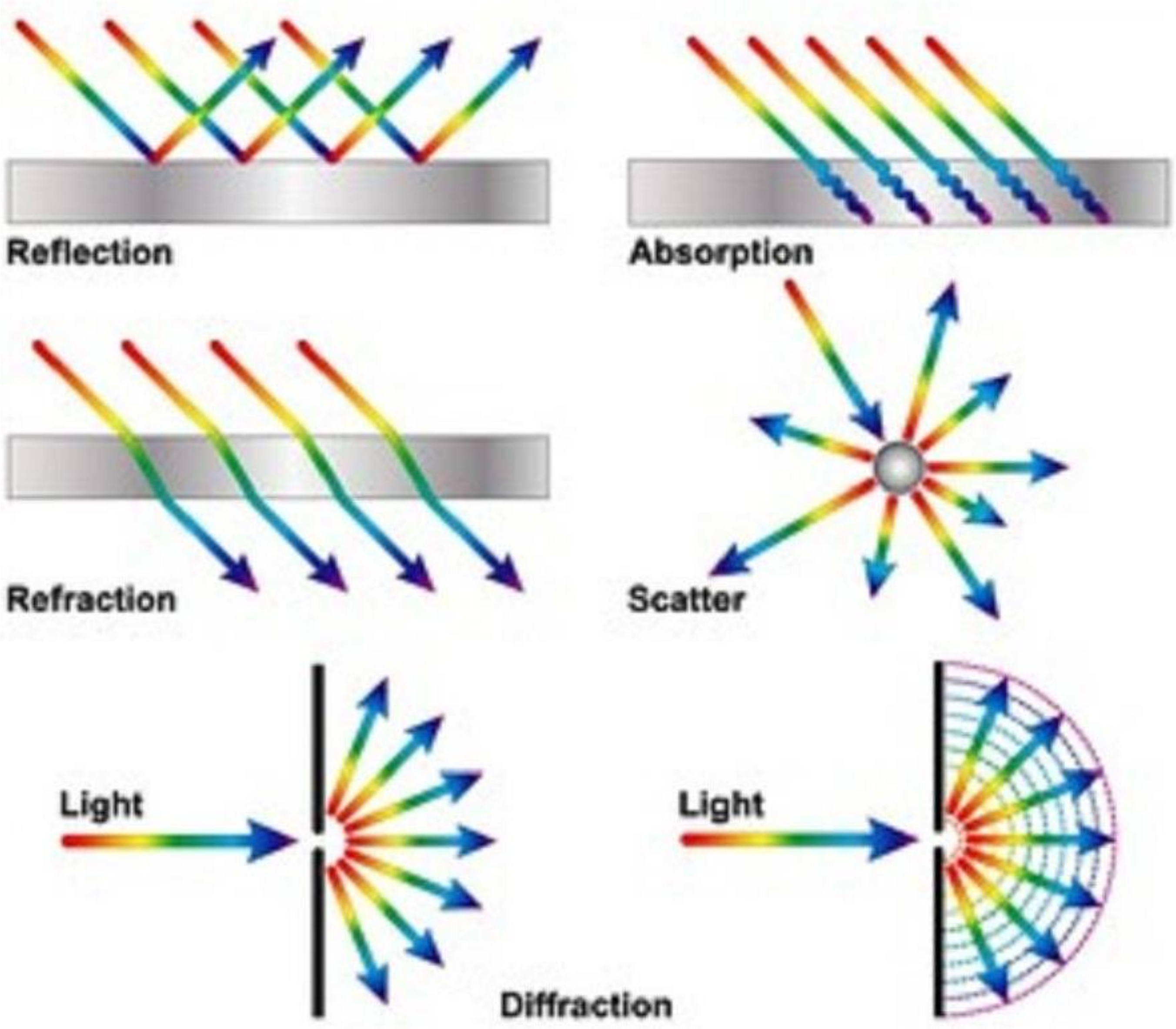
Figure 3.15
Bullington's construction of an equivalent knife edge [From [Bul47] © IEEE].

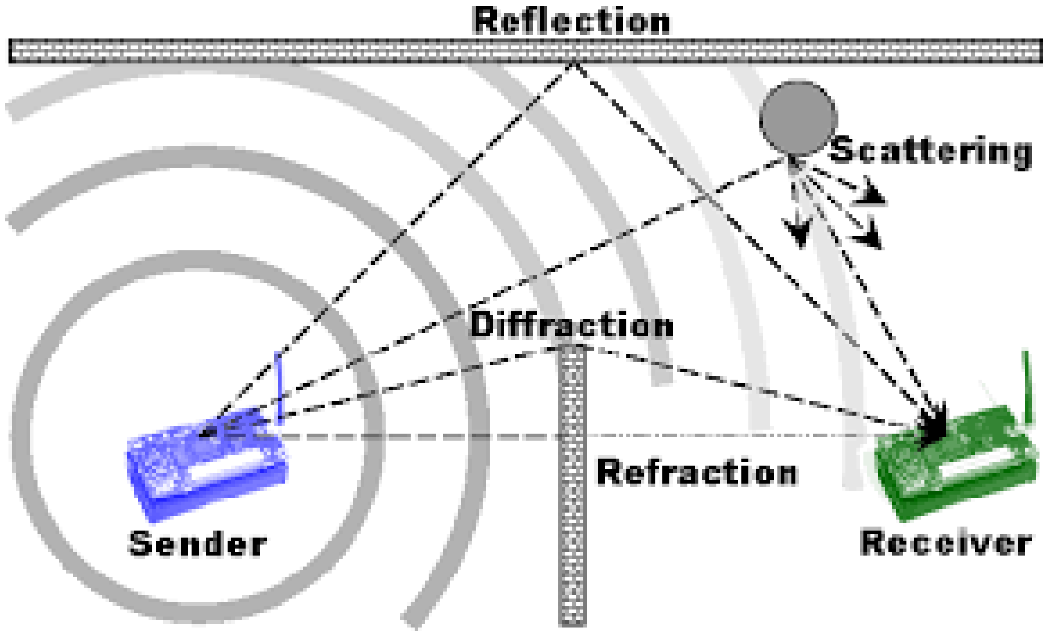
6. Scattering

- Scattering occurs when medium has objects that are smaller or comparable to the wavelength of the signal.
- Scattered waves are produced by droplets, rough surfaces, rain drops, snow, small objects, or by irregularities in the channel, foliage, street signs etc.




Reflection
Absorption
Refraction
Scatter
Light
Light
Diffraction

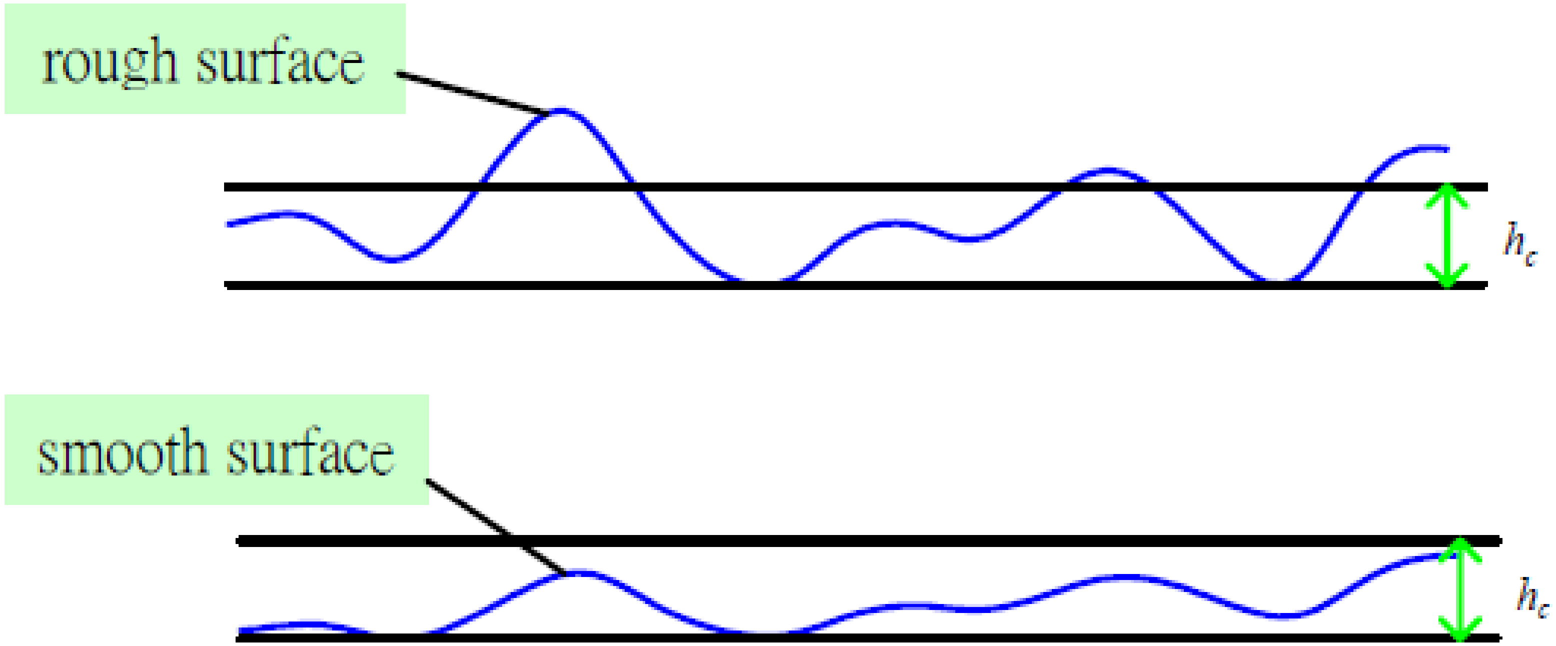

Reflection
Scattering
Diffraction
Refraction
Sender
Receiver

- The actual received signal in a mobile radio environment is often stronger than what is predicted by reflection and diffraction models alone.
- This is because when a radio wave impinges on a rough surface, the reflected energy is spread out (diffused) in all directions due to scattering.
- Objects such as lamp posts and trees tend to scatter energy in all directions, thereby providing additional radio energy at a receiver.
- Flat surface may be modeled EM reflection (one direction).
- Rough surface may be modeled EM scattering (many directions).

- Surface roughness is often tested using the Rayleigh criterion which defines a critical height (h_c) of surface protuberances for a given angle of incidence θ_i, given by

$$h_c = \frac{\lambda}{8\ Sin\ \theta_i}$$

- If minimum to maximum roughness height $h < h_c$ then the surface is smooth
- If minimum to maximum roughness height $h > h_c$ then the surface is rough



- Ament assumed that the surface height h is a Gaussian distributed random variable with a local mean and found Scattering loss factor ρ_s to be given by:

$$\rho_s = exp\left[\left(-8\left(\frac{\pi\sigma_h \, sin\, \theta_i}{\lambda}\right)\right)^2\right]$$

where σ_h is the standard deviation of the surface height about the mean surface height.

- The scattering loss factor derived by Ament was modified by Boithias to give better agreement with measured results, and is given by:

$$\rho_s = exp\left[\left(-8\left(\frac{\pi\sigma_h \, sin\, \theta_i}{\lambda}\right)\right)^2\right] I_o\left[\left(8\left(\frac{\pi\sigma_h \, sin\, \theta_i}{\lambda}\right)\right)^2\right]$$

where I_o is the Bessel function of the first kind and zero order.

- For rough surface, the flat surface reflection coefficient is multiplied by scattering loss factor ρs to account for diminished electric field

- The reflected E-fields for $h > h_c$ can be solved for rough surfaces using a modified reflection coefficient given as:

$$\Gamma_{rough} = \rho_s \, \Gamma$$

6.1 Radar Cross Section Model

- In radio channels where large, distant objects induce scattering. So the knowledge of physical location of such objects can be used to accurately predict scattered signal strengths.

- The *radar cross section (RCS) of a scattering object* is defined as the *ratio of the power density of the signal scattered in the direction of the receiver to the power density of the radio wave incident upon the scattering object, and has units of square meters.*

- For urban mobile radio systems, the bistatic radar equation based models may be used to compute the received power due to scattering in the far field.

- The bistatic radar equation describes the propagation of a wave traveling in free space which impinges on a distant scattering object, and is then reradiated in the direction of the receiver, given by

$$P_R(dBm) = P_T(dBm) + G_T(dBi) + 20\log(\lambda) + RCS[dBm^2]$$
$$-30\log(4\pi) - 20\log d_T - 20\log d_R$$

Where

d_T distance from the scattering object to the transmitter

d_R distance from the scattering object to the receiver

- The above Equation may be applied to scatterers in the far-field of both the transmitter and receiver and is useful for predicting receiver power which scatters off large objects, such as buildings, which are for both the transmitter and receiver.

7. Practical Link Budget Design using Path Loss Models

- The early models (like two-ray, knife-edge diffraction) are oversimplified.
- Most radio propagation models are derived using a combination of analytical and empirical models.
  - Empirical method is based on collecting measurement data and fitting into curves
  - Analytical methods model the propagation mechanism mathematically and derive equations for path loss

$$P_r(\mathrm{d}) = P_t - \overline{PL}\,(d)$$

- Over many years, some classical propagation models have been developed, which are used to predict large-scale coverage for mobile communication system design.

- By using path loss estimation models to estimate the received signal level as a function of distance, it becomes possible to predict the SNR for a mobile communication system.

  ✓ Log-Distance Path Loss Model

  ✓ Log-Normal Shadowing model

7.1. Log-Distance Path Loss Model

- The average large-scale path loss for an arbitrary T-R separation is expressed as a function of distance by using a path loss exponent, *n*.

$$\overline{PL}(d) \propto \left(\frac{d}{d_o}\right)^n$$

or

$$\overline{PL}(d)[dB] = \overline{PL}(d_o)[\text{dB}] + 10n\log\left(\frac{d}{d_o}\right)$$

$$\text{Path loss at } d_0 = P_T / P(d_0) = \overline{PL}(d_0)$$

$$\text{Path loss at } d = P_T / P(d) = \overline{PL}(d)$$

$$\frac{\overline{PL}(d)}{\overline{PL}(d_0)} = \left(\frac{d}{d_0}\right)^n$$

Where

n: path loss exponent which indicates the rate at which the path loss increases with distance,

d_o: reference distance which is determined from measurements close to the transmitter,

d: T-R separation distance. More is the distance *d* more is the path loss $\overline{PL}$

- The bars in equations denote the *ensemble average of all possible path loss values* for a given value of *d*.
- *When plotted on a log-log scale, the modeled path loss is a straight line with a slope equal to 10n dB per decade.*
- *The value of n depends on the specific propagation environment. For example, in free space, n is equal to 2, and when obstructions are present, n will have a larger value.*

Table 4.2 Path Loss Exponents for Different Environments

Environment	Path Loss Exponent, n
Free space	2
Urban area cellular radio	2.7 to 3.5
Shadowed urban cellular radio	3 to 5
In building line-of-sight	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3

7.2. Log-Normal Shadowing Path Loss Model

- Log-distance path loss gives only the average value of path loss. It does not consider the shadowing effects.
- Surrounding environment may be vastly different at two locations having the same T–R separation *d*.
- More accurate model includes a random variable to account for change in environment.

$$PL(d)[\mathrm{dB}] = \overline{PL}(d)[\mathrm{dB}] + X_\sigma = \overline{PL}(d_0)[\mathrm{dB}] + 10n\log\left(\frac{d}{d_0}\right) + X_\sigma$$

and
$$P_r(d)[\mathrm{dBm}] = P_t(d)[\mathrm{dBm}] - PL(d)[\mathrm{dB}]$$

X_σ : Zero mean Gaussian random variable (dB)

σ : Standard deviation (dB)

- The log-normal distribution describes the random shadowing effects which occur over a large number of measurement locations which have the same T-R separation. This phenomenon is referred to as log-normal shadowing.
- Q function or error function (erf) can be used to determine the probability that the received signal will exceed (or fall below) a particular level.

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_{z}^{\infty} \exp\left(-\frac{x^2}{2}\right) dx = \frac{1}{2}\left[1 - erf\left(\frac{z}{\sqrt{2}}\right)\right]$$

$$Q(-z) = 1 - Q(z)$$

$$Q(0) = 0.5$$

Outage probability:

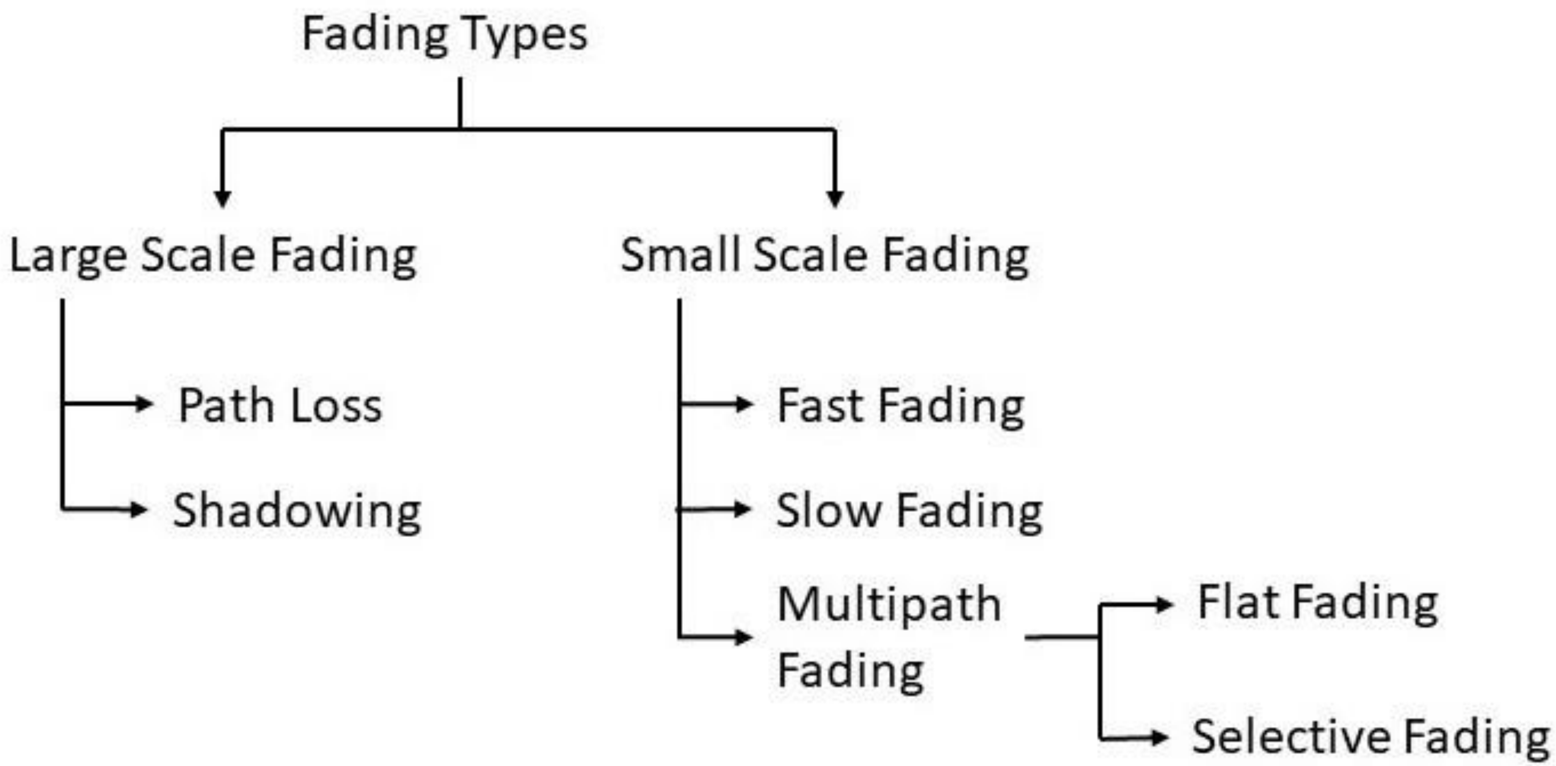
- The probability that the received signal level (in dB power unit) will exceed a certain value $\boldsymbol{\gamma}$ can be calculated from the cumulative density function as:

$$\Pr[P_r(d) > \gamma] = Q\left(\frac{\gamma - \overline{P_r(d)}}{\sigma}\right)$$

- The probability that the received signal level will be below $\boldsymbol{\gamma}$ can be calculated from:

$$\Pr[P_r(d) < \gamma] = Q\left(\frac{\overline{P_r(d)} - \gamma}{\sigma}\right)$$

7.3. Determination of Percentage of Coverage Area


Fading Types
Large Scale Fading
Small Scale Fading
Path Loss
Shadowing
Fast Fading
Slow Fading
Multipath Fading
Flat Fading
Selective Fading

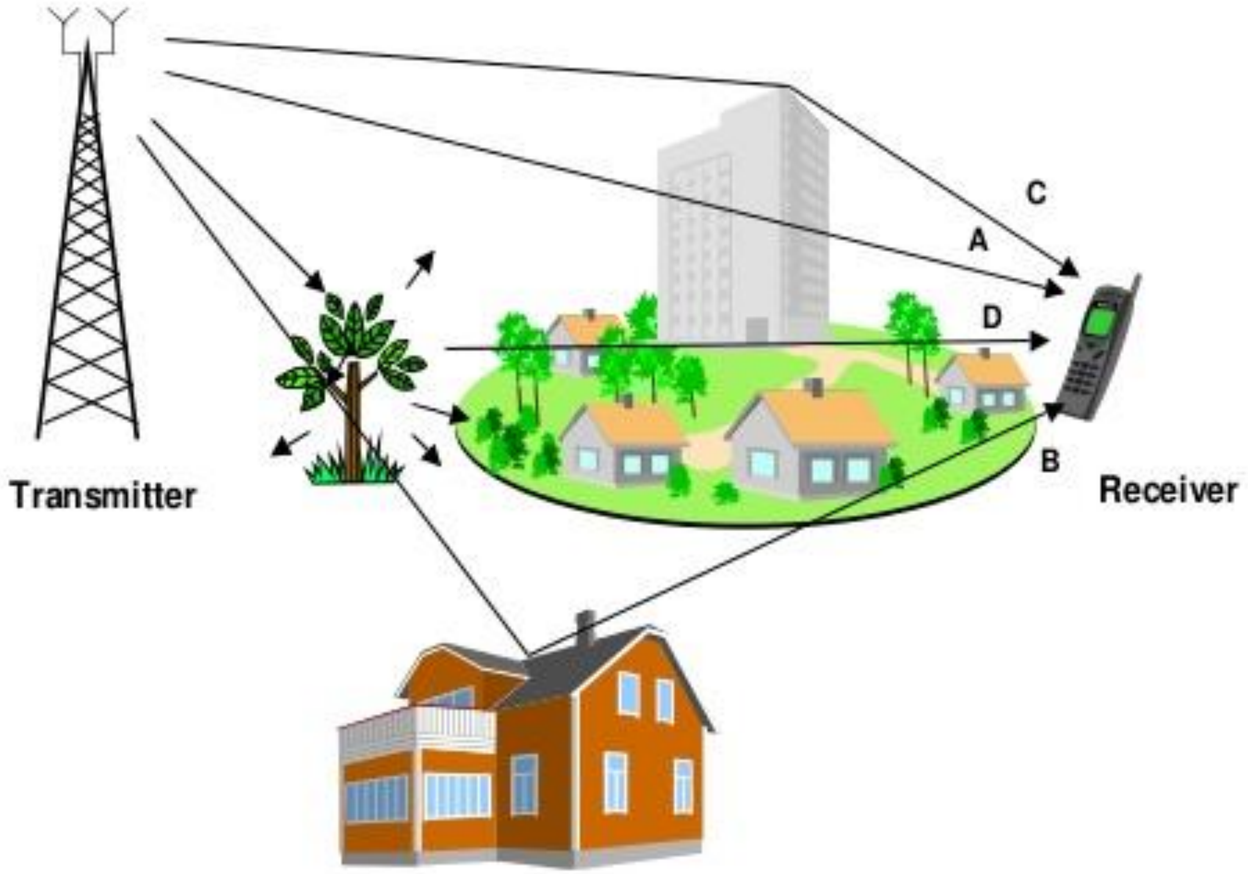
II. Small-Scale Fading

Small Scale Fading: Multipath Propagation, Parameters of mobile multipath channel, Types of small scale fading: Fading effects due to multipath time delay spread and Doppler spread.

Introduction to Small-scale fading

- Small-scale fading, or simply *fading, is used to* describes the rapid fluctuation of the amplitudes, phases, or multipath delays of a radio signal over a short period of time or travel distance.
- Fading is caused by interference between two or more versions of the transmitted signal (out of phase) which arrive at the receiver at slightly different times.
- These waves, called **multipath waves**, **combine vectorially at the receiver antenna** to give a resultant signal which can vary widely in amplitude and phase, **produce a fade or distortion**.
- At a receiver the radio waves generated by same transmitted signal may come
    I. From different directions
    II. With different propagation delays
    III. With different amplitudes
    IV. With different phases

Multi path propagation effect



A: free space
B: reflection
C: diffraction
D: scattering

1. Small-scale multipath propagation

- Multipath in the radio channel creates **small-scale fading effects**.
- The three **most important fading effects** are
  I. Rapid changes in signal strength over a small travel distance or time interval.
  II. Random frequency modulation due to varying Doppler shifts on different multipath signals
  III. Time dispersions (echos) caused by multipath propagation delays

- **In built-up urban areas, fading occurs because:**
  I. the height of the mobile antennas are well below the height of the surrounding structures (So NLOS to the base station)
  II. Even in LOS exists, reflection from the ground and surrounding structures
  III. Even when a mobile receiver is stationary, the received signal may fade due to a non-stationary nature of the channel (reflecting objects can be moving)

2. Factors Influencing Small-scale Fading

- The following physical factors in the radio propagation channel influence small-scale fading:

I. **Multipath propagation:**
   - The presence of reflecting objects and scatterers in the space between transmitter and receiver creates a constantly changing channel environment that dissipates signal energy in amplitude, phase, and time.
   - Causes the signal at receiver to fade or distort.

II. **Speed of the mobile receiver:**
   - The relative motion between the transmitter and receiver results in a random frequency modulation due to different Doppler shifts on each of the multipath signals. (The shift in received signal frequency due to motion is called the Doppler shift)
   - Doppler shift may be positive or negative depending on whether the mobile receiver is moving toward or away from the base station.

3. **Speed of the surrounding objects:**

- If the objects in the radio channel are in motion, they introduce a time varying Doppler shift on multipath components.
- If the speed of surrounding objects is greater than mobile, then this effect dominates the small-scale fading.
- If the surrounding objects are slower than the mobile, then their effect can be ignored.

4. The transmission bandwidth of the signal:

- Coherent bandwidth: bandwidth of the multipath channel.
- The received signal will be distorted if the transmission bandwidth is greater than the channel coherent bandwidth.

3. Doppler shift

- Due to the relative motion between the mobile receiver and base station, each multipath wave experiences an apparent shift in frequency.

- The shift in received signal frequency due to motion is called the Doppler shift.

- It is directly proportional to
  1. the velocity of the mobile and
  2. the direction of motion of the mobile with respect to the direction of arrival of the received wave.

- Consider a mobile moving at a constant velocity *v*, along a path segment having length *d* between points *X* and *Y.*
- The mobile receives signals from a remote source *S* as illustrated in figure.
- The difference in path lengths traveled by the wave from source *S* to the mobile at points X and Y is called ***path length difference***.

$$\Delta l = d\cos\theta = v\Delta t\cos\theta$$

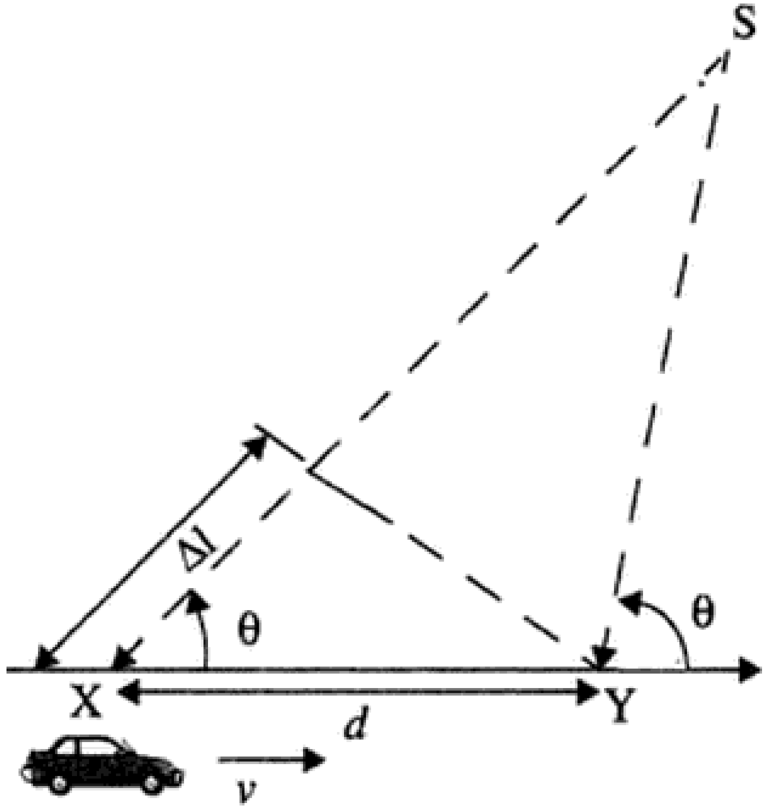
where **Δt** is the time required for the mobile to travel

from X and Y

θ angle of arrival of the wave,

is assumed to be same at X and Y since the

source is very far away from.



- The ***phase change*** in the received signal due to the path length differences is therefore:

$$\Delta\phi = \frac{2\pi\Delta l}{\lambda} = \frac{2\pi v\Delta t}{\lambda}\cos\theta$$

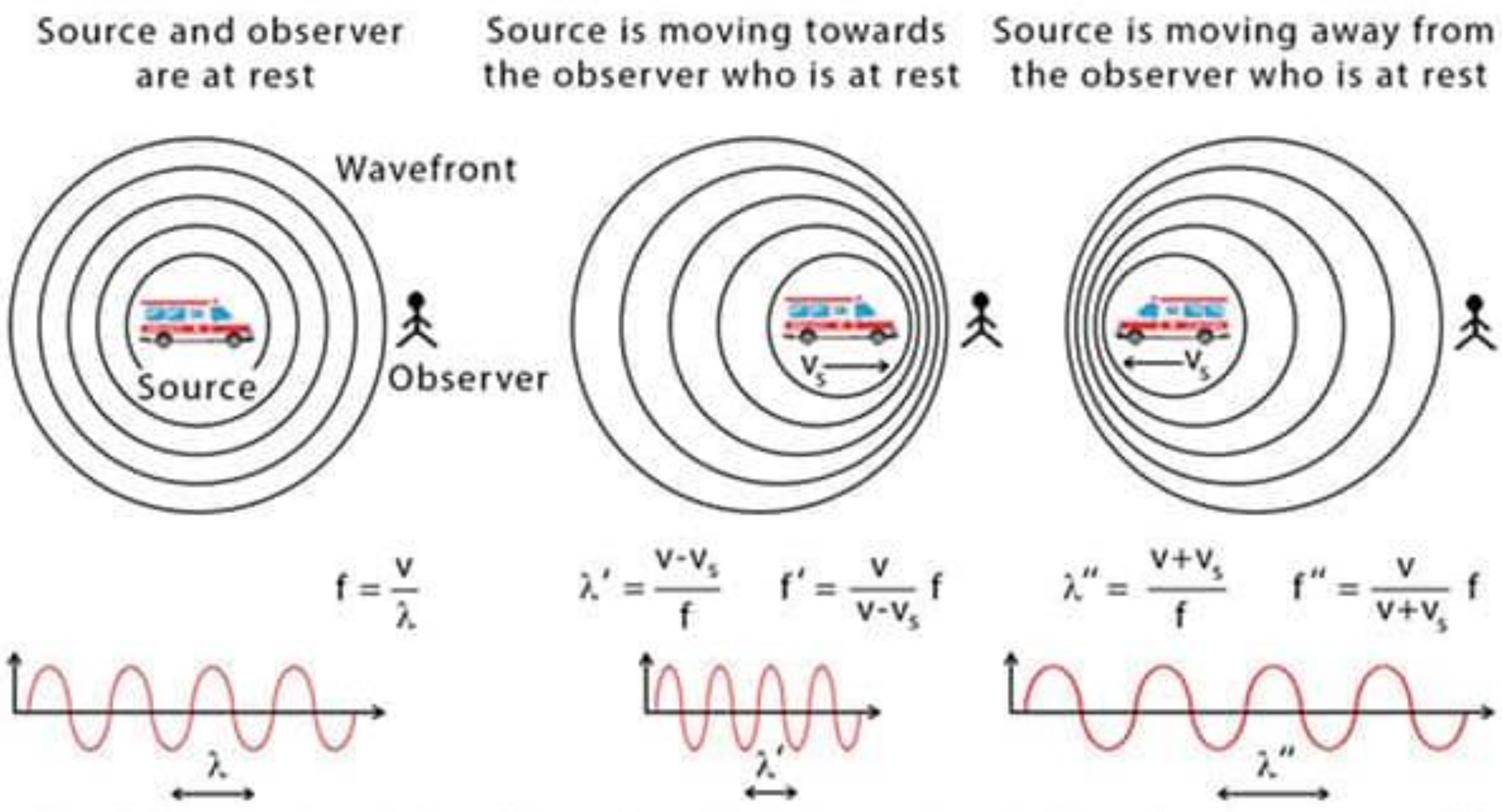
- And the apparent change in frequency, or ***Doppler shift***, is given by ***fd***:

$$f_d = \frac{1}{2\pi}\cdot\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}\cos\theta$$

∴ ΔØ=Δ*w* x Δt

ΔØ=Δ2*πf* x Δt

- If **mobile is moving towards the direction of arrival** of the wave, the Doppler shift is **positive** (apparent received frequency is increased i.e. ***fc+fd***).
- If mobile is **moving away from the direction of arrival of the wave**, the Doppler shift is **negative**(apparent received frequency is decreased i.e. ***fc-fd***).



Wavelength (λ) and frequency (f) of sound waves emited by the source, and are moving with a velocity v

Motion of the source that is moving with velocity v_s relative to the observer alters the wavelength (λ', λ'') and frequency (f', f'') of sound waves

Example 5.1

Consider a transmitter which radiates a sinusoidal carrier frequency of 1850 MHz. For a vehicle moving 60 mph, compute the received carrier frequency if the mobile is moving (a) directly toward the transmitter, (b) directly away from the transmitter, and (c) in a direction which is perpendicular to the direction of arrival of the transmitted signal.

Solution

Given:

Carrier frequency $f_c = 1850$ MHz

Therefore, wavelength $\lambda = c/f_c = \dfrac{3 \times 10^8}{1850 \times 10^6} = 0.162$ m

Vehicle speed $v = 60$ mph $= 26.82$ m/s

(a) The vehicle is moving directly toward the transmitter.

The Doppler shift in this case is positive and the received frequency is given by Equation (5.2)

$$f = f_c + f_d = 1850 \times 10^6 + \frac{26.82}{0.162} = 1850.00016 \text{ MHz}$$

(b) The vehicle is moving directly away from the transmitter.

The Doppler shift in this case is negative and hence the received frequency is given by

$$f = f_c - f_d = 1850 \times 10^6 - \frac{26.82}{0.162} = 1849.999834 \text{ MHz}$$

(c) The vehicle is moving perpendicular to the angle of arrival of the transmitted signal.

In this case, $\theta = 90°$, $\cos\theta = 0$, and there is no Doppler shift.

The received signal frequency is the same as the transmitted frequency of 1850 MHz.

1 mph = 1 mile per hour

$= \dfrac{1 \text{ mile}}{1 \text{ hour}}$

$= \dfrac{1609.34 \text{ meters}}{3600 \text{ seconds}}$

$= 0.447 \dfrac{\text{meters}}{\text{seconds}}$

$= 0.447$ m/s

Hence,

mph $\xrightarrow{0.447}$ m/s

In the U.S. digital cellular system, if $f_c = 900$ MHz and the mobile velocity is 70 km/hr, calculate the received carrier frequency if the mobile (a) directly toward the transmitter (Positive Doppler Shift), (b) directly away from the transmitter (Negative Doppler Shift), and (c) in a direction perpendicular to the direction of the arrival of the transmitted signal.

Solution

Given:

Carrier frequency $f_c = 900$ MHz

Therefore, wavelength $\lambda = c/f_c = \dfrac{3 \times 10^8}{900 \times 10^6} = 1/3\,\text{m} = 0.33$ m

Vehicle speed, $v = 70 \times 1000/60 \times 60 = 19.44$ m/s

(a) The vehicle is moving directly toward the transmitter.

The received frequency is

$$f = f_c + f_d = 900 \times 10^6 + \frac{19.44}{0.33} = 900.0000589 \text{ MHz}$$

(b) The vehicle is moving directly away from the transmitter.

The received frequency is given by

$$f = f_c - f_d = 900 \times 10^6 - \frac{19.44}{0.33} = 899.9999411 \text{ MHz}$$

(c) The vehicle is moving perpendicular to the angle of arrival of the transmitted signal.

In this case, $\theta = 90°$, $\cos\theta = 0$, and there is no Doppler shift.

The received signal frequency is the same as the transmitted frequency of 900 MHz.

Example: A vehicle is travelling at 60km/hr towards a BS of height 30m. The MS is at 1 km from BS and the frequency of operation is 900 MHz. What is the received frequency at the MS?

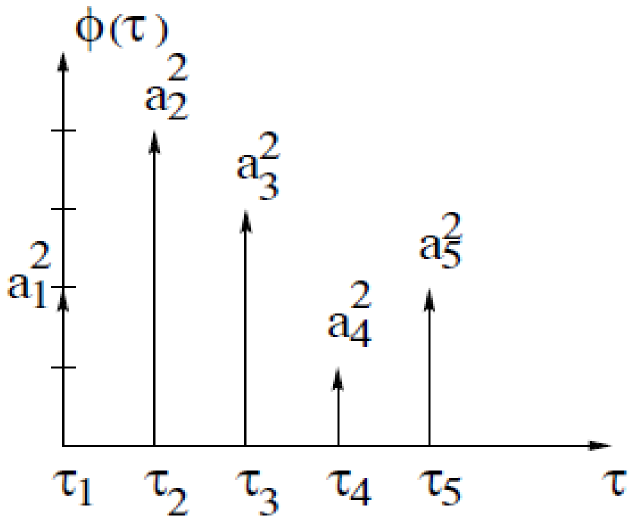
- $\lambda = (3 \times 10^8)/(900 \times 10^6) = 0.33$ m
- $v = 60 \times 1000/3600 = 16.67$ m/s
- $\theta = \tan^{-1}(30/1000) = 1.72^{\circ}$
- $f_D = 16.67 \times \cos(1.72^{\circ})/0.33 = 50.49$ Hz
- $f = f_c + f_D = 900 \times 10^6 + 50.49$ Hz.

Example: If the geometric distance between the top of the tower and mobile is 25km and the tower height is 90 meter, find the received frequency at the mobile for the following cases assuming that the operative frequency is 920MHz and velocity of the mobile is 20km/hr.

A) mobile is moving towards the direction of arrival of the wave
B) moving away from the direction of arrival of the wave
C)at an angle of 45 degrees to the direction of arrival of the wave

4. Parameters of mobile multipath channels

- Multipath channel parameters are used to compare different multipath channels and to develop some general design guidelines for wireless systems.
- Power delay profiles are:
  i. used to derive many multipath channel parameters
  ii. represented as plots of relative received power (a_k^2) as a function of excess delay (τ) with respect to a fixed time delay reference.



Multipath Power Delay Profile

- Instantaneous multipath power delay profile is given by

$$|r(t_0)|^2 = \sum_{k=0}^{N-1} a_k^2(t_0)$$

- The total receiving power is related to the sum of the powers in the individual multipath components.
- **Power delay profiles are found by averaging instantaneous power delay profile measurements over a local area in order to determine an average small-scale power delay profile.**
- Assuming that the received power from the multipath components forms a random process where each component has a random amplitude and phase at any time *t*, the average small-scale received power is:

$$E_{a,\theta}[P_{WB}] = E_{a,\theta}\left[\sum_{i=0}^{N-1} |a_i \exp(j\theta_i)|^2\right] = \sum_{i=0}^{N-1} \overline{a_i^2}$$

The average small-scale received power is simply the sum of the average powers received in each multipath component

Parameters of mobile multipath channels

1. The time dispersion parameters:

- These mobile multipath channel parameters can be determined from a power delay profile. The time dispersion mobile multipath channels are:

  i. *mean excess delay* ($\bar{\tau}$)
  ii. *rms delay spread* (σ_τ)
  iii. *excess delay spread (X dB)*

- The time dispersive properties of wide band multipath channels are most commonly quantified by their mean excess delay ($\bar{\tau}$) and rms delay spread (σ_τ)

i. Mean excess delay:

- The **mean excess delay** ($\bar{\tau}$) is the first moment of the power delay profile and is defined as

$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}$$

Where

$$P(\tau_k) = \frac{a_k^2}{\sum_i a_i^2}$$

- The central moment is the moment of probability distribution of a random variable about its mean.

ii. RMS Delay Spread:

- The rms delay spread (σ_τ) is the square root of the second central moment (variance) of the power delay profile and is defined to be

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\overline{\tau})^2}$$

Where

$$\overline{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)}$$

- The *rms delay* spread parameter is used to characterize the multipath channel in time domain

- The mean excess delay and rms delay spread are measured relative to the first detectable signal arriving at the receiver at τ_0=0.
- $\bar{\tau}$ and $\overline{\tau^2}$ do not rely on the absolute power level, but only the relative amplitudes of the multipath components.
- Typical values of rms delay spread are on the order of
    i. microseconds in outdoor mobile radio channel
    ii. nanoseconds in indoor radio channels

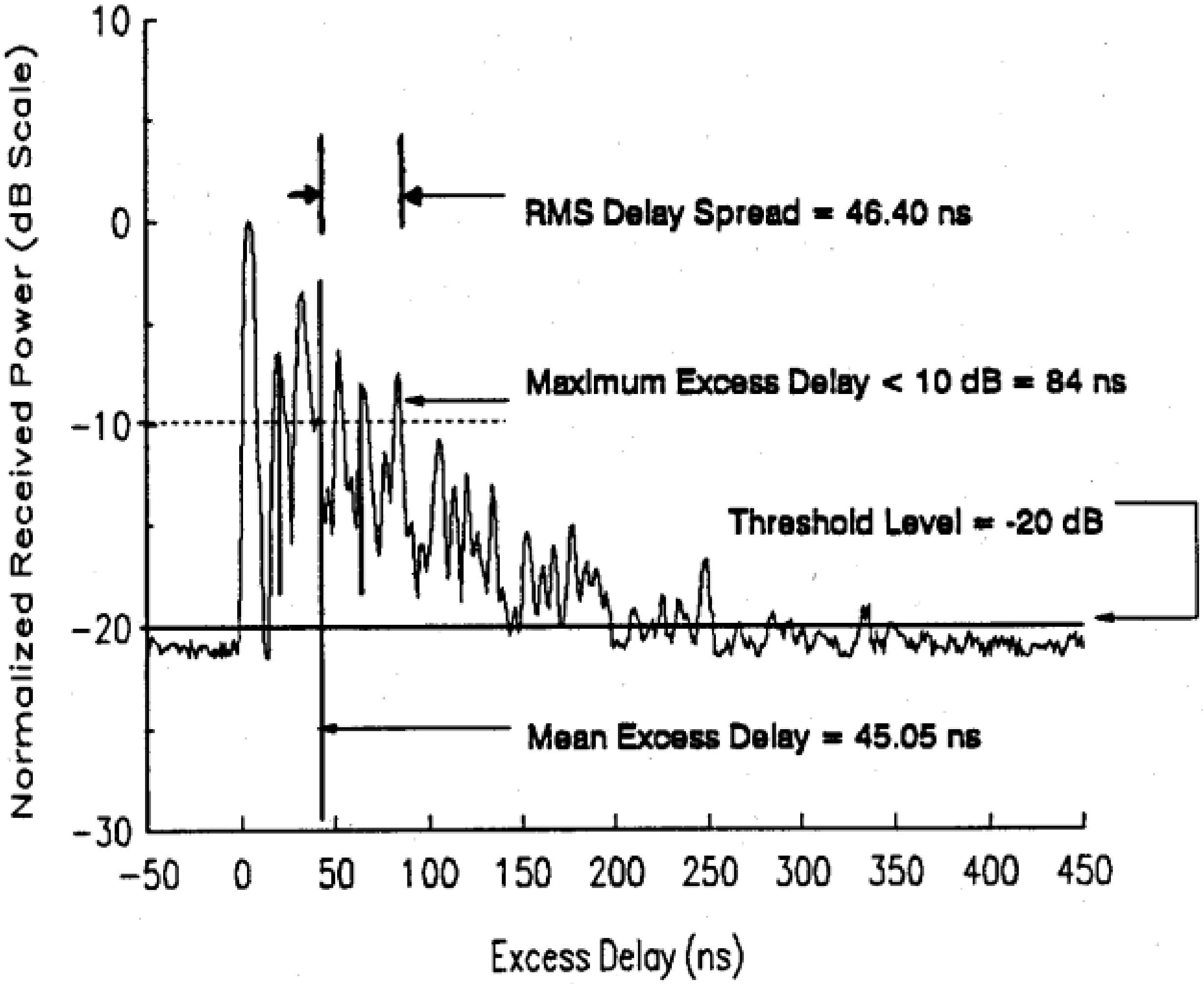
iii. Maximum Excess Delay

- The maximum excess delay (X dB) of the power delay profile is defined to be the time delay during which multipath energy falls to X dB below the maximum.

- If τ_0 is the first arriving signal and τ_X is the maximum delay at which a multipath component is with X dB of the strongest multipath signal (which does not necessarily arrive at τ_0), then the maximum excess delay is defined as

$$\tau_{\max}(X\,\mathrm{dB}) = \tau_X - \tau_0$$

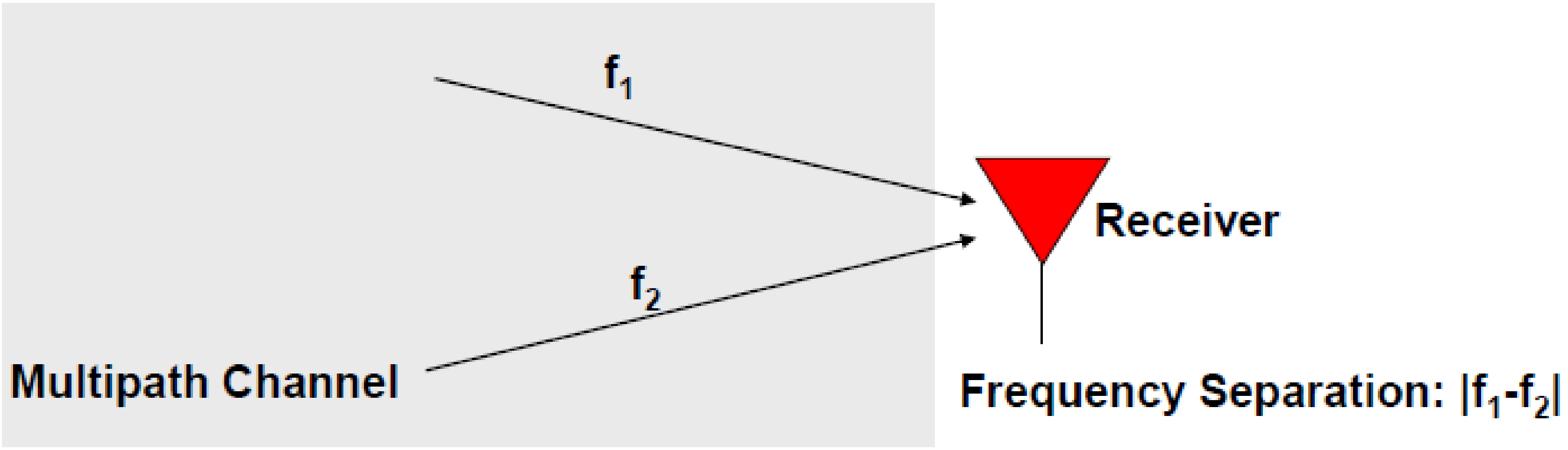
Example of an indoor power delay profile; rms delay spread, mean excess delay, maximum excess delay (10dB), and the threshold level are shown



ii. Coherence Bandwidth

- The *rms delay* spread parameter is used to characterize the multipath channel in time domain, *coherence bandwidth* is used to characterize the multipath channel in frequency domain.
- The rms delay spread and coherence bandwidth are inversely proportional to one another, although their exact relationship is a function of the exact multipath structure.
- **Coherent bandwidth, *Bc*, is a statistic measure of the range of frequencies over which the channel can be considered to be "flat".**
- **In other words, coherence bandwidth is the range of frequencies over which two frequency components have strong correlation between amplitudes.**
- **Flat channel** means a channel which passes all frequency components with approximately equal gain and linear phase.

- Two sinusoids with frequency separation greater than *Bc* are affected quite differently by the channel.



- If the coherent bandwidth is defined as the bandwidth over which the frequency correlation function is above 0.9, then the coherent bandwidth is approximately

$$B_c \approx \frac{1}{50\sigma_\tau}$$

where σ_τ is the rms delay spread

- If the definition is relaxed so that the frequency correlation function is above 0.5, then the coherence bandwidth is approximately

$$B_c \approx \frac{1}{5\sigma_\tau}$$

Important

- This is called 50% coherence bandwidth.

Example:

For a multipath channel, rms delay spread σ_τ is given as 1.37ms. Then what is the 50% coherence bandwidth?

Solution: 146kHz.

- This means that, for a good transmission from a transmitter to a receiver, the range of transmission frequency (channel bandwidth) should not exceed 146kHz, so that all frequencies in this band experience the same channel characteristics (gain and linear phase).

iii. Doppler Spread and Coherence time

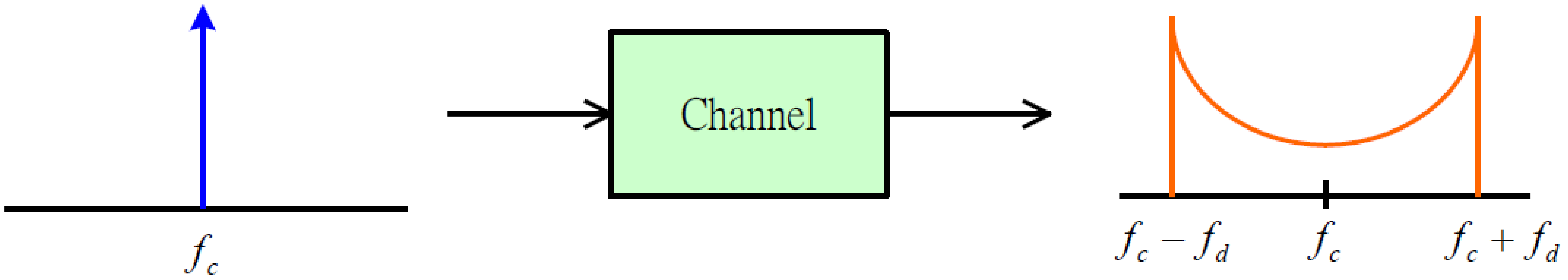
- RMS delay spread and coherence bandwidth parameters describe the *time dispersive nature* of the channel in a local area.

- However, they do not offer information about the time varying nature of the channel caused by relative motion of transmitter and receiver.

- Doppler Spread ***BD*** and Coherence time ***Tc*** are parameters which describe the time varying nature of the channel in a small-scale region.

- Time varying nature of channel caused either by relative motion between BS and mobile or by motions of objects in channel are categorized by ***BD*** and ***Tc***

- Doppler Spread ***BD*** is a measure of spectral broadening caused by motion.
- When a pure sinusoidal tone of ***fc*** is transmitted, the received signal spectrum, called the Doppler spectrum, will have components in the range ***fc-fd*** and ***fc+fd***, where ***fd*** is the Doppler shift.
- The amount of spectral broadening depends on Doppler shift ***fd***. And ***fd*** is a function of the relative velocity of the mobile, and the angle of arrival of the scattered waves



- Coherence time ***Tc*** is the time domain dual of Doppler spread and is used to characterize the time varying nature of the frequency dispersiveness of the channel in the time domain.

$$T_C \approx \frac{1}{f_m}$$

fm : maximum Doppler shift given by *fm= v/λ* = $f_{d\max}$

v : speed of the mobile

λ: speed of the light

- **If the coherent time is defined as the time over which the time correlation function is above 0.5**, then

$$T_C \approx \frac{9}{16\pi f_m}$$

Ex. 2. Calculate coherence time if it is defined as the time over which the time correlation function is above 0.5 and Doppler spread if carrier frequency 1900MHz and velocity of mobile receiver is 50meters/sec.

Solution:

$$T_C \approx \frac{9}{16\pi f_m} = \frac{9\lambda}{16\pi v} = \frac{9c}{16\pi v f_c} = \frac{9 \times 3 \times 10^8}{16 \times 3.14 \times 50 \times 1900 \times 10^6}$$

$$T_C = 565 \mu s$$

The Doppler spread is $B_D = f_m = \frac{v f_c}{c} = \frac{50 \times 1900 \times 10^6}{3 \times 10^8} = 316.66 \text{ Hz}$

5. Types of Small-Scale Fading

- **Type of fading depends on the nature of the transmitted signal (such as bandwidth, symbol period etc.) with respect to the characteristics of the channel (such as rms delay spread, Doppler spread etc.)**

A. **Multipath delay spread:** leads to *time dispersion* and *frequency selective fading.*

B. **Doppler spread:** leads to *frequency dispersion* and *time selective fading*.

- Multipath delay spread and Doppler spread are independent of one another.

Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading

1. BW of signal < BW of channel
2. Delay spread < Symbol period

Frequency Selective Fading

1. BW of signal > BW of channel
2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

Fast Fading

1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than baseband signal variations

Slow Fading

1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

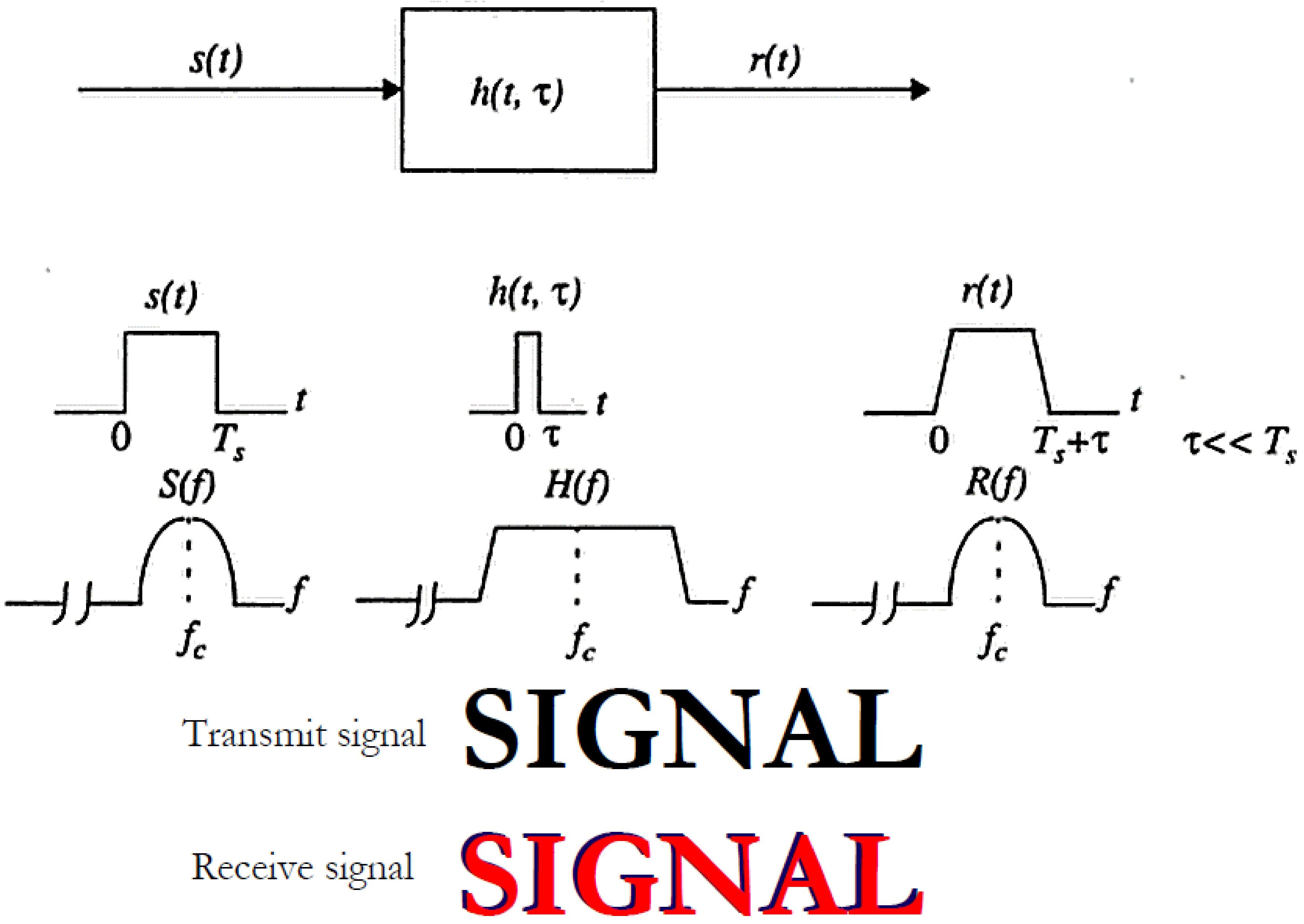
6. Fading effects due to multipath delay spread

- *Time dispersion due to m*ultipath delay spread leads to flat fading or frequency selective fading.

1. Flat Fading:

- If the channel has a constant gain and linear phase response over a bandwidth which is greater than the bandwidth of the transmitted signal, the received signal will undergo flat fading.
- The multipath structure of the channel is such that the spectral characteristics of the transmitted signal are preserved at the receiver.
- However, the received signal strength changes with time due to fluctuations in the gain of the channel caused by multipath.

Flat fading channel characteristic:



- Flat fading channel is also called ***amplitude varying channel***.
- Also called ***narrow band channel***: bandwidth of the applied signal is *narrow* as compared to the channel bandwidth.
- A signal undergoes flat fading if

$$B_S << B_C$$

and

$$T_S >> \sigma_\tau$$

Common rule of thumb

$$T_s \geq 10\,\sigma_\tau$$

T_S : reciprocal bandwidth (symbol period)

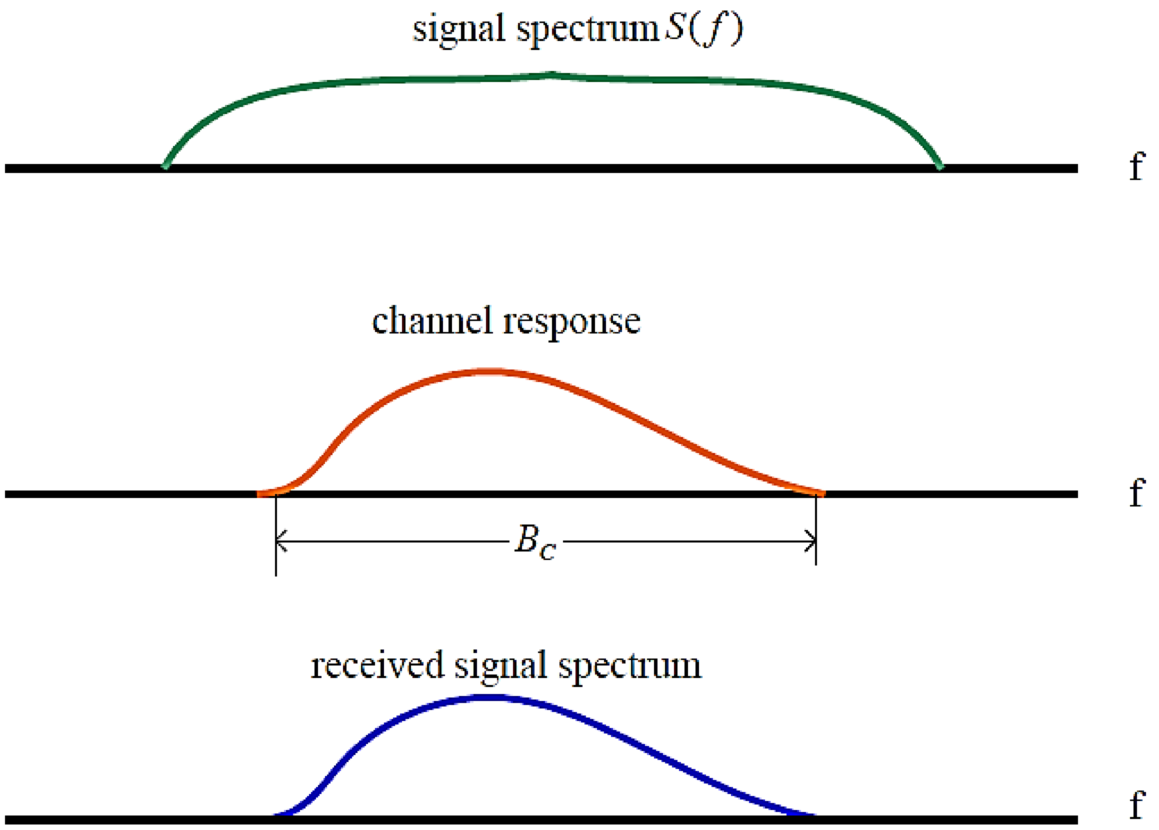
B_S : bandwidth of the transmitted signal

B_C : coherent bandwidth

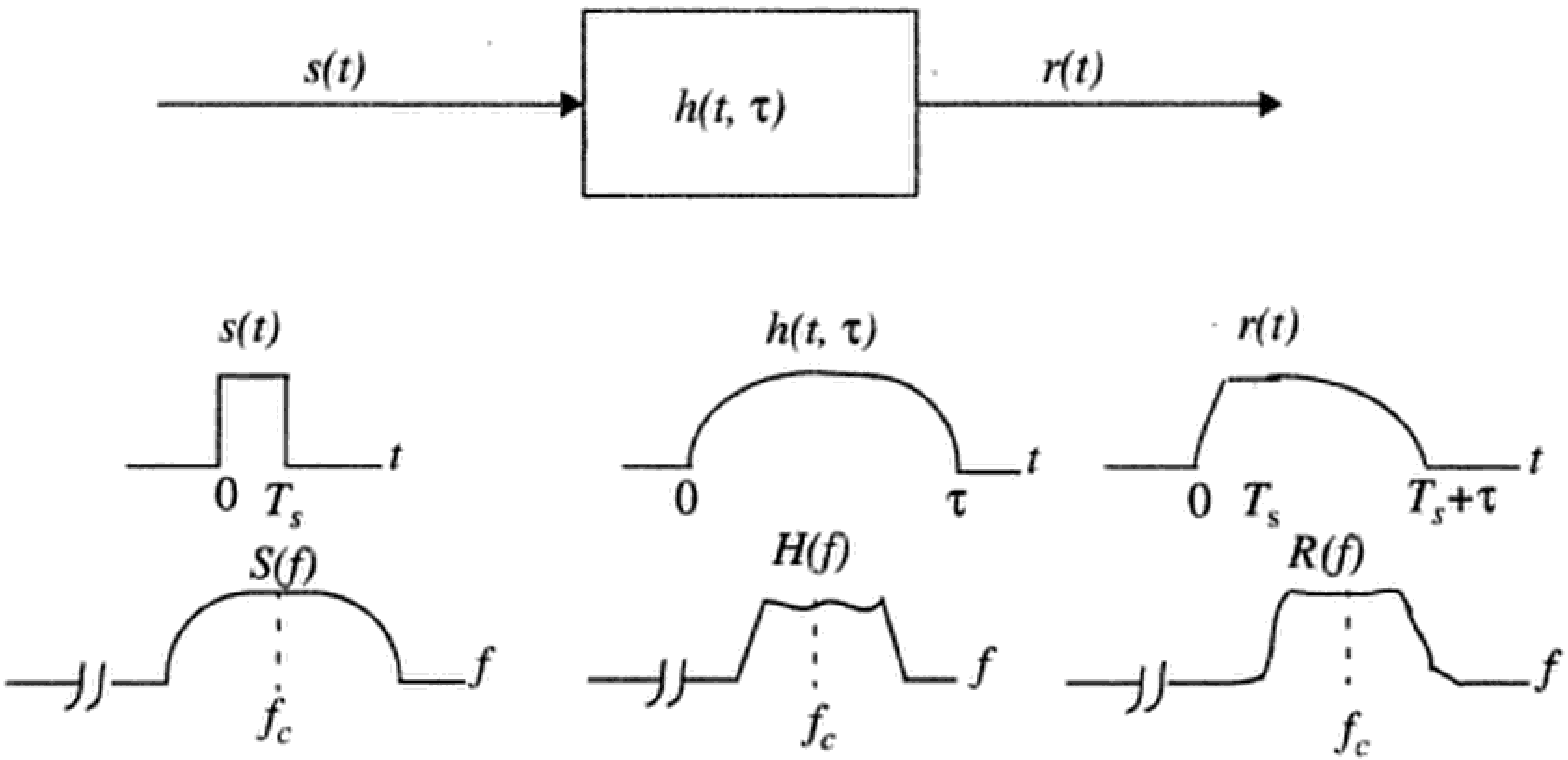
σ_τ : rms delay spread

2.Frequency-Selective Fading

- If the channel possesses a constant-gain and linear phase response over a bandwidth that is smaller than the bandwidth of transmitted signal, then the channel creates **frequency-selective fading** on the received signal.
- The received signal includes *multiple versions* of the transmitted signal which are attenuated (faded) and delayed in time, and hence received signal is distorted.



Frequency selective fading channel characteristic:



- Frequency selective fading channels are also known as ***wideband channels*** since the bandwidth of the signal is wider than the bandwidth of the channel impulse response

- As time varies, the channel varies in gain and phase across the spectrum of transmitted signal resulting in time varying distortion in the received signal.
- Frequency-selective fading is due to time dispersion of the transmitted symbols within the channel.
  - – Waveform is distorted by inter-symbol interference (ISI)
  - – Equalization is required
- For frequency-selective fading

$$B_S > B_C$$

$$T_S > \sigma_\tau$$

Common rule of thumb

$$T_s < 10\ \sigma_\tau$$

- Frequency-selective fading channels are much more **difficult to model** than flat fading channels.

B. Fading Effects Due to Doppler Spread

- Depending upon how rapidly the transmitted baseband signal changes as compared to the rate of change of the channel, a channel may be classified either as a **fast fading or slow fading channel**.

1. Fast Fading

- In a fast fading channel, the channel impulse response changes rapidly within the symbol duration. i.e. the coherent time (Tc) of the channel is smaller than the symbol period (Ts) of the transmitted signal.
- This causes frequency dispersion (also called time selective fading) due to Doppler spreading, which leads to signal distortion.
- A signal undergoes fast fading if

$$T_S > T_C$$

and

$$B_S < B_D$$

2. Slow Fading

- In a slow fading channel, the channel impulse response changes at a rate much slower than the transmitted baseband signal. i.e. The Doppler spread of the channel is much less then the bandwidth of the baseband signal.
- A signal undergoes slow fading if

$$T_S << T_C$$

and

$$B_S >> B_D$$